The K–12 Math Test Conundrum

This is a story with disasters, well-intentioned efforts gone wrong, villains, and no quick fix. There are constructive small steps, however.

Tests

High-stakes K–12 tests seem to be both necessary and an abomination. High school math skills are falling, ranking on international tests is an embarrassment, colleges' math offerings are dominated by remedial courses. None of this will change without high-stakes tests to provide discipline and accountability. On the other hand, they distort teaching: they are high stakes for schools as well as students, so administrators apply pressure to teachers. Teachers pass this on to students. As a matter of survival, teachers teach to the test, so things skipped on tests are left out of courses. And if "understanding" doesn't help scores, then tricks and mechanical drill are the order of the day.

The tests themselves are antiques: one-shot tests with multiple-choice questions. Questions are mediocre, ambiguous, or even wrong. Even the answer format has consequences. Questions are often numerical, in part to avoid giving clues in answer choices: πr^2 is obviously the area of a circle, but 16.6 is not obviously the area of a circle of radius 2.3. This and calculators have shifted the focus to numerical work and led to a significant decline in abstract and symbolic thinking.

The antique-test problem, at least, has an explanation and some prospect of relief. Each edition of a traditional test is made essentially from scratch, so it is necessary to weight scores to keep outcomes consistent. Determining this weighting is the main expense in test development, and the testing industry has an enormous investment in the necessary expertise. However, this is irrelevant to modern computer-based tests. Each student gets a slightly different test, and if these vary a bit in difficulty (or the student has a bad day), the remedy is to retake it. It seems unlikely the traditional industry will make this transition, since it would mean discarding their major asset. However, a new industry is developing, and soon choices will be available to departments of education with the courage to break with their traditional partners.

Standards

Each U.S. state has a standards document for mathematics education. In principle this organizes teaching, texts, and tests and ought to ease problems with tests by providing an environment designed to prepare students for them. The dysfunctional educational situation would thus seem to be due to dysfunctional standards documents, and consequently much of the debate is focused on standards. They do leave much to be desired.[1]

Most states' standards documents have not been reviewed by an academic mathematician and are rife with inaccuracies. Many are so ambitious they cannot have been reviewed for realism by classroom teachers. It is common to describe material in "bands" of several grades. It is then consistent with the standard to promote students who have learned very little in the first year of the band, guaranteeing a lot of repetition in the last year. Sometimes material is described in topic-centered "threads" with no due date at all.

Most standards are largely concerned with process or exposure not expected to result in testable skills. The word "understand" for instance is clearly distinguished from "able to work problems with". This lets teachers promote students who "understand" material even if they can't work problems. But this ostensibly higher-level knowledge is not recognized by high-stakes tests.

Even perfect standards documents are not the full solution, since tests would still supplant them as de facto standards.[2]

What We Can Do

The state of U.S. math education is a national emergency, and academic mathematicians have a crucial role in resolving it. It is a delicate role, since there is no agreement on the nature or even the existence of the emergency. There are politicians who feel that their election to high office after failing algebra demonstrates that math is not necessary for success. The K–12 community has been shifting toward qualitative and imaginative "understanding" for decades, and a good deal of the estrangement from the academic community results from our reactionary attachment to testable skills and pedantic precision.

Some suggestions:

- If you have an opportunity to participate in development of state standards, take it but be prepared to yield on many issues. Remember, this is an incremental process, and if our viewpoints are correct, then the discipline imposed by high-stakes tests give us a long-term advantage.
- If you have an opportunity to discuss K–12 pedagogy, pass. There are exceptions to this of course, but at the moment alienation is more likely than progress. Again, the discipline of high-stakes tests should help as the issue becomes more a struggle against a common enemy and less a difference of opinion or philosophy.
- Think about tests. Tests will be driving standards and curriculum in the near future, since they provide concrete objectives and quantitative measures of success. Bad tests drive the process in bad directions, so the current test-development process is a serious weak point. Large collections of mathematically "wholesome" sample problems would be a great resource. There is a particular need for symbolic word problems.

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[1]See `http://www.edexcellence.net/foundation/publication/publication.cfm?id=338&pubsubid=1118` for a recent Fordham Foundation report.

[2]See `http://www.math.vt.edu/people/quinn/education/CourseDefinition.pdf` for an attempt to address this.

Letters to the Editor

Mathematical Jokes

I enjoyed the article of Renteln and Dundes on mathematical folk humor in the January *Notices*. However, I would like to suggest a couple corrections: one in interpretation, the other in one of the jokes.

Firstly, I do not think that the use of "Zorn's Lemma/Lemming/Lemon" in the well-known riddle needs to be interpreted as betraying antichoice sentiments or guilt. The usage is adequately explained by the fact that Zorn's Lemma is about the only "lemma" (it seems only a historical accident that it did not pass into general usage as "Zorn's Axiom" or "Zorn's Theorem") to be really well known and to have both a familar name and a succinct restatement.

Contrary to the authors' assertions, this wordplay would not work on just any lemma. A test case:

Q: What's yellow and asserts that exponents of commuting elements of a Banach algebra have the multiplicative property?

A: Lemon 2.12 of Douglas' Banach Algebra Techniques in Operator Theory.

Occam's Razor tells us, then, not to waste time looking for psychological or political explanations: whoever made up the riddle hadn't (pause...) any Choice.

Secondly, surely it is obvious that a compact city is one which can lay off all but a finite number of its police force, no matter how shortsighted they are.

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(Received January 10, 2005)

Origins of an Anecdote

I thoroughly enjoyed Paul Renteln and Alan Dundes' paper "Foolproof: A Sampling of Mathematical Folk Humor", especially the section "Making Fun of Mathematicians", which was the only one whose humour my beloved mediaeval historian wife was able to share, and a little too readily at that. I remember laughing twice at the classical "Abelian grape" joke: once when I first heard it at my high school math club and then again that night after I'd read enough about group theory to get the joke.

I take issue, however, with the laxness of citation of one joke in particular, appearing on p. 32, described as "an interesting metajoke."

I was the original author of that joke, which first appeared in the moderated USENET newsgroup rec.humor.funny in April 1993 (see http://groups-beta.google.com/group/rec.humor.funny/msg/7e8a721f677ef749) and can be found in Google a modest twenty-seven times.

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(Received January 17, 2005)

Dundes and Renteln Reply

We are generally pleased with the response to our survey of mathematical humor but somewhat surprised by the number of mathematicians who emailed us with considerable proprietary fervor claiming the authorship of one or more of the jokes. These claimants demonstrated a basic lack of knowledge of the nature of folklore. Once A tells a joke to B and B transmits it to C and D and the geometric progression begins, A loses forever his control over the subsequent dissemination and appearance of his creation. Presumably there was an originator of such contemporary proverbs as "You snooze, you lose" or "Use it or lose it", but we do not know who these creative spirits were. It is not easy to create new traditions. If one had as an exercise to create an entirely new folk song or an entirely new curse, it would not be an easy task. In fact, the authors of new mathematical jokes ought to feel gratified that their creation has enjoyed some acceptance by members of their peer group. We found, for example, that John Chew's metajoke was to be found on at least fifteen different websites, in all cases without formal attribution.

The Dawson comment reveals a lack of understanding of the aims of many social scientists when he recommends that they should not "waste time looking for psychological or political explanations." This unthinking slight of social science suggests that psychologists and political scientists, who surely look for psychological or political explanations for various phenomena, are wasting their time. Social scientists might equally well suggest that mathematicians who devote months or years trying to solve an arcane mathematical problem are wasting their time.

The goal of our essay was to suggest that there is such a thing as mathematical mentality or world view and that mathematical humor provides a bona fide and revealing window on that way of thinking. Now it may well be that our delineation of that mentality is in error, but the fact that such a mentality exists is surely not at issue. The jokes we report are in circulation, and all of them were in theory created by some ingenious individual, even if, after the fact, we are not always able to properly recognize that individual's efforts. The proof of the pudding is in the eating, so the jokes we report are meant to be savored and enjoyed by those who best understand them.

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—Paul Renteln
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(Received January 20, 2005)

Correction

The following website was inadvertently omitted from the list of references in our article: http://www.math.uchicago.edu/~conrad/amused.html. We thank Justin Sinz for bringing this to our attention.

—Paul Renteln and Alan Dundes

Learned Societies and the War on Terror

"Mathematician Caught in 'War on Terror' Dragnet". This is the lead story

in the October 2004 issue of the *CAUT Bulletin* (CAUT = Canadian Association of University Teachers, counterpart of the American Association of University Professors). The full story is available free online. Open `http://www.caut.org`. Click on "Bulletin online", then "archives", finally "October". The first item to pop up is this story. Well worth reading.

Professor James Lewis of the University of Alberta was en route to a mathematics meeting in Chicago when he was harrassed by U.S. authorities, as had been the case, he discovered, with two men each named John Lewis. It doesn't pay to have a common name, as Senator Ted Kennedy had already learned under similar circumstances.

CAUT learned that such episodes were frequent. This case was distinguished only by our colleague's willingness to go public. It is worthwhile reading the *CAUT Bulletin*'s report on his comments. A letter column does not offer sufficient space to do them justice. The implications are scary and lasting.

They apply beyond individual human rights and have possible consequences for the functioning of learned societies and for the content of their meetings. Participants from anywhere in the world could be excluded at the U.S. border, no matter how long their voyages, to the detriment of other participants as well as to themselves.

The U.S. authorities interfere also for purely political purposes, having nothing to do with the "war on terror". Quite recently, a Latin American Studies Association held an international conference in Las Vegas. At the very last moment, the U.S. government refused visas to all sixty-one Cuban specialists who had been invited, including several who had made previous scholarly visits to the U.S. This damaged the professional content of the conference for all participants.

Also recently, the U.S. Treasury Department forbad translating or abstracting publications emanating from several countries under the Trading with the Enemy Act. This would have criminalized at least *Mathematical Reviews*. Loud protests and refusals to comply by various affected societies has forced Treasury to put this into abeyance.

Such behavior on the part of U.S. authorities is not new. It was common during the cold war. One example: The International Congress of Mathematicians resumed activity after World War II in 1950 at Harvard. Jacques Hadamard, then 85, had been designated Honorary President. Initially the U.S., mindful of his left views, said he would not be admitted, although he had spent the war years in the U.S. Other French mathematicians threatened a boycott. There was talk of moving the congress to Canada. Under this pressure, the U.S. authorities relented—somewhat. Hadamard was given a visa, but valid only for Cambridge, Massachusetts. He was not even allowed transit to Mexico after the congress.

Nonetheless, securing his participation and leadership in the congress was an important victory for the vigorous protest which took place and for the success of the congress. The current suspension of the Treasury Department's ban on processing scientific publications from the countries specified is a current notable victory, hopefully lasting. If we are as alert and vigorous and public on similar issues, we may not need to despair.

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(Received January 13, 2005)