Chapter 1

Fermi Problems

1.1   Hailing a Ride

On May 1, 2018 a headline in *The Boston Globe* read "There were nearly 100,000 Uber and Lyft rides per day in Boston last year". The article began:

> It's not your imagination: There are an awful lot of Uber and Lyft cars in Boston traffic.
>
> The ride-hailing companies provided nearly 35 million trips in Boston in 2017, or an average of about 96,000 every day, according to data released by Massachusetts officials Tuesday.
>
> Boston accounted for more than half of the 65 million rides Uber and Lyft provided across all of Massachusetts last year, according to data the Department of Public Utilities collected from the ride-hailing companies under a new state law regulating the industry. Every minute in the city there were, on average, 67 Uber and Lyft rides underway. [R1]

There are several numbers here: 100,000, 35 million, 96,000, 65 million, and 67. Do these numbers make sense?

In particular, should you believe Uber and Lyft accounted for "35 million trips in Boston in 2017"?

Note the question. "Should you believe?" not "Do you believe?". To decide whether or not to believe we will think about what that number says in a context where you can see whether (or not) it makes common sense. Imagining a year's worth of trips is hard. Trips per day or per hour or per minute might be easier.

Start by checking the arithmetic in the second paragraph. If we work with 350 days in a year it's easy to divide the 35 million rides among the days. Cancelling the 35 and counting zeroes gives 100,000 rides per day. That's exactly what's in the headline, and is for all common sense purposes the same as the 96,000 daily rides in the quotation.

We could have used a calculator to discover that

$$\frac{35,000,000}{365} = 95890.4109589.$$

But why waste the time? Since all we know to start with is that the numerator is "nearly 35,000,000" all but the first two digits rounded to 96 make no sense at all.

Should we believe 100,000 rides per day? Boston has about 600,000 people. If 50,000 of them commuted to work and back with Uber or Lyft that would come to 100,000 rides per day.

You can argue about some of the assumptions - not all the rides are to and from work, 50,000 is probably not a very a good estimate for the number of workers - but the report of 35 million rides per year makes sense: it's in the right ballpark. 10,000 rides per day would clearly be too small while 1,000,000 rides per day would be too large.

Another way to look at 100,000 rides per day is to think about the number of rides per hour, and then per minute. There are about 25 hours in a day, so about 4,000 rides per hour. There are 60 minutes in an hour, and $40/6$ is about 7, so there are about 70 rides per minute. That's believable if you imagine people all over Boston looking for rides.

We've checked that the 35 million rides per year is a reasonable number. Should we believe 67 rides underway at any particular time? It feels too small. We've estimated 70 rides per minute. If each of ride lasted just one minute there would be 70 rides underway at any time. So we suspect the 67 can't be right.

The last paragraph of the article provides some data to confirm our suspicion.

> The average ride-hailing trip in Massachusetts lasts about 15 minutes and travels about 17 miles an hour, according to the state data. That's heavily influenced by the clustering of trips in Boston, where the average ride-hailing trip travels at 16 miles an hour.

A 15 minute ride at 16 miles per hour is a 4 mile ride. That's probably about right for a trip in the city, or to or from a suburb.

Since the average ride lasts 15 minutes and 70 rides start each minute there must be about $15 \times 70 \approx 1,000$ rides underway (on average) at any time. That's not in the same ballpark as the 67. Should we believe it?

Googling "how many Uber drivers in Boston" finds this quote from 2015 at `www.americaninno.com/boston/uber-driver-data-boston-has-10000-uber-drivers/`:

> There are nearly 10,000 Uber drivers in Boston, according to data the ridesharing company released Thursday morning. Assuming that figure is accurate, that means Boston Uber drivers currently outnumber licensed city taxis by a ratio of almost five-to-one. [R2]

You can't believe everything you read on the internet, but this site says the 10,000 driver figure comes from Uber. It's probably reliable. That was several years before the 2017 report. Updating and adding Lyft drivers suggests there might have been 20,000 drivers then. If 5 percent of them had passengers at any moment that would account for a believable 1,000 rides on the road.

Where might the 67 have come from? A clue is how close it is to our estimate that about 70 rides start each minute. In fact it's just what you get when you carefully convert 96,000 rides per day into

$$\frac{96{,}000 \text{ rides}}{\text{day}} \times \frac{1 \text{ day}}{24 \text{hours}} \times \frac{1 \text{ hour}}{60 \text{minutes}} = \frac{66.666\ldots \text{ rides}}{\text{minute}}.$$

So the author meant to say

> Every minute there were, on average, 67 Uber and Lyft rides *starting*.

rather than

> Every minute there were, on average, 67 Uber and Lyft rides *underway*.

If we had discovered this error when the article first appeared we could have written the author asking for a correction in the on line story.

What have we learned? First, that almost all the numbers in this article are consistent with one another: they fit together. Only the 67 rides on the road is wrong. Second, though 35,000,000 Boston rides per year might at first seems unbelievably large, it is in fact reasonable. Finally, thinking with the numbers rather than skipping over them led us to discover and correct a mistake and helped us focus on the ideas that the article is meant to convey.

R1 A. Vaccaro, There were nearly 100,000 Uber and Lyft rides per day in Boston last year, *The Boston Globe*, May 1, 2018, `www.bostonglobe.com/business/2018/05/01/there-were-nearly-uber-and-lyft-rides-day-boston-streets-last-year/yzOWJ9PdVg8KKQMQSKSF2K/story.html` (last visited January 9, 2019).

R2 N. Boroyan, Here's How Many Uber Drivers There Are in Boston, BostINNO, January 22, 2015, `www.americaninno.com/boston/uber-driver-data-boston-has-10000-uber-drivers/` (last visited January 12, 2019).