A Centennial Celebration of Two Great Scholars:

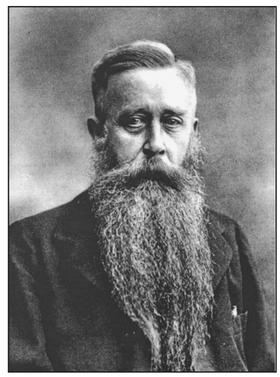
Heiberg's Translation of the Lost Palimpsest of Archimedes—1907
Heath's Publication on Euclid's Elements—1908

Shirley B. Gray

The 1998 auction of the "lost" palimpsest of Archimedes, followed by collaborative work centered at the Walters Art Museum, the palimpsest's newest caretaker, remind *Notices* readers of the herculean contributions of two great classical scholars. Working one century ago, Johan Ludvig Heiberg and Sir Thomas Little Heath were busily engaged in virtually "running the table" of great mathematics bequeathed from antiquity. Only World War I and a depleted supply of manuscripts forced them to take a break. In 2008 we as mathematicians should honor their watershed efforts to make the cornerstones of our discipline available to even mathematically challenged readers.

Heiberg

In 1906 the Carlsberg Foundation awarded 300 kroner to Johan Ludvig Heiberg (1854–1928), a classical philologist at the University of Copenhagen, to journey to Constantinople (present day Istanbul) to investigate a palimpsest that previously had been in the library of the Metochion, i.e., the "daughter" or "sharing" house of the Church of the Holy Sepulcher in Jerusalem. Heiberg's proposal to the foundation was to make photographs of a "damaged and readable, but very hard to decipher" Byzantine Greek work on vellum. Over its thousand year history, the palimpsest had journeyed between these two Greek Orthodox church libraries. Indeed, the overscript is a euchologion most likely written near the turn of the thirteenth century by a priest of the church. Today the palimpsest contains four illuminated plates, presumably of Matthew, Mark, Luke, and John.



Johan Ludvig Heiberg.
Photo courtesy of The Danish Royal Society.

Heiberg was eminently qualified for support from a foundation. His stature as a scholar in the international community was such that the University of Oxford had awarded him an honorary doctorate of literature in 1904. His background in languages and his publications were impressive. His first language was Danish but he frequently published in German. He had publications in Latin as well as Arabic. But his true passion was classical Greek. In his first position as a schoolmaster and principal, Heiberg insisted that his students learn Greek and Greek mathematics—in Greek. Indeed, he made his debut into public life at the age of thirty when he led a movement to require the study of Greek in all Danish schools. He even founded a Greek social society that continued until after his death.

He had entered Copenhagen University as a young man of fifteen and immediately took up mathematics, Greek, and classic philology. He completed his doctoral work at the inordinately young age of twenty-five with a thorough, very meticulous chronology of the works of Archimedes. He was especially attracted to *The Sand-Reckoner* problem dealing with the number of grains of sand needed to fill the universe. His capstone graduation trip was naturally to Italy to investigate more deeply the background of his dissertation, *Quœstiones Archinedeœ*.

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The author thanks Nigel Wilson (Lincoln College, Oxford), Adrian Ponce (Jet Propulsion Laboratory, Pasadena, CA), Akif Tezcan (University of California, San Diego) and Erik Petersen (Det Kongelige Bibliotek).

Photograph courtesy of the author.

The Metochion faces the main courtyard and entrance to the Church of the Holy Sepulcher in Jerusalem.

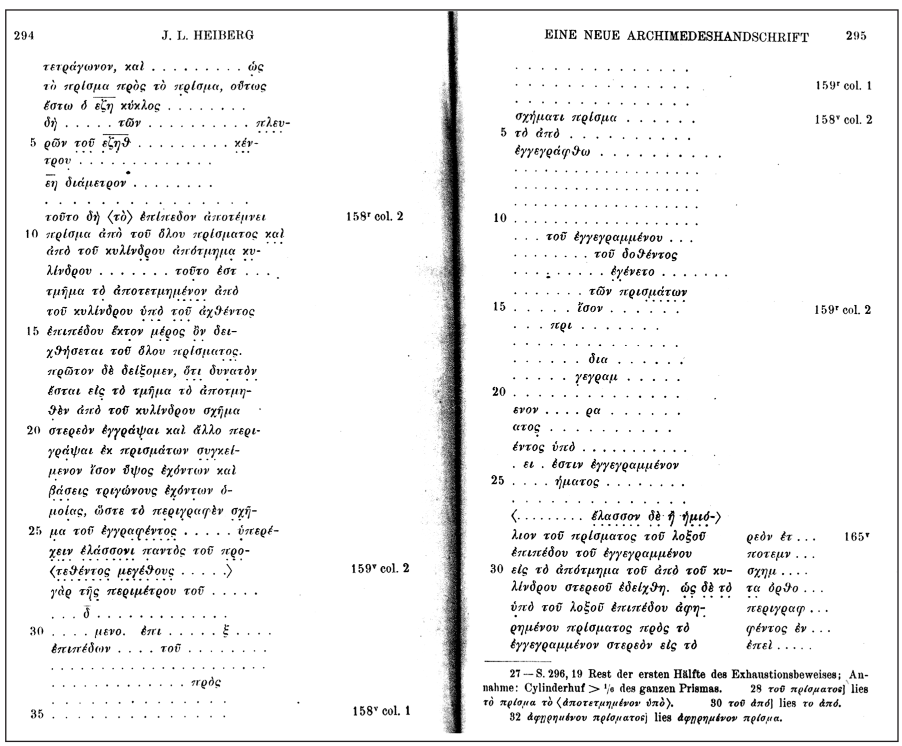
Upon returning to Denmark he initiated a career of translating classical manuscripts. While not abandoning Archimedes, his first academic recognition came through a parallel interest in various editions of Euclid. Dijksterhuis, the Dutch scholar, would later write that "If a reader was to become acquainted with the work of a writer of Archimedes' level, an understanding of the *Elements* of Euclid was—and still is—a prerequisite, though by no means a sufficient preparation." Heiberg's publications underscore his multiple talents. In addition to mathematics, he practiced his textual skill by putting the 1269 Latin works of Flemish Dominican Willem van Moerbeke back into their original Greek. Heiberg's work was recognized by his appointment (1896) to be professor at the University of Copenhagen.

For over two decades and in several publications, Heiberg refined the work on classical Archimedean manuscripts initiated in his doctoral dissertation. When he wrote his proposal to the Carlsberg Foundation, Heiberg was aware of two major unexplored references. First, there was mention of a palimpsest in a travel book, *Reise in der Orient* (Leipzig 1846), written by the German biblical scholar Constantine Tischendorf and translated into English by W. E. Shuckard (London 1847). A half century later, A. I. Papadopoulos-Kerameus cataloged 890 Greek manuscripts at the church in Constantinople. This catalog, which was published in 1899, came to the attention of H. Schöne in Germany, who then alerted Heiberg in Denmark of the possibility of finding additional Greek mathematics palimpsests.

Following the award of the Carlsberg grant and subsequent travel, Heiberg arrived in Constantinople in 1906 for the first of two trips. He painstakingly transcribed the original Greek underscript using only a magnifying glass and various light sources. Judging by the proposal, one feels that Heiberg probably left Copenhagen thinking he could successfully photograph the palimpsest and then later read the prints at his leisure upon returning home to Denmark. Being a meticulous scholar, Heiberg returned to Constantinople in 1908 to refine and check his earlier work. Then satisfied with his notes, he was joined by a German colleague, H. G. Zeuthen, who assisted him in verifying a transcription of the text. This watershed discovery was announced in *Eine neue Archimedes-Handschrift*, Hermes XLII (1907), pp. 235–297.

This author attended the 1998 auction at Christie's in New York and has examined the palimpsest both prior to and after its Christie's sale; also, the one stolen page now in the Cambridge University Library, Add. 1879.23. In addition, the 65 photos taken by Heiberg in 1906 were examined both when on loan to the Walters Art Museum in Baltimore and in their home in Copenhagen's Royal Library. The condition of the palimpsest is poor. Mildew over the past century has marked some pages with

Photograph below courtesy of Det Kongelige Bibliotek.


294 J. L. HEIBERG

τετράγωνον, καὶ . . . . ὡς
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ἔστω ὁ εζη κύκλος . . . .
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εη διάμετρον . . . .
. . . .
τοῦτο δὴ ⟨τὸ⟩ ἐπίπεδον ἀποτέμνει — 158ʳ col. 2
πρίσμα ἀπὸ τοῦ ὅλου πρίσματος καὶ
ἀπὸ τοῦ κυλίνδρου ἀπότμημα κυ-
λίνδρου . . . . τοῦτο ἐστ . . . .
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ἐπιπέδου ἕκτον μέρος ὂν δει-
χθήσεται τοῦ ὅλου πρίσματος.
πρῶτον δὲ δείξομεν, ὅτι δυνατὸν
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στερεὸν ἐγγράψαι καὶ ἄλλο περι-
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μενον ἴσον ὕψος ἐχόντων καὶ
βάσεις τριγώνους ἐχόντων ὁ-
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⟨τεθέντος μεγέθους . . . . .⟩ — 159ᵛ col. 2
γὰρ τῆς περιμέτρου τοῦ . . . . .
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. . . .
. . . . πρὸς
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EINE NEUE ARCHIMEDESHANDSCHRIFT 295

. . . .
. . . . — 159ʳ col. 1
. . . .
σχήματι πρίσμα . . . . . . — 158ᵛ col. 2
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ἐγγεγράφθω . . . .
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. . . τοῦ ἐγγεγραμμένου . . .
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. . . . ἐγένετο . . . .
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. . . . . ἴσον . . . . . . — 159ʳ col. 2
. . . πρι . . . .
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⟨. . . . . ἔλασσον δὲ ἢ ἡμιό-⟩
λιον τοῦ πρίσματος τοῦ λοξοῦ — ρεὸν ἐτ . . . — 165ᵛ
ἐπιπέδου τοῦ ἐγγεγραμμένου — ποτεμν . . .
εἰς τὸ ἀπότμημα τοῦ ἀπὸ τοῦ κυ- — σχημ . . . .
λίνδρου στερεοῦ ἐδείχθη. ὡς δὲ τὸ — τα ὀρθο . . .
ὑπὸ τοῦ λοξοῦ ἐπιπέδου ἀφη- — περιγραφ . . .
ρημένου πρίσματος πρὸς τὸ — φέντος ἐν . . .
ἐγγεγραμμένον στερεὸν εἰς τὸ — ἐπεὶ . . . . .

27 — S. 296, 19 Rest der ersten Hälfte des Exhaustionsbeweises; Annahme: Cylinderhuf > 1/6 des ganzen Prismas. 28 τοῦ πρίσματος] lies τὸ πρίσμα τὸ ⟨ἀποτετμημένον ὑπὸ⟩. 30 τοῦ ἀπὸ] lies το ἀπό. 32 ἀφηρημένου πρίσματος] lies ἀφηρημένον πρίσμα.

Heiberg's original publication (1907) draws attention to the missing portions of Propositions 14 and 15 in the *Method*. Even with the most advanced technology, it may not be possible to image these critical sections, thus leaving mathematicians to speculate on Archimedes' approach.

a purple stain. Browning around the edges suggests that in addition to being quite aged, the palimpsest may narrowly have escaped a fire. Far worse today, much of the text that Heiberg tediously translated is no longer visible. Thus the photos taken in Constantinople one century earlier have become a valuable guide for contemporary scholars. The photos, as enlargements of the original leaves, are easier to read than the Archimedean underscript visible in 2008. In translating the Greek mathematics, Heiberg sequenced the pages and identified the original book, i.e., *Equilibrium of Planes, Floating Bodies, Method of Mechanical Theorems, Spirals, On the Sphere and Cylinder, Measurement of the Circle,* and *Stomachion*. Of these seven texts, four (see later sections, each marked with an asterisk) were already known from other sources. Also, readers might note that the *The Sand-Reckoner* entered our literature from other sources and is not in the palimpsest.

In the 1450s Pope Nicholas V commissioned a new translation of Archimedes. This richly illuminated manuscript, considered to be one of the Vatican's treasures, ends with "Finis librorum Archimedis quos transcrivi iussit dominus Franciscus Burgensis sempre deo laus". Basilese, 1544. (Note the small cone, sphere, and cylinder in Archimedes' left hand.)

Photograph above courtesy of the Vatican Library.

There is an additional challenge. When the thirteenth century scribe set about to scrape off the original Archimedean manuscript to reuse the parchment for a prayer book, he simply selected the leaf he felt was in the best condition. Thus today sequential pages of the overscript in the legible prayer book represent randomly selected leaves of unorganized mathematics in the underscript. One can imagine the chaos of mixing the pages of this issue of the *Notices* translated into a foreign language and then trying to read 178 pages of mathematics overwritten by religious text. The ancient Greek mathematics in the underscript, if visible, is faded, tedious to translate, and randomly organized when compared to its original source. In brief, this might be called a scholar's nightmare. Fortunately, the very best are hard at work on the translation. Nigel Wilson at Lincoln College, Oxford, and Reviel Netz, Department of Classics, Stanford University, are the principal scholars involved in the task.

Another factor emerges. Nigel Wilson makes the point that while Euclidean geometry was a significant portion of the quadrivium and thus widely studied in the Middle Ages, Archimedes was primarily a source of interest only to the connoiseur of mathematics (Wilson, 1999). There are numerous manuscripts and editions of Euclid in European libraries, but the textual transmission of Archimedes across the centuries "hangs by a slender thread." There are not many occasions when today's translators can catch a glimpse of past experts at work on Archimedes. Among Heiberg's papers in Copenhagen is a hand-written manuscript listing where he found references to copies of Archimedes and, all too frequently, their later disappearance. Primarily his sources, bridging centuries of time, were found in the Vatican, other Italian libraries, and Basel. For example, Heiberg wrote that Leonardo da Vinci had found one copy in the library of the Bishop of Padua, and noted Renaissance men truly did seek ancient Greek manuscripts to translate mathematics. The Dutch Archimedean scholar E. J. Dijksterhuis continued Heiberg's "philological investigations" of manuscripts and printed editions. Wilson, Heiberg, and scholars before them have noted where they were unable to trace a missing copy.

As mathematicians, readers of the *Notices* will recall classic problems from each of the books. We briefly remind readers of their contents.

Equilibrium of Planes*

Archimedes, in writing on rectilinear figures, followed the century-earlier investigations of Aristotle, but constructed propositions knowing the mathematical dictates of Euclid. This treatise uses the method of exhaustion to concentrate on the center of gravity of triangles, trapezoids, and parabolic segments. We thus find a veiled suggestion of modern calculus as well as the principle of moments, also known as the law of levers. Archimedes has been called the "Father of Classical Mechanics".

Floating Bodies

The palimpsest is the only known surviving copy of *Floating Bodies* in the original Greek. In this text we find the principle of buoyancy. Familiarity with these investigations was probably a factor in Archimedes' legendary naked run through the streets of Syracuse yelling the famous "Eureka, Eureka!" Similarly, his understanding of hydrostatics and levers combined to supposedly move a floundered ship built for King Hiero. This led to "Give me a place to stand and I can move the Earth." Travelers to third world countries may still see the Archimedean water screw being used to irrigate fields, drain marshes, or empty bilge water from a boat.

Spirals*

Unlike many expressions in mathematics, the Spiral of Archimedes $r = a\theta$ seems to be properly named. He appears to be the very first of a long list of distinguished mathematicians, e.g., Descartes, Fermat, Bernoulli, and Euler, to investigate its special properties. In particular the area bounded by the Spiral is one of 28 propositions in this text. He wrote, "The space bounded by the spiral and the initial line after one complete revolution is equal to one-third of the circle described from the fixed extremity as center, with radius that part of the initial line over which the moving point advances in one revolution." [$A = \frac{1}{3}\pi(2\pi a)^2$].

On the Sphere and Cylinder*

George F. Simmons calls Archimedes' discovery of the formula for the volume of a sphere "one of the greatest mathematical achievements of all time" (Simmons, 1992). There are several levels of importance. Here Archimedes described to Eratosthenes, his friend at the Museum in Alexandria, a "most wonderful demonstration" that the sum of slices of cross sections of a sphere and a cone would counterbalance the sum of slices of a related cylinder. Then, by his choice of words in Greek, we are certain that Archimedes recognized the difference between a heuristic investigation, $\theta\epsilon\omega\rho\epsilon\iota\nu$, and the very cornerstone of mathematical reasoning—proof or $\alpha\pi o\delta\epsilon\iota\kappa\nu\upsilon\nu\alpha\iota$. Moreover, he was applying the principle of the lever balancing a center of gravity from classical mechanics. By thinking in terms of solids being dissected into slices, he was again on the brink of modern calculus. Heiberg translated the same passages into German as "……dass er sie vorher durch die Mechanik gefunden, nachher geometrisch bewiesen habe" (Heiberg, 1908).

The one leaf in the Cambridge Library, now known to be taken from *On the Sphere and the Cylinder*, is thought to have been stolen by Tischendorf, for it was sold by his executors in 1876. This leaf is embedded in air-tight plastic and its condition is strikingly better than the heavily mildewed manuscript auctioned at Christie's. Both the leaf and the manuscript are scorched at the edges and thus must have been in a fire. We owe the identification and transcription of this leaf to Nigel G. Wilson who also prepared much of the text for the Christie's catalog.

Measurement of the Circle *

The most significant contribution of this particular text is Archimedes' method for approximating a value for π. (Both he and the early Chinese mathematician Liu Hui (ca. 250 AD) used only words, not a symbol, to express the ratio of the circumference of a circle to its diameter. The symbol π did not appear until much later in the eighteenth century.) Once again, his approach was the method of exhaustion. This time he calculated a difference in areas by successively inscribing and circumscribing regular polyhedra of 6, 12, 24, 48, and 96 sides about a circle.

Stomachion

Heiberg recovered only the opening sentences though other references (Dijksterhuis, 1987, pp. 408–412) to this text have become known as the *loculus Archimedius* (Archimedes' box), a kind of game or puzzle consisting of fourteen pieces of ivory in various triangular shapes cut from a square. It is assumed that these pieces were to be reassembled into other forms, e.g., a ship, sword, or tree. After much thought, Netz concluded that Archimedes asked the modern combinatorial question, "In how many ways can you put the 14 pieces together to form a square?" Answer: 17,152. Netz headed a team of Persi Diaconis, Susan Holmes, Ronald Graham, Fan Chung, and William Cutler who systematically counted all possibilities and then validated their calculation with a computer program.

Method of Mechanical Theorems

The *Method* holds a very special niche among pure mathematicians and classicists. The book opens with a letter sent by Archimedes in Sicily to Eratosthenes in Alexandria. For decades mathematicians have savored its closing:

> I am convinced that it (the method) will prove very useful for mathematicians; in fact, I presume there will be some among the present as well as future generations who by means of the method here explained, will be enabled to find other theorems which have not yet fallen to our share.
>
> *—(Dijksterhuis, pp. 313–315, et al)*

This has to be one of the more provocative statements ever written by a great mathematician. Clearly Heiberg, Heath, and dozens of other scholars were and are intrigued. Wilbur Knorr wrote, "It is…extraordinary that fully eight decades later there is still no adequate translation of this work into English; for the version by Heath is based on Heiberg's text." Alas, a scholar cannot translate that which cannot be seen; thus, the importance of new imaging techniques for the current quest to translate the palimpsest. While presenting meticulous details about other propositions, Dijksterhuis basically skips Propositions 14 and 15 (pp. 332–335) due to gaps in the visible evidence.

In particular, the interpretation of methods in Proposition 14 and Proposition 15 is at stake. Be reminded that the shoulders of giants stood on by Archimedes surely included the following:

1. From Eudoxus (408-355 BC) he knew the concept of a "method of exhaustion".

2. From Hippocrates (460-380 BC) he knew the special case for finding the quadrature of the circle (the area of a curvilinear lune equals the area of one-half of a 45° -45° -90° triangle inscribed in a semicircle).

3. From Euclid (323-285 BC) he surely had learned that in two similar solids (a) the lateral and surface areas have the same ratio as the square of any pair of corresponding segments; (b) the

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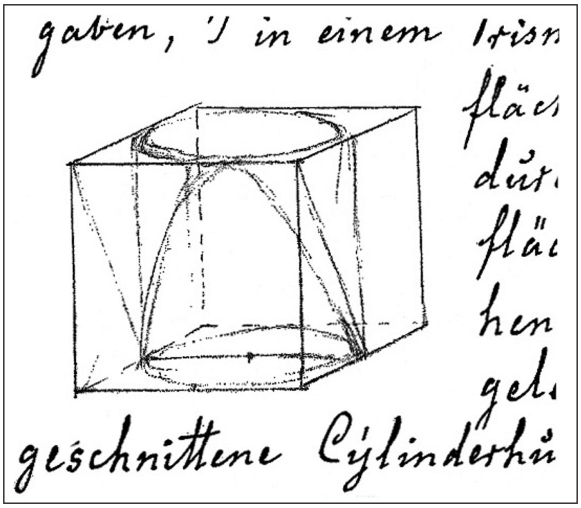
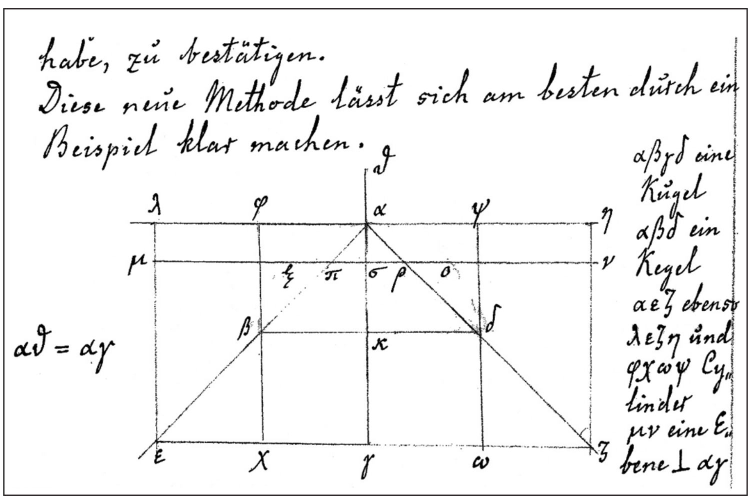


The geometrical illustrations in the original palimpsest were drawn with a finer stylus than the pen used for text. In several instances Heiberg and Heath were forced to guess at their reconstruction. This illustration of the "Cylinderhüf" or "cylinder hoof" was drawn by Heiberg (Heiberg, 1908, p. 7).



Heiberg's illustration of Archimedes' signature investigations of related volumes of cones, spheres, and cylinders (Heiberg, 1908, p. 5).

volumes have the same ratio as the cube of any pair of corresponding segments.

4. Archimedes experimentally determined that the areas of cross sections of two bodies have an intimate relationship: the length of the lever arm times the area of one cross section equals a constant times the area of a cross section of another solid.

In translating the letter from Archimedes to Eratosthenes, Heiberg used the German verb "knüpfen" or "tying together" for what Archimedes hoped to achieve with their collaborative knowledge of mathematical truths in the *Method*.

In Proposition 14 Archimedes deals with a proof introduced earlier regarding the volume of a cylinder "hoof"; thus, he deals with cross sections of a paraboloid to conclude that the volume of the "hoof" is two-thirds of that of the prism and consequently one-sixth of that of the cube circumscribed about the cylinder. His method now avoids the concept of equilibrium, but continues with visualizing a solid as the sum of its parallel intersections. To quote Dijkersterhuis, "This proof therefore does not yet satisfy the requirement of exactness."

Proposition 15 was "lost" one century ago, but appeared to Heath to be the same posited in Proposition 14, but with a different method to drawing the same conclusion. "As however the two propositions are separately stated, there is no doubt that Archimedes' proofs of them were distinct." (Well, maybe; the gaps are enormous.) Dijksterhuis asserts "the concept of volume as being the sum of the areas of parallel intersections is abandoned" and then takes a giant scholarly leap in publishing "the method of the indirect limiting process is applied." He inserts his personal proof by contradiction. Continuing on Proposition 15, he refers readers to Heath's reconstruction (p. 48) but fails to elaborate. Heath writes, "There is no doubt that Archimedes proceeded to, and completed, the rigorous geometrical proof by the method of exhaustion" (p. 51).

Did Archimedes really grasp integration as we think of it today? Several points are to be made. In London, Heath, nearing completion of his monumental work on Euclid, perused the investigations of his colleagues' research from the Middle East to write, "One of the two geometrical proofs is lost, but fragments of the other are contained in the manuscript which are sufficient to show that the method was the orthodox *method* of exhaustion in the form in which Archimedes applies it elsewhere, and to enable the proof to be *reconstructed*." A purist would remark that a reconstruction is not a verbatim or literal translation. The missing portions, the lacunae, are important.

The four premises known to Archimedes have been refined over their 2,000 year history. To the method of exhaustion have been added modern concepts of limit, infinity, and convergence. Today we speak of the center of gravity and calculate the moment of a system about an axis. Cross sections of thickness dx must be parallel to one another and perpendicular to an axis. Equations and notation are far less cumbersome than doing mathematics expressed in literal sentences. Heath informally observed, the "dx" thickness of perpendicular cross sections cancels on both sides of Archimedes' expression of equilibrium.

Heiberg summarized the evolution of Archimedes with different emphases: "Even stranger is the boldness with which the assumption operated that

(1) a plane is filled with lines and that (2) a body consists of or is filled with circles. This is, in fact, our infinity-based calculus, which Archimedes already used as a method while the meaning of infinity in Greek mathematics was usually strongly rejected as exactly not comprehensible and therefore dangerous. That is why Archimedes emphasizes that these 'Raisonnements' cannot be proven and instead only yield likely suspicions that require a strict form of exhaustion, the geometric proof" (Heiberg, 1908).

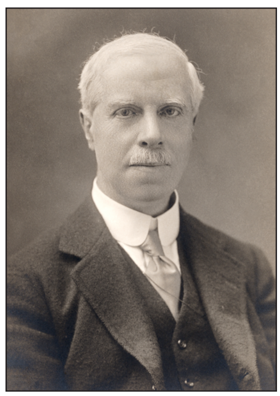
C. H. Edwards writes "While it is true that Archimedes' work ultimately gave birth to the calculus, three indispensable ingredients of the calculus are missing in his method" (Edwards, 1979). Briefly, (1) the explicit introduction of limit concepts, (2) a computational algorithm for the calculation of areas and volumes, and (3) a recognition of the inverse relationship between area and tangent problems. Any instructor teaching calculus can recognize the symbiotic relationship between Archimedes and chapters we routinely teach today. Yet there is an incompleteness to the thoughts of Archimedes that will be debated even after the palimpsest project finishes its imaging and translation.

2008 Technology

The state of the art for photography was not sufficiently developed in 1906 to enhance the pale, stone-scrubbed underscripted ink, though Heiberg translated approximately 80 important mathematical illustrations that could not be seen. Of the 177[1] leaves noted by Heiberg, Heath later reported 29 leaves that were destitute of any trace of underscript, and 9 pages that had been "hopelessly washed off; on a few more leaves, only a few words can be made out; and again some 14 leaves have old writing upon them in a different hand and with no division into columns."

Both Heiberg and Heath were captives of the technology of their generation. Imaging is in a far more advanced state today. Mathematicians and paleographers are intrigued by the wish to see the missing late twelfth or early thirteenth century illustrations and proof—not their reconstruction. We have no need to question the work of the great scholars at the turn of the last century. But today we have far more imaging options. Powerful optical spectroscopic methods can be employed along with digital enhancement techniques to reveal features in the palimpsest that hitherto have not been legible. (The fluorescence and other spectroscopic properties of pre-Columbian inks used by North American Indians are well known.) Strikingly characteristic emission spectra of different inks observed under ultraviolet excitation can be exploited, but care must be exercised in such experiments, since UV light has the potential to bleach the delicate traces of ink in the thousand-year-old manuscript. Recall the darkened libraries and art museums prescribed by conservators.

The most recent imaging technique has been to scan the palimpsest using rapidly emitted, finely focused x-ray beams that excite fluorescence in the latent inks. Thus the metal atoms in ancient inks come alive. Related spectroscopic methods have long been used by physicists and chemists, especially at the Department of Energy's Stanford Linear Accelerator Center (SLAC), to determine the precise separations of atoms in biological and other complex molecular structures. The intensity of the fluorescence in key regions can be amplified to produce clear images if any trace of the ink remains in previously hidden writings. (Mathematicians will enjoy discussion with colleagues in the physical and biological sciences concerning the relative merits of steady-state fluorescence and the newer time-resolved absorption-emission methods that employ femtosecond and picosecond x-ray pulses.)



Thomas Little Heath.
Photo courtesy of the Royal Society London.

We wish to see the text and geometrical illustrations from *The Method* that may or may not validate our fondest wish that Archimedes did, in fact, have the thoughts required for modern techniques of finding volumes by integral calculus. We want to see and to read all of the world's oldest copy of Archimedes' method.

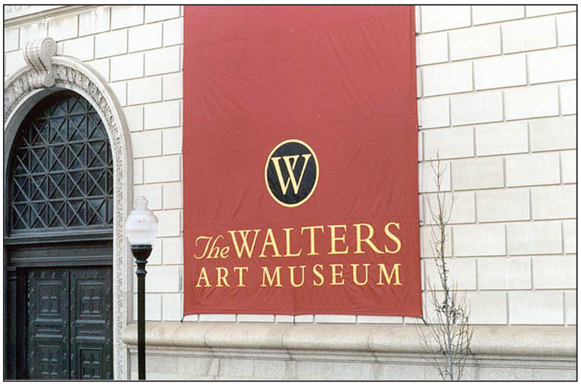
Heath

Heiberg's announcement of finding the palimpsest (1907) and his subsequent publications appearing in German must have come as a great surprise to Heath. He immediately set about to update *The Works of Archimedes* (1897) with a supplement, *The Method of Archimedes Recently Discovered by Heiberg* (1912). In English, this publication found in many university libraries has become an indispensable source for scholars of Propositions 14 and 15.

Heath was truly in his element when writing about Heiberg's translation. Like Newton, Heath was born in Lincolnshire and educated at Trinity College, Cambridge. Following his "first class" on the tripos, he went up to London to become a civil servant in the Treasury. He was quick, accurate, neat, and thorough in all written work. His honesty was never questioned. However, the

[1] *In examining the palimpsest in 1999 Nigel Wilson counted 178 leaves. In 1906 Heiberg made 65 photos, some of a single page, others of an opening.*

Photograph courtesy of the author.



The Palimpsest is now under the watchful care of the Walters Art Museum in Baltimore.

very qualities that would merge into his becoming one of the world's greatest classical scholars would also lead to his demise in the office place. Victorian to the hilt, Heath opposed women in the office, telephones, and oral briefings; moreover, he cut budgets. Heath was at ease with the stuffy, candle-saving formality of the old Treasury prior to World War I. But modernization reforms under Lloyd George resulted in his being pushed aside. Those who cut budgets are seldom popular. Possibly his pedantic stiffness was unsettling to those less dedicated in their work.

Heath was one of a small number of British civil servants who managed to use leisure time to make a major impact on scholarship. Thus, he achieved in two simultaneous careers, one as a scholar and the other as a government employee. While working from 1884 until his retirement at the age of 65 he published major treatises on Diophantus (1885), Apollonius (1896), his original Archimedes (1897), and seven other publications. His style was to use modern mathematical notation while faithfully translating the literal expositions from difficult Greek and Medieval Latin. He rendered the text to be readily understood by all contemporary mathematicians. His polished, meticulous prose with modern mathematical notation culminated in the monumental three-volume *non pareil* edition of Euclid's *Elements* (1908). Though published one century ago, his work on Euclid remains in print and is in virtually every university library. In particular, his treatment of rational and irrational magnitudes in Book X is considered a masterpiece. Heath became the indispensable gatekeeper for opening the works of ancient Greek mathematics to modern students. Readers of the *Notices* also know *A History of Greek Mathematics* (1921) as the definitive two-volume work on the subject.

Heath had a passion for trains and rock climbing. He was an unerring sight-reading pianist. His wife, Lady Ada Heath, wrote "Music served as a solvent of difficulties which arose in the writing of his books on Greek Mathematics. He would wander over to the piano and play one or more of Bach's '48'. Watching him, it was obvious at what point his difficulty was cleared up, when he would go back to his desk and continue his writing." He even published Euclid in the original Greek (1920), hoping his notes would fascinate schoolboys into doing more proofs.

Heiberg and Heath after Constantinople

Heiberg was awarded two additional honorary doctorates, one from Leipzig (1909) and one in medicine from Berlin (1910). Throughout his career he had been both a prolific author and an editor, but largely turned to Greek medicine in his final years. He also served as rector of the University of Copenhagen (1915–1916). As a shy young man, few would have predicted he would become a gregarious social leader, much loved by his students and colleagues. While his friends were strongly religious, he attempted to remove both Oriental and mystic influences in his writings. He had little interest in Roman contributions. Outside of antiquity, he admired Kierkegård. Travelers to Copenhagen today may visit his grave in Holmens and see his portrait in the restored Renaissance-style Frederiksborg Castle on a lake near Hillerød.

Like Heiberg, honors filled Heath's career. Undoubtedly his thoroughness in translating Euclid led to his being knighted (K.C.B., 1909; K.C.V.O., 1916) and named a Fellow of the Royal Society (1912). He joined Heiberg in being awarded an honorary degree from the University of Oxford (1913). His college (Trinity) at Cambridge named him an honorary fellow (1920). His professional service included being on the council of the Royal Society and serving as president of the Mathematical Association. American readers will find it unusual that his wealth at death (£18.427 9s 2d) in 1940 is published information (resworn probate, April 25, 1940, *CGPLA Eng. & Wales*).

Archimedes and the Palimpsest at the Year 2008

Archimedes can become addictive. Sherman Stein (p. ix) wryly notes that each time he teaches the history of mathematics he puts more time on Archimedes. Finally on his fifth time, seven of his twenty lectures were devoted to Archimedes, while Newton and Leibniz received hasty treatment. He also asserts Heath and Dijksterhuis are hard to read, while Heiberg is available only in German or translation. Chris Rorres has shared his lifelong passion for collecting Archimedean materials and was one of the very first to launch a mathematics website (1995). See `http://www.mcs.drexel.edu/~crorres/Archimedes/contents.html`.

Reviel Netz remarks, "I am always humbled by Heiberg" but has the reservation that when Heiberg inserted bracketed passages that are not clearly Achimedes' own, Heiberg might have been too fast in his judgment. By modern standards, we know not to second guess an ancient translator or author too quickly. Yet, a scholar's opinion has value.

Following the Christie's auction, the current owner placed the "lost" palimpsest in the hands of the Walters Art Museum in Baltimore; William Noel is heading the project of restoration, imaging, and translation. Both the seller and the buyer, exchanging US$2 million "under the hammer", strongly wish to remain anonymous. This is highly understandable—indeed, as a priest who leads pilgrimages to the Patriarchate in Jerusalem and to Istanbul remarked to this author, "The palimpsest of Archimedes is to the Greek Orthodox Church what the Elgin Marbles are to the Greek government." Both the *New York Times* and the *KATHIMERINI* of Athens report the owner has pledged not to limit access to the ancient manuscript.

Concluding Remarks

In 2008, one century after publication, Heath's edition of Euclid's *Elements* is still being printed and sold throughout the world. Only a handful of mathematics publications can make this claim. A visitor to the National Portrait Gallery in London can see Heath's photograph, along with DeMorgan and Boole, but not Hardy and Littlewood. Without Heath's translation, Heiberg would be largely unknown in a world now dominated by English. What is more, we should not forget to pay tribute to Heiberg. Were it not for his determined efforts to find grant money, to make difficult journeys across Europe, and to publish demanding scholarly work, we as mathematicians would be much the poorer. It is altogether fitting that we pay tribute to both Heiberg and Heath on the centennial of their landmark work.

References

Modern sources in many university libraries:

[1] M. W. Browne, Ancient Archimedes text turns up, and it's for sale, *New York Times*, October 27, 1998.

[2] _____ , Archimedes text sold for $2 million, *New York Times (national)*, October 30, 1998.

[3] Christie's, The Archimedes Palimpsest, Sale catalog, October, 29, 1998.

[4] E. J. Dijksterhuis, *Archimedes*, Princeton University Press, 1987, 44–45; 408-412.

[5] C. H. Edwards Jr., *The Historical Development of the Calculus*, Springer-Verlag, 1979, 68–76.

[6] S. H. Gould, The method of Archimedes, *American Mathematical Monthly* **62** (1955), 473–476.

[7] E. Spang-Hanssen, *Filologen Johan Ludvig Heiberg 1854–1928*. 2nd ed. 1969. [Contains a list of Heiberg's MS. and letters.]

[8] M. F. Headlam, Sir Thomas Little Heath, *Proceedings of the British Academy*, xxvi (1940), 424–438. [Contains a complete list of Heath's publications.]

[9] T. L. Heath, *The Thirteen Books of Euclid's Elements: Translated from the Text of Heiberg*, 2nd ed., Dover, 1956.

[10] W. R. Knorr, *The Method of Indivisibles in Ancient Geometry, in Vita Mathematica*, (R. Calinger, ed.), Mathematical Association of America, 1996, 71–74.

[11] G. Kolata, In Archimedes' puzzle, a new eureka moment, *New York Times*, December 14, 2003.

[12] D. C. MacGregor II, Mathematics and physical science in classical antiquity, in *Chapters in the History of Science*, (C. Singer, ed.) Oxford University Press, 1922.

[13] R. Netz, K. Saito, and N. Tchernetska, A new reading of Method Proposition 14: Preliminary evidence from the Archimedes Palimpsest (Part I), *Sources and Commentaries in Exact Sciences* **2**, April, 2001.

[14] L. D. Reynolds and N. G. Wilson, *Scribes and Scholars: A Guide to the Transmission of Greek and Latin Literature*, Oxford University Press, 1968.

[15] S. K. Ritter, Imaging in 2020: Seeing is believing, *Chemical and Engineering News* **77** (45), November 8, 1999, 30–35.

[16] G. F. Simmons, *Calculus Gems: Brief Lives and Memorable Mathematics*, McGraw-Hill, 1992, 242–245.

[17] S. Stein, *Archimedes: What Did He Do Besides Cry Eureka?*, Mathematical Association of America, 1999.

[18] D'Arcy W. Thompson, *Obituary notices of fellows of the Royal Society* **3** (9), January, 1941, 408–426.

[19] N. G. Wilson, Archimedes: The palimpsest and the tradition, *Byzantinische Zeitschrift* **92** (1999), 89–101.

[20] _____ , *From Byzantium to Italy: Greek Studies in the Italian Renaissance*, Duckworth, 1992.

Rare sources or sources in languages other than English:

[1] I. Boserup, ed., Københavns Universitet 147-9-1979: Klassisk filologi efter 1800 **8** (1992), 367–374.

[2] J. L. Heiberg, Eine neue Archimedeshandschrift (Nebst einer Tafel), Hermes: Zeitschrift für Philologie **42** (1907), 235–303.

[3] _____ , Mathematici graeci minores, historisk-filologiske meddelelser, *Det Kgl Danske Videnskabernes Selskab* **13**, 1926.

[4] _____ , Fra Hellas til Italien II. 1929, 390–419.

[5] _____ , Archimedes, seine Entwickelung und die Wirkung seiner Schriften, MS in Det Kongelige Bibliotek. NKS 3643 4°, 1908.

[6] T. L. Heath, *The Method of Archimedes: Recently Discovered by Heiberg: A Supplement to "The Works of Archimedes 1897"*, Cambridge University Press, 1912.

Relevant Websites:

http://www.cs.drexel.edu/~crorres/Archimedes/contents.html

http://archimedespalimpsest.org/

http://www.thewalters.org/

http://www.slac.stanford.edu/gen/com/slac_pr.html