Hans Grauert: Mathematiker Pur

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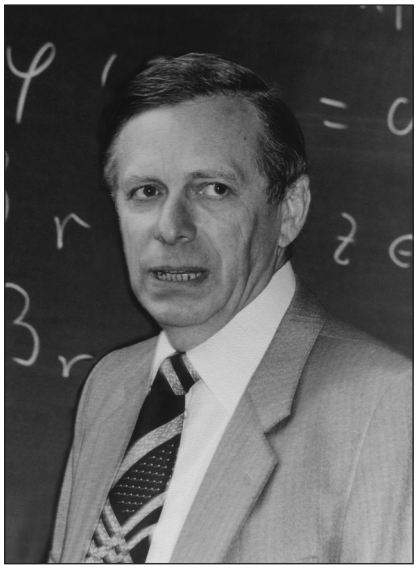

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Hans Grauert in Utrecht (1980).

On April 25, 2008, on the occasion of the Gauß–Vorlesung in Bonn, Hans Grauert was presented the *Ehrenmitgliedschaft* of the Deutsche Mathematiker-Vereinigung (DMV, German Mathematical Society). Given the opportunity to describe even superficially the contributions of this most distinguished colleague, I was pleased when asked to write this article for the *Mitteilungen* of the DMV. Only a few months after that article appeared,[1] on September 15, 2008, Grauert was awarded the prestigious Cantor-Medaille of the DMV at its annual meeting, which took place in Erlangen. The inaugural Cantor-Medaille was awarded in 1990, on the occasion of the 100th anniversary of the founding of the DMV, to Karl Stein who, like Grauert, devoted most of his scientific life to the subject of several complex variables. The Medaille has been awarded on a regular basis since that time, being presented to J. Moser (1992), E. Heinz (1994), J. Tits (1996), V. Strassen (1999), Y. Manin (2002), F. Hirzebruch (2004), and H. Föllmer (2006).

It is *presumptuous* for a mere mortal to even attempt to write a laudation for Hans Grauert. The easy part is to construct a representative list of the various important stations of his life, and of his accomplishments and honors, and to put them in a timeline. I have indeed integrated a rough outline of these data in this article, but to me this is just a small part of a story that I feel is very important.

Hans Grauert was born in Haren-Ems in 1930. At his retirement festival in Göttingen he recalled how he struggled with mathematics as a schoolboy until a teacher told him it was acceptable to think abstractly, he didn't necessarily need deal with numbers. No more than fifteen years later he was introducing spaces without points, just structure!

After beginning his studies in the summer semester of 1949 in Mainz, Grauert transferred to Münster, starting in the winter semester of 1949–50. There he was integrated into an exciting, energized mathematical atmosphere with friends and teachers of all ages and experience. Among these was Reinhold Remmert, who would become his lifelong friend and main collaborator. The mathematics guru was Karl Stein. Heinrich Behnke was well connected to the outside mathematical world, in particular to H. Hopf and H. Cartan, and had a very good feeling for the important directions in complex analysis.

After a brief sojourn in Zürich, Grauert received his Dr. rer. nat. in Münster in 1954. Starting with his dissertation, Grauert contributed fundamental results that lie at the heart of a field of mathematics that was in an infantile state when he started and was at a refined and incredibly high level less than ten years later. Let us now think back to the time when he began his studies!

There were indeed the deep, perhaps mysterious ideas of Oka on the table. Stein understood these in his own way and was, for example, attempting to understand the role of topology in complex analysis, in particular for noncompact spaces. Hirzebruch had received his doctorate in 1950 in Münster and was on the path toward his

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[1] *The present article, reprinted with the kind permission of the Deutsche Mathematiker-Vereinigung is a mildly modified version of the article by Huckleberry that appeared in the* DMV Mitteilungen, *volume 16, number 2, 2008.*

fundamental book *Neue Topologische Methoden in der Algebraischen Geometrie*. The power of the developing Cartan-Serre theory cannot be underestimated. However, the foundations of what is now called *several complex variables* were simply not there!

The works of Grauert and Remmert, together with the input of Cartan and Serre (the postive impact of the Münster-Paris connection is well-documented), *are* these foundations. *Komplexe Räume* and *Bilder und Urbilder analytischer Garben* are two of numerous examples of their prolific joint work, which is basic for our subject.

Perhaps also because they have jointly written basic books on several complex variables, *Stein Theory* and *Coherent Analytic Sheaves*, one might tend to overlook their different viewpoints. One can see that in this beginning phase Remmert is interested in analytic sets, their continuations, their properties with respect to holomorphic and meromorphic maps.

Grauert seems to be guided by problems involving complex analytic objects on these sets. His *Oka Principle*, which in terms that certainly hide the true depth of this work, states that the category of holomorphic vector bundles on a Stein space is the same as the category of topological bundles, is a perfect example. The same is true of his solution to the *Levi Problem* where he constructs holomorphic functions with given polar data at the boundary of a domain by proving the finite-dimensionality of a certain obstruction space using a Fredholm theorem in a Fréchet context. His proof of the optimal version of *Theorem B* and his solution (with Docquier) of the Levi Problem for weakly pseudoconvex domains in Stein manifolds shows his deep understanding for approximation theorems of Runge type.

Shrinking coverings and understanding subtle properties involving restriction operators can be found at the top of the list of Grauert's important methods. The most complicated and perhaps most famous of his results where such arguments appear is his *direct image theorem*. Here one starts with a proper holomorphic map $F : X \to Y$ between complex spaces where one knows that the image $F(X)$ is an analytic subset of Y (Remmert's Theorem). Proving a theorem that is in a sense in another universe, Grauert shows that direct images of coherent sheaves on X are coherent on Y. One cannot think of working in global complex geometry without the availability of this result! To obtain some feeling for the order of magnitude of this work and for other interesting information we recommend reading Remmert's article in ([R]).

The last of the above-mentioned works appeared in 1960, but it is not at all clear to me when Grauert proved these theorems. It seems that at a certain point he understood *everything*, and it was just a matter of finding the time and energy

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H. Grauert, K. Stein, R. Remmert.

to write the papers. In any case he chose the Oka Principle as his *Habilitationsschrift* and around the time of completing his *Habilitation*, continuing the postwar tradition that opened the world to numerous outstanding young German mathematicians of that generation, Grauert left Münster for the Institute for Advanced Study, where he spent the winter semester of 1957–58. I know from direct discussions with others who were there at the time that the richness, depth and breadth of his ideas, which he presented both formally and informally, were nothing short of startling.

In 1959 Grauert became professor in Göttingen and remained in this position until becoming emeritus in 1996. Once he told me he didn't want to be away from Göttingen for more than two weeks. But in fact he did travel widely. For example, most likely due to the connection to Wilhelm Stoll, Grauert, Remmert, and Stein visited Notre Dame for extended periods. I know how important this was for that faculty and of course for me personally!

Grauert also invited distinguished foreign guests to Göttingen, among them Aldo Andreotti. In the winter semester of 1968–69 at Stanford I was introduced to Grauert's work in the lectures of Andreotti. Imagine being the only student in a course given by the most wonderful of lecturers discussing results of his friend and coworker that are even in hindsight some of the most beautiful in complex geometry. Their joint work was certainly one of the highlights: the Andreotti-Grauert theorems on finiteness and vanishing of cohomology for q-pseudoconvex manifolds, and their jewel "Algebraische Körper von automorphen Funktionen", where they show how to use pseudoconcavity to prove the finite-dimensionality of spaces of automorphic forms. However, what I remember most is Andreotti's explaining Grauert's elegant solution

J. Heinze, photographer.

DMV chairman Günter Ziegler presenting the Cantor-Medaille to Grauert's daughter Ulrike Peternell, 2008.

of the Levi problem and applications to Kodaira-type vanishing and embedding theorems.

These last mentioned results are in a sense just snippets of Grauert's remarkable paper "Über Modifikationen und exzeptionelle analytische Mengen". There, answers to fundamental questions such as "When can you blow down a variety?" are given. Concepts such as plurisubharmonicity, bundle curvature and signature of intersection forms flow together. A new, important criterion for projective embeddings is proved. After reading this work, I was sure that this is the way mathematics should be!

On Grauert's research timeline we have now reached a point around 1963. Of course the ideas kept coming! There was a phase when he was thinking about parameter spaces of complex analytic objects (deformation theory). Here his two basic *Inventiones* papers should be mentioned ("Über die Deformation isolierter Singularitäten analytischer Mengen" (1972) and "Der Satz von Kuranishi für kompakte komplexe Räume" (1974)). At the time when he was concentrating on vector bundles (see for example his paper with Mülich, "Vektorbündel vom Rang 2 über dem n-dimensionalen komplex-projektiven Raum" (1975)) , I remember a young mathematician asking him a general question about what would nowadays be the most important direction of research in mathematics. Typifying how Grauert focused: "Vector bundles on $\mathbb{P}_3$"!

On the more analytic side there is the important work with his student Ramirez in the late 1960s and then with Lieb on integral kernel representations. The Grauert-Riemenschneider vanishing theorem ("Verschwindungssätze für analytische Kohomologiegruppen auf komplexen Räumen" (1970)) can also be regarded as being at home in complex analysis.

More recently Grauert turned back to his old interests in holomorphic and meromorphic equivalence relations. I remember he and Stein discussing these topics with great animation just a few years before Stein's passing. His most recent work in that direction appeared in 1987. Finally, one should not forget that *hyperbolicity* has been in the background for many years. One sees this in his work in 1965 with Reckziegel, his 1985 paper with Ulrike Peternell, née Grauert, and in his final paper that was devoted to mathematics research "Jetmetriken und hyperbolische Geometrie" (1989).

At this point I could begin to be more precise about the details of Grauert's work. However, I hope that the above is sufficient for the interested bystander. For those whose appetites might have been whetted, Grauert should have the final word: Please take a look at his collected works ([G]) with its interesting annotations written by Yum-Tong Siu and Grauert himself.

As we all know, research is an extremely important part of our academic lives, but there are other aspects that must be emphasized. Here in Germany there is the classical notion of *Akademischer Lehrer*, which encompasses everything that a professor should be. Nowadays there seem to be new interpretations being propagated. Grants, research clusters, elite universities, etc., are the buzzwords. However, we don't need new words to describe Grauert's contributions. Let me expand on this.

His work in administration of science must be commended, in particular his involvement with projects of the Deutsche Forschungsgemeinschaft (German Science Foundation) and his role on editorial boards, for example in bringing the *Mathematische Annalen* back to its historic high standards. Nevertheless, when I think of Grauert I think of him *in* the science not above it. This includes his lectures, which may seem dry and minimal, sometimes even formal, but you should listen very carefully. There are always deep ideas that should be followed! From the undergraduate student in his *Funktionentheorie* course to the researcher being advised in a private conversation, every listener should take every word seriously. The same is true of his vast written work. The reader must take the time to understand what is meant by every sentence! This holds just as well for his research monographs as it does for his textbooks. While reading a Grauert-proof of Stokes' Theorem, you should keep in mind that he has seriously thought about it! These textbooks range from the basic

analysis sequence written with Fischer and Lieb to the new version of Grauert-Fritzsche where even new ideas in complex analysis are introduced.

Speaking of Grauert's minimality, I can't resist an anecdote. Whenever he lectures he carries the *Konzept* of the lecture with him on a three by five card. He will most often start his lecture writing *Let X be a complex space* . . . on the board and meticulously checking his Konzept to make sure he got it right. Given that he and Remmert originally *defined* the notion of a complex space, this is a beautiful sight! Gossip has it that when giving a two-semester course on several complex variables he never changed the little piece of paper, but for the second semester did in fact turn it over!!

I have no idea how many students (*Diplom*, *Staatsexam*, *Promotion*) have done their work with Grauert. In any case it is a large number! We who are working in areas near their works see the strong positive influence of the master teacher, and I know that Grauert is proud of them all.

A researcher of the highest quality, a teacher at all levels with relevant fundamental new ideas always in the background, an author with a style where every word has a meaning, an important participant in and leader of academic societies, a cultured intellectual in the sense of Humboldt, and a very kind gentleman, Grauert personifies the true notion of *Akademischer Lehrer*.

References

[G] H. Grauert, *Selected Papers I. and II.*, Springer-Verlag, 1994.

[R] R. Remmert, *Mathematical Intelligencer* **17**(2) (1995), 4–11.