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THE LÉVY-LINDEBERG CENTRAL LIMIT THEOREM IN ORLICZ SPACES L_Φ

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ABSTRACT. An $L_\Phi(T, \mathfrak{F}, m)$-valued random element X, where $\Phi(t^{1/2})$ is equivalent to a concave function, satisfies the Lévy-Lindeberg central limit theorem if and only if it is centered and pre-Gaussian; that is, if and only if $EX(t) = 0$ m-a.e. and $\{EX^2(t)\}^{1/2} \in L_\Phi$.

1. Introduction and preliminaries. It is well known that an L_p-valued random element X, for $1 \leqslant p \leqslant 2$, satisfies the Lévy-Lindeberg central limit theorem (CLT) if and only if X is pre-Gaussian (i.e. $EX(t) = 0$ m-a.e. and $\{EX^2(t)\}^{1/2} \in L_p$). This fact is a consequence of L_p being of cotype 2—see e.g. [**1**, §3.8]. Recently Giné [**9**] proved the above theorem for L_p-valued random elements ($0 < p < 1$). Since L_p is a special type of Orlicz space (where $\Phi(t) = t^p$) it is therefore natural to ask for which class of Orlicz spaces the above theorem is true. In [**10**] Gorgadze and Tarieladze proved that when $\Phi(t^{1/2})$ is equivalent to a convex function $\tilde{\Phi}$ (i.e. $\Phi(c_1 t^{1/2}) \leqslant \tilde{\Phi}(t) \leqslant \Phi(c_2 t^{1/2})$ for some $c_1, c_2 \in (0, \infty)$ and all $t > 0$), then L_Φ is of type 2, and, therefore, the condition $\{EX^2(t)\}^{1/2} \in L_\Phi$ is not sufficient to guarantee that the L_Φ-valued random element X satisfies the CLT. In [**10**] it was also proved that for a convex function Φ, such that $\Phi(t^{1/2})$ is equivalent to a concave function $\tilde{\Phi}$, L_Φ is of cotype 2. Therefore [**8** and **9**], we can expect, and it was conjectured by Giné, that any L_Φ-valued random element, where $\Phi(t^{1/2})$ is equivalent to a concave function $\tilde{\Phi}$, satisfies the CLT if and only if it is pre-Gaussian. It is not difficult to realize that under the last assumption about Φ, all L_p spaces, $0 < p \leqslant 2$, are a special case of the L_Φ spaces which are under consideration.

The proof of the Lévy-Lindeberg CLT in L_p, $0 < p < 1$, in [**9**] is based on some theorems whose validity is unknown for general L_Φ spaces and also depends on the p-homogeneity of the seminorm. The author's proof is based on the correspondence between measures on L_Φ spaces and measurable stochastic processes with sample paths in L_Φ spaces, and makes no assumption on the p-homogeneity of the seminorm.

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Let $(T, \mathfrak{F}, m)$ be an arbitrary σ-finite measure space with σ-algebra $\mathfrak{F}$ and a separable measure m. Let S be the space of equivalence classes in m of all real-valued $\mathfrak{F}$ measurable functions.

By Φ let us denote a continuous, nonnegative, nondecreasing function defined for $u \geqslant 0$ such that $\Phi(u) = 0$ if and only if $u = 0$. We assume additionally that $\Phi(u)$ satisfies the so-called Δ_2 condition, i.e., there is a positive constant k such that for any u, $\Phi(2u) \leqslant k\Phi(u)$. For $x \in S$, let us put

$$R_\Phi(x) = \int_T \Phi(|x(t)|) m(dt) \cdot$$

and let L_Φ be the set of all $x \in S$ such that $R_\Phi(ax) < \infty$ for a positive constant a. The set L_Φ is a linear space under the usual addition and scalar multiplication. Moreover it becomes a complete, separable metric space [**14**] under the (usually nonhomogeneous) seminorm $\|\cdot\|_\Phi$:

$$\|x\|_\Phi = \inf\{c : c > 0, R_\Phi(c^{-1}x) \leqslant c\}.$$

The space $(L_\Phi, \|\cdot\|_\Phi)$ is called an *Orlicz space*.

By the same arguments as in [**9**] we can show that the restrictions concerning the space $(T, \mathfrak{F}, m)$ do not influence the generality of the theorem.

For convenience, let us recall the necessary facts concerning probability measures on $(L_\Phi, \mathfrak{B}(L_\Phi))$, where $\mathfrak{B}(L_\Phi)$ is the Borel σ-algebra in L_Φ.

A. For each probability measure μ on $(L_\Phi, \mathfrak{B}(L_\Phi))$, a measurable stochastic process $\zeta = \{\zeta(t) : t \in T\}$ can be constructed with simple paths in L_Φ such that $\tilde{\zeta}(x) = x$ μ-a.e. ($\tilde{\zeta}$ is the random element generated by the stochastic process ζ), the induced measure μ_ζ is equal to μ, and, conversely, each jointly measurable stochastic process $\zeta(t, \omega)$ defined on T, with almost all its sample paths in L_Φ, induces an $L_\Phi(T, \mathfrak{F}, m)$-valued random element [**4**].

B. Let ζ_i be some measurable stochastic processes with sample paths in L_Φ, $i = 1, 2$, and let μ_i denote the measures induced by ζ_i on $\mathfrak{B}(L_\Phi)$, $i = 1, 2$. Then $\mu_1 = \mu_2$ if and only if there is a $T_0 \in \mathfrak{F}$, $m(T_0) = 0$, such that the corresponding finite-dimensional distributions of ζ_1 and ζ_2 based on points $T \setminus T_0$ are equal [**3**].

C. An L_Φ-valued r.e. $\tilde{\zeta}$ (or p.m. μ on $(L_\Phi, \mathfrak{B}(L_\Phi))$ is *Gaussian* if for any pair of independent copies of $\tilde{\zeta}$, X_1 and X_2, the random elements $X_1 + X_2$ and $X_1 - X_2$ are independent; this definition is equivalent to: the process ζ with sample paths in L_Φ is Gaussian if and only if there exists a measurable subset T_0, $m(T_0) = 0$, such that for all finite sets $\{t_1, \ldots, t_k\} \subset T \setminus T_0$ the random vector $\langle \zeta(t_1), \ldots, \zeta(t_k) \rangle$ is Gaussian [**4**].

D. Let $\zeta = \{\zeta(t) : t \in T\}$ be a measurable Gaussian process and let $\theta(t) = E\zeta(t)$, $K(s, t) = E(\zeta(s) - \theta(s)) \cdot (\zeta(t) - \theta(t))$. Then for almost every ω, $\zeta(\cdot, \omega) \in L_\Phi$ if and only if $\theta \in L_\Phi$ and (for the diagonal of K) $K^{1/2} \in L_\Phi$. If almost all sample paths of the process ζ belong to the space L_Φ then the measure μ_ζ induced by ζ on $(L_\Phi, \mathfrak{B}(L_\Phi))$ is Gaussian [**5**].

E. An L_Φ-valued r.e. X is pre-Gaussian if and only if $\{EX^2(t)\}^{1/2} \in L_\Phi$ [**12**].

F. Let Z_n, $n \in N$, be L_Φ-valued symmetric Gaussian r.e.'s such that $\mathfrak{L}(Z_n) \underset{w}{\rightarrow} \mathfrak{L}(Z)$. Then Z is also Gaussian and symmetric [**9**].

G. Let M be a complete separable metric space and let $Y_{n,\varepsilon}$, Z_n, be M-valued r.e.'s ($\varepsilon > 0$, $n \in N$) such that: (a) $\mathfrak{L}(Y_{n,\varepsilon}) \underset{w}{\rightarrow} \mathfrak{L}(Y_\varepsilon)$ for some r.e.'s Y_ε; and (b) for each $\varepsilon > 0$ there exists $n(\varepsilon) < \infty$ such that for all $n > n(\varepsilon)$, $d_{\text{pr}}(Z_n, Y_{n,\varepsilon}) < \varepsilon$, where d_{pr} is any distance metrizing convergence in probability. Then w-$\lim_n \mathfrak{L}(Z_n)$ and w-$\lim_{\varepsilon \downarrow 0} \mathfrak{L}(Y_\varepsilon)$ exist and are equal (w-lim denotes limit in the weak topology) (Billingsley [**2**] and Pisier [**13**]).

H. Let $x_k, x \in L_\Phi$. Then $\|x_k - x\|_\Phi$ as $k \to \infty$ if and only if $R(|x_k - x|) \to 0$ as $k \to \infty$ [**14**].

PROPOSITION. *Let $\{X_n\}$ be a sequence of L_Φ-valued r.e.'s such that:*

(1) *$\mathfrak{L}(X_n)$ is a tight family of p.m.*

(2) *There exists a measurable subset T_0, $m(T_0) = 0$, such that for any finite set $\{t_1, \ldots, t_k\} \subset T \setminus T_0$,*

$$\mathfrak{L}(X_n(t_1), \ldots, X_n(t_k)) \underset{w}{\rightarrow} \mu_{t_1, \ldots, t_k}.$$

Then the sequence $\{\mathfrak{L}(X_n)\}$ is weakly convergent to some p.m. μ on $(L_\Phi, \mathfrak{B}(L_\Phi))$ and if $\zeta = \{\zeta(t) : t \in T\}$ is any measurable stochastic process with almost all sample paths in L_Φ such that $\mu_\zeta = \mu$, then there exists a measurable subset T_1, $m(T_1) = 0$, such that for any finite subset $\{t_1, \ldots, t_k\} \subset T \setminus T_1$, $\mathfrak{L}(\zeta(t_1), \ldots, \zeta(t_k)) = \mu_{t_1 \cdots t_k}$.

PROOF. For the proof it is sufficient to show that there exists a p.m. μ on $(L_\Phi, \mathfrak{B}(L_\Phi))$ such that each subsequence of $\{\mathfrak{L}(X_n)\}$ contains a subsequence which is weakly convergent to μ.

Let $\{\mathfrak{L}(X_m)\}$ be an arbitrary subsequence of $\{\mathfrak{L}(X_n)\}$. Since $\{\mathfrak{L}(X_n)\}$ is a tight family, $\{\mathfrak{L}(X_m)\}$ contains a subsequence $\{\mathfrak{L}(X_{m_l})\}$, such that $\mathfrak{L}(X_{m_l}) \underset{w}{\rightarrow} \gamma$, for some p.m. γ. By Skorohod's theorem there exist L_Φ-valued random elements G_{m_l}, G with laws $\mathfrak{L}(X_{m_l})$, γ, respectively, such that $G_{m_l} \to G$ a.e. This implies that for some subsequence (m_i), $G_{m_i}(t, \omega) \to G(t, \omega)$ $(m \times P)$-a.e. From this and fact B it follows that there exists a measurable subset T_0, $m(T_0) = 0$, such that for any finite set $\{t_1, \ldots, t_k\} \subset T \setminus T_0$,

$$\langle G_{m_i}(t_1), \ldots, G_{m_i}(t_k) \rangle \to \langle G(t_1), \ldots, G(t_k) \rangle \quad \text{in } P;$$

therefore $\mathfrak{L}(G_{m_i}(t_1), \ldots, G_{m_i}(t_k)) \to \mathfrak{L}(G(t_1), \ldots, G(t_k))$. Since

$$\mathfrak{L}(G_{m_i}(t_1), \ldots, G_{m_i}(t_k)) = \mathfrak{L}(X_{m_i}(t_1), \ldots, X_{m_i}(t_k)),$$

then, by assumption (2) of the Proposition,

$$\mathfrak{L}(G(t_1), \ldots, G(t_k)) = \mu_{t_1, \ldots, t_k},$$

and from fact B it follows that there exists a p.m. μ on $(L_\Phi, \mathfrak{B}(L_\Phi))$ such that each subsequence of $\{\mathfrak{L}(X_n)\}$ contains a subsequence which converges weakly to μ. Let ζ be as in the assumption of the Proposition. Then by B there exists a measurable subset T_1, $T_0 \subset T_1$, $m(T_1) = 0$, such that for any finite set $\{t_1, \ldots, t_k\} \subset T \setminus T_1$,

$$\mathfrak{L}(\zeta(t_1), \ldots, \zeta(t_k)) = \mathfrak{L}(G(t_1), \ldots, G(t_k)) = \mu_{t_1, \ldots, t_k}.$$

2. The Central Limit Theorem. We assume throughout that $\Phi(t^{1/2})$ is equivalent to a concave function $\tilde{\Phi}$ (i.e. $\Phi(c_1 t^{1/2}) \leqslant \tilde{\Phi}(t) < \Phi(c_2 t^{1/2})$ for some $c_1, c_2 \in (0, \infty)$ and all $t > 0$), and Jensen's inequality implies, for any random variable ζ,

$$E\tilde{\Phi}(|\zeta|) \leqslant \tilde{\Phi}(E|\zeta|). \tag{1}$$

THEOREM. (a) *Let X be a centered $L_\Phi = L_\Phi(T, \mathfrak{F}, m)$-valued r.e., $\Phi(t^{1/2})$ is equivalent to $\tilde{\Phi}$-concave, and let X_i, $i \in N$, be independent copies of X. If*

$$\{EX^2(t)\}^{1/2} \in L_\Phi \tag{2}$$

then

$$\mathfrak{L}\left(\sum_{i=1}^{n} \frac{X_i}{n^{1/2}}\right) \underset{w}{\rightarrow} \gamma, \tag{3}$$

where γ is the symmetric Gaussian p.m. determined by the covariance of X, i.e. such that

$$\int_{L_\Phi} x(s)x(t)\, d\gamma(x) = EX(s)X(t) \tag{4}$$

for all s and t outside a set of m-measure zero.

(b) *Conversely, if X is an L_Φ-valued r.e. such that $\{\mathfrak{L}(\sum_{i=1}^n X_i/n^{1/2})\}_{n=1}^\infty$ is weakly convergent, then X is centered and* (2) *holds* (*hence also* (3) *and* (4)).

PROOF. For the proof of (a), let $T_r \nearrow T$, $r \in N$, with $T_r \in \mathfrak{F}$, $m(T_r) < \infty$. Let $\sigma(t) = K^{1/2}(t, t)$. For each T_r and for each $c > 0$ let us define

$$Y_c^r(t, \omega) = \begin{cases} X(t, \omega) & \text{if } t \in T_r \text{ and } \sigma(t) < c, \\ 0 & \text{otherwise,} \end{cases}$$

$$Z_c^r(t, \omega) = X(t, \omega) - Y_c^r(t, \omega),$$

and, similarly, $Z_{i,c}^r$, $Y_{i,c}^r$ for the r.e.'s X_i. Clearly Y_c^r, Z_c^r are still mean zero r.e.'s. Since

$$\{E|Y_c^r(t, \omega)|^2\}^{1/2} \begin{cases} \leqslant c & \text{if } t \in T_r, \\ = 0 & \text{otherwise,} \end{cases}$$

this ensures that Y_c^r satisfies CLT in $L_2(T, \mathfrak{F}, m)$ and therefore (a fortiori) in $L_\Phi(T, \mathfrak{F}, m)$. Since

$$\begin{aligned} ER_\Phi\left(c_1 c_2^{-1} n^{-1/2} \sum_{i=1}^n Z_{i,c}^r\right) &\leqslant ER_{\tilde{\Phi}}\left(c_2^{-2} n^{-1}\left(\sum_{i=1}^n Z_{i,c}^r\right)^2\right) \\ &\leqslant R_{\tilde{\Phi}}\left(c_2^{-2} n^{-1} E\left(\sum_{i=1}^n Z_{i,c}^r\right)^2\right) = R_{\tilde{\Phi}}\left(c_2^{-2} E\left(Z_c^r(t)\right)^2\right) \\ &\leqslant R_\Phi\left(\left\{E\left(Z_c^r(t)\right)^2\right\}^{1/2}\right) = \int_{T/T_r} \Phi(\sigma(t)) m(dt) + \int_{T_r} \Phi\left(\sigma(t) 1_{\{t:\, \sigma(t) > c\}}\right) m(dt) \\ &\rightarrow 0 \end{aligned}$$

uniformly in n as $r \wedge c = \min(r, c) \rightarrow \infty$, then there exists a subsequence $r' \wedge c'$, such that

$$R_\Phi\left(n^{-1/2} \sum_{i=1}^n Z_{i,c'}^{r'}\right) \rightarrow 0$$

almost everywhere uniformly in n as $r' \wedge c' \to \infty$. By fact H,

$$\left\| n^{-1/2} \sum_{i=1}^{n} Z_{i,c'}^{r'} \right\|_{\Phi} \to 0 \tag{5}$$

almost everywhere uniformly in n as $r' \wedge c' \to \infty$. Since Y_c^r satisfies CLT in $L_\Phi(T, \mathfrak{F}, m)$, then facts F, G and (5) imply

$$\mathfrak{L}\left(n^{-1/2} \sum_{i=1}^{n} X_i \right) \underset{w}{\to} \gamma,$$

where γ is a symmetric Gaussian p.m. on $(L_\Phi, \mathfrak{B}(L_\Phi))$. There exists a measurable subset T_0, $m(T_0) = 0$, such that $EX^2(t) < \infty$ for $t \subset T \setminus T_0$, and the finite-dimensional CLT implies that for any finite subset $\{t_1, \dots, t_k\} \subset T \setminus T_0$,

$$\mathfrak{L}\left(\sum_{i=1}^{n} \frac{X_i(t_1)}{n^{1/2}}, \dots, \sum_{i=1}^{n} \frac{X_i(t_k)}{n^{1/2}} \right) \underset{w}{\to} \gamma_{t_1, \dots, t_k},$$

k-dimensional Gaussian measure with the covariance function $\{EX(t_i)X(t_j) : i, j = 1, \dots, k\}$. Then by the Proposition it follows that

$$\int_{L_\Phi} x(s)x(t)\, d\gamma(x) = EX(s)X(t) \tag{6}$$

for all s and t outside a set of m-measure zero.

PROOF OF (b). Let X' be an independent copy of X, and let $\tilde{X} = X - X'$. Then

$$\gamma_n = \mathfrak{L}\left(\sum_{i=1}^{n} \frac{\tilde{X}_1}{n^{1/2}} \right) \underset{w}{\to} \gamma.$$

There exist Gaussian r.e.'s G_n, G with laws γ_n, γ, $n \in N$, such that $G_n \to G$ a.e. This implies there exists a subsequence (n') such that, for almost every t, $G_{n'}(t, \omega) \to G(t, \omega)$ in P, therefore in law. By fact B there exists a measurable subset T_0, $m(T_0) = 0$, such that for any finite subset $\{t_1, \dots, t_k\} \subset T \setminus T_0$,

$$\mathfrak{L}(G_{n'}(t_1), \dots, G_{n'}(t_k)) = \mathfrak{L}\left(\sum_{i=1}^{n'} \frac{\tilde{X}_i(t_i)}{(n')^{1/2}}, \dots, \sum_{i=1}^{n'} \frac{\tilde{X}_i(t_k)}{(n')^{1/2}} \right)$$

and

$$\mathfrak{L}(G_{n'}(t_1), \dots, G_{n'}(t_k)) \underset{w}{\to} \mathfrak{L}(G(t_1), \dots, G(t_k)).$$

By the finite-dimensional CLT it follows that for any $\{t_1, \dots, t_k\} \subset T \setminus T_0$, $(G(t_1), \dots, G(t_k))$ is a symmetric Gaussian vector with the covariance function $\{E\tilde{X}_i(t)\tilde{X}(t_j) : i, j = 1, \dots, k\}$. Then facts C and D imply that γ is a centered Gaussian measure on $(L_\Phi, \mathfrak{B}(L_\Phi))$ such that

$$\int_{L_\Phi} x(s)x(t)\, \gamma(dx) = E\tilde{X}(s)\tilde{X}(t)$$

for all s and t outside a set of m-measure zero and $\{E\tilde{X}^2(t)\}^{1/2} \in L_\Phi$. Therefore for some $a > 0$,

$$ER_\phi\left(ac_1c_2^{-1}|X(t) - EX(t)|\right) \leqslant ER_{\tilde{\phi}}\left(a^2c_2^{-2}|X(t) - EX(t)|^2\right)$$
$$\leqslant R_{\tilde{\phi}}\left(a^2c_2^{-2}E\left(X(t) - EX(t)\right)^2\right) \leqslant R_\phi\left(a\left\{E\left(X(t) - EX(t)\right)^2\right\}^{1/2}\right) < \infty,$$

and the process $X(t) - EX(t)$ has almost all its sample paths in L_Φ so the function (class of functions) $EX(t)$ is in L_ϕ. Since $\{E(X(t) - EX(t))^2\}^{1/2} \in L_\phi$, by the first part of the theorem $X - EX$ also satisfies the CLT, but this implies the functions $n^{1/2}EX(t)$ are L_ϕ bounded. Hence, $EX(t) = 0$ m-a.e. and $\{EX^2(t)\}^{1/2} \in L_\phi$, hence (b) is proved.

Let us denote by $D_\phi = D_{L_\phi}[0, 1]$ the space of all functions defined on the unit interval into L_ϕ which are right-continuous and have left-hand limits endowed with the Skorohod topology induced by the metric d_0 [**2**, **6**].

Let $\{\xi_n\}_{n=1}^\infty$ be a sequence of i.i.d. L_ϕ-valued random elements, $\Phi(t^{1/2})$ equivalent to $\tilde{\Phi}$-concave, defined on a common probability space (Ω, Σ, P). Define $S_0 = 0$, $S_n = \xi_1 + \cdots + \xi_n$ and $X_n(t) = S_{[nt]}/n^{1/2}$. Obviously, X_n is a sequence of D_ϕ-valued random elements. By applying our theorem to Corollary 1 of [**6**] we immediately get the necessary and sufficient conditions for the Invariance Principle for L_ϕ-valued random elements ($\Phi(t^{1/2})$ equivalent to $\tilde{\Phi}$-concave).

COROLLARY 1. *The sequence of D_ϕ-valued random elements $X_n(t) = S_{[nt]}/n^{1/2}$ converges in distribution to the Wiener process W_ξ if and only if $\{E\xi_1^2(t)\}^{1/2} \in L_\phi$.*

Let S_n and X_n be defined as before on (Ω, Σ, P). By ν_n let us denote a sequence of integer-valued random variables defined on (Ω, Σ, P), and by a_n, a sequence of positive reals tending to infinity. Define

$$Y_n = \left(1/\sqrt{\nu_n}\right)S_{[nt]}, \qquad Z_n = \left(1/\sqrt{a_n}\right)S_{[nt]}.$$

By Theorems 2 and 3 from [**6**] and our Theorem we get the following corollary concerning the random change of time.

COROLLARY 2. *Assume $\{E\xi^2(t)\}^{1/2} \in L_\phi$. If*:

(i) *ν_n/a_n converges in probability to a positive constant θ, then $Y_n \underset{w}{\to} W_\xi$ and $Z_n \underset{w}{\to} \theta^{1/2}W_\xi$.*

(ii) *ν_n/a_n converges in probability to a positive random variable θ then $Y_n \underset{w}{\to} W_\xi$ and $Z_n \underset{w}{\to} \theta_0^{1/2}W_\xi$, where θ_0 is independent of W_ξ and has the same distribution as θ.*

(iii) $(1/\sqrt{\nu_n})S_{\nu_n} \underset{w}{\to} \xi$ and $(1/\sqrt{a_n})S_{\nu_n} \underset{w}{\to} \theta_0^{1/2}\xi$.

REFERENCES

1. A. Araujo and E. Giné, *The central limit theorem for real and Banach valued random variables*, Wiley, New York, 1980.

2. P. Billingsley, *Convergence of probability measures*, Wiley, New York, 1968.

3. T. Byczkowski, *Gaussian measures on L_p spaces*, $0 \leqslant p \leqslant \infty$, Studia Math. **59** (1977), 249–261.

4. ______, *Some results concerning Gaussian measures on metric linear spaces*, Lecture Notes in Math., vol. 656, Springer-Verlag, 1978, pp. 1–16.

5. ______, *Norm convergent expansion for L_Φ-valued Gaussian random elements*, Studia Math. **64** (1979), 87–95.

6. T. Byczkowski and T. Inglot, *The invariance principle for vector valued random variables with applications to functional random limit theorems*, preprint, 1980.

7. N. Dunford and J. T. Schwartz, *Linear operators*, Vol. I, Interscience, New York, 1958.

8. E. Giné and J. Zinn, *Central limit theorems and weak laws for large numbers in certain Banach spaces*, preprint, 1981.

9. E. Giné, *The Levy-Lindeberg central limit theorem in L_p*, $0 < p < 1$, Proc. Amer. Math. Soc. **88** (1983), 147–153.

10. T. G. Gorgadze and V. I. Tarieladze, *On geometry of Orlicz spaces*, Lecture Notes in Math., vol. 828, Springer-Verlag, 1980, pp. 47–52.

11. A. Kufner et al., *Function spaces*, Noordhoff, Leyden, 1977.

12. A. T. Lawniczak, *Gaussian measures on Orlicz spaces and abstract Wiener Spaces*, preprint, 1981.

13. G. Pisier, *Le théorème limite central et la loi du logarithme itére dans les espaces de Banach*, Sém. Maurey-Schwartz (1975–1976), Exp. III et IV.

14. S. Rolewicz, *Metric linear spaces*, PWN, Warsaw, 1972.

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