

Correction to “From the Fundamental Theorem of Algebra to Astrophysics...”

In our paper “From the fundamental theorem of algebra to astrophysics: A ‘harmonious’ path”, *Notices* 55 (6), 2008, 666–674 (the paragraph preceding Corollary 1 and the last sentence of Corollary 1 on page 670), we cited a parity result on the number of lensed images versus the number of lensing masses and credited S. H. Rhie. This result was actually found earlier by Arlie O. Petters: page 1918 of his paper “Morse theory and gravitational microlensing”, *J. Math. Phys.* 33 (5), 1992, 1915–31, and page 432 of the book *Singularity Theory and Gravitational Lensing* by A. O. Petters, H. Levine, and J. Wambsganss, Birkhauser, Boston (2001). We are grateful to Petters for pointing this out and we take this opportunity to correct this error.

—Dmitry Khavinson
University of South Florida
dkhavins@cas.usf.edu

Genevra Neumann
University of Northern Iowa
neumann@math.uni.edu

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Editor’s Note: More on Cake Cutting

Theodore P. Hill has found “serious mathematical errors” in the article “Better ways to cut a cake” by Steven J. Brams, Michael A. Jones, and Christian Klamler (BJK) which appeared in the December 2006 *Notices* and supplied analysis and detailed counterexamples in a document entitled “Cutting cakes correctly”, not reproduced here, now available at <http://arXiv.org>. Hill’s counterexamples and comments, which have been checked by an expert referee, verify the following mistakes:

1. The well-known “cut-and-choose” procedure is not Pareto-optimal, as BJK claims, although it is if one assumes 3 additional hypotheses:

(a) the cake is one dimensional,
(b) all the player measures are absolutely continuous with respect to each other, and

(c) all player measures are absolutely continuous with respect to Lebesgue measure.

2. There are problems in the definition of two of BJK’s procedures: their Surplus Procedure (SP) and Equitability Procedure (EP). These may not be well-defined. Also according to the definition of Pareto-optimal given by BJK, SP and EP are not Pareto-optimal.

3. BJK’s definition of “strategy-vulnerable” and “strategy-proof” are flawed. The player measures might all be identical. Because BJK did not take this into account when making their definition, every procedure happens to be “strategy-proof”. Indeed, BJK’s proof of Theorem 1 assumes implicitly that the player measures are different (their argument only works if the 50–50 points a and b are different).

(The above comments are a paraphrase of the report the referee gave the *Notices*.)

Brams and his colleagues have examined Hill’s analysis and, with the referee’s permission, the latter’s report on that analysis, and offered the following corrections:

1. While the model we develop on p. 1315 (after the introduction on p. 1314) does explicitly assume a one-dimensional cake, we did not explicitly assume that player measures are absolutely continuous with respect to each other and to Lebesgue measure, which are necessary for some of our theoretical results to hold.

2. The latter two assumptions are also necessary for our two cake-cutting procedures, the Surplus Procedure (SP) and the Equitability Procedure (EP), to be well-defined.

3. Our definition of strategy-proofness implicitly assumes that player measures are not identical. In the exceptional case in which they are, SP and EP may be strategy-vulnerable. Our omission of this exceptional case can be rectified by substituting “do at least as well and

sometimes better” for “do assuredly better” in our definition of strategy-vulnerability on p. 1316.

The *Notices* is grateful to Hill for his work, to our anonymous referee for carefully checking that work and the original article, and to Brams *et al* for supplying their updates.

And a final editorial comment: precision and exposition are not necessarily incompatible, although when they do conflict the *Notices* tends to favor the second, or rather encourage authors to do so. And the *Notices* also encourages correspondence from readers setting the mathematical record straight.

—Andy Magid
Editor