Changing Teaching of Future Teachers

Some years ago, one of us taught mathematics courses for future elementary teachers much as he did other college mathematics courses—lecture, drill, and test. Connections to school mathematics were not part of the course because they were not part of his education and experience. After reading *The Mathematical Education of Teachers* (MET), edited by Cathy Kessel (lead editor), Judith Epstein, and Michael Keynes (CBMS Issues in Mathematics Education, vol. 11, AMS and MAA, 2001) and Liping Ma's *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States* (Lawrence Erlbaum Associates, 1999) and while working with mathematics educators on the Mathematical Association of America's Preparing Mathematicians to Educate Teachers (PMET) project, he is again teaching these courses—but in a very different way.

This conversion illustrates the goal of PMET and like-minded efforts to help mathematics faculty improve the mathematical preparation of future teachers. Workshops, mini-courses, and conferences for college and university faculty are PMET's main vehicles for accomplishing this goal. During the past three years, thirteen PMET workshops have been held with over 300 participants. Nine new PMET workshops are scheduled for summer 2005, including two at historically black institutions, one at the Southwest Indian Polytechnic Institute, and one at Park City Mathematics Institute. One initiative similar to PMET workshops is the Michigan-Georgia NSF Center for Proficiency in Teaching Mathematics summer institutes.

Are these efforts improving the mathematical education of teachers? Preliminary evidence suggests they are. Aimed at changing college faculty understanding and behavior, PMET is farther removed from improving K–12 learning than are projects directed at K–12 teachers. So an ultimate evaluation of impact will require considerable time. But as an intermediate step, we contacted participants in the 2003 workshops to see if they have changed how they teach future teachers.

Certainly, they overwhelmingly say they have. Ninety-five percent of those contacted report more emphasis on group work and collaborative approaches. Eighty-eight percent report requiring students to explain reasoning when solving problems—for example, through writing in and out of class, asking for oral explanations in class, and group discussion. And the changes were apparent to students as well. As one participant reported, "after a couple of days…I had one student ask me if we were only going to do 'word problems'…another asked if we were ever going to do 'a page full of calculations'…this seems to be unlike any math course they have ever had before, given their previous beliefs of what it was like to 'do math.'"

Workshop participants also told us they had changed how they teach particular mathematical concepts. These covered a wide array of topics—from greater emphasis on the concept of "the whole" in interpreting fractions, through materials for teaching algebraic reasoning, to particular approaches to teaching topics in measurement and geometry. Others reported they are much more aware of the need to instill in their students an understanding of the mathematical knowledge that K–12 students need at every grade level to be successful. One noted, "even in Beginning Algebra I spend more time discussing the various meanings of a fraction…it helps my students gain a deeper understanding of something they felt they had already 'learned.'" Finally, participants told us they thought students learned better after these changes. One said that for the first time her students "were able to make the connection between fractions, decimals, and percents without my making it for them." Others cited differences in how students handled problems—both with respect to the explanations they provided of their own reasoning and the representations they used to construct their answers. On the latter, one noted, "units no longer disappear at the beginning of the work only to mysteriously reappear as part of the solution."

Changing teaching of future teachers can improve K–12 mathematics, and early evidence suggests the PMET workshops are effectively inducing mathematics faculty to make appropriate changes. But few mathematicians from research universities have so far participated in these workshops. In addition to the fact that their presence would enrich the experience, we believe there are several reasons why more should do so.

- Research institutions prepare future teachers, and research faculty should be involved in the development and teaching of courses for future teachers. In the words of the MET Report, "All mathematicians should be concerned about teacher education, and all have a role to play in the education of teachers."
- The mathematics required for teaching, in the words of MET, is "quite different from that required by students pursuing other mathematics-related professions." Most of us know some bits and pieces of the mathematics of K–12 teaching, but this mathematics needs more structure if we are to incorporate it coherently into college courses.
- Research mathematicians wield tremendous influence on future college and university faculty members, and the priorities and attitudes of research faculty influence all college and university faculty.
- Mathematics education is a major public policy issue, and mathematicians should be positioned to steer policy toward improving education. Research mathematicians can help add unity and credibility to this case for improvement.

In the September 2004 *Notices*, Lynn Arthur Steen wrote that to contribute to K–12 education "mathematicians should focus first on the mathematical education of teachers." We reinforce his call with some encouraging reports of changes being made through PMET, noting, as he did, that *everybody* in the mathematics community needs to be involved.

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Letters to the Editor

More on the Prisoner's Dilemma

In his letter (August 2004, page 735) commenting on Steven E. Landsburg's remark about the Prisoner's Dilemma ("Quantum game theory", April 2004), Dr. Marcus perpetuates the fallacious inference of Douglas R. Hofstadter in his justly admired 1985 book *Metamagical Themas*. The general issue is what hypotheses assure socially desirable choices when individual choices are relatively penalized unless all players choose to act for the social good. Clear thinking about this is important, since many crucial situations resemble the Prisoner's Dilemma, notably the need to limit births as we near the carrying capacity of our planet.

Before getting to the crux of the fallacy, let me repair one obvious deficiency of the argument. The hypotheses that the players are rational and selfish are what *we* know about them, but to impel a player to act, *he* must know this about the other, and then know that the other knows that he knows, and so *ad infinitum*. This may be a little awkward to describe, but there is no difficulty in annexing this iterated hypothesis of mutual complete knowledge (and Hofstadter essentially did so). These hypotheses remain insufficient.

Hofstadter weaves a magical spell about the theme that with symmetry selfish players will each cooperate as a rational choice. Ironically, it is the symmetry that makes it easy to refute this. A selfish player can rationally decide on cooperation only on the basis of an inference that the other player will choose cooperation—the inference must precede the decision. The symmetry guarantees that neither gets the necessary precedence. The argument never gets beyond the magic to a valid conclusion. It is the finitary nature of logic that prevents such a leap.

A simple analogy may allay any remaining doubts. The valid part of the symmetry argument is like proving that the tails of the sequence {1/n} become arbitrarily small. Hofstadter's conclusion is like the assertion that lim{1/n} exists and is 0. But the latter conclusion requires independent existence hypotheses about the presence of 0 and the nature of its neighborhood system. Choosing defection in the game contradicts none of Hofstadter's hypotheses. The reasoning is inconclusive, and rational choices are not based on inconclusive reasoning.

What the symmetry argument *does* accomplish is to demonstrate that additional hypotheses *can* consistently be added to entail mutual cooperation. What could such hypotheses be? A pre-arranged agreement? That is safe, but more than is necessary. The minimum is to assume, along with knowledge of the symmetry, a disposition to choose cooperation in a situation where all know that mutual cooperation will have a payoff in excess of any other. This, of course, is just a restatement of the conclusion. Explicitly adopting such a hypothesis renders the whole question trivial. Further, it is clearly a moral hypothesis, in the same category as selfishness, and not at all akin to rationality.

Since the behavior of the players is not determined by the hypotheses, it is clear that any claim that "Rational selfish players always choose to defect" is also incorrect—rational selfish players need not choose the Nash equilibrium in every case. Such claims as Landsburg's are epimathematical—imprecise heuristics that provide a perspective to ease the assimilation of the mathematics and to suggest applications of it. In this role they do little harm. It is when they are separated from this role and exalted into exaggerated claims that they are objectionable. Mathematicians cherish rationality and elegant arguments, and imputing to rationality more than it can deliver is sure to damage these values.

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Underrepresentation

In his article on doctoral degrees earned by ethnic minorities, Herbert Medina points out that "Blacks, Hispanics/Latinos, Native Americans are underrepresented in earning doctoral degrees in the mathematical sciences. That is, these ethnic groups do not earn doctorates comparable to the percentage of the population they comprise." Most of the article is concerned with various attempts to try to reduce if not eliminate this underrepresentation. The final section of the article begins, "Supposing that we aim to solve the underrepresentation by the year 2025." From the discussion that follows it is clear that the problem will be "solved" when each of these groups is earning degrees at a rate roughly proportional to its percentage of the population.

I agree strongly that it is desirable to try to provide conditions that might increase the doctoral percentages of the study's underrepresented groups, but I seriously question the way the eventual goal is formulated. Do we really want to make our objective "to each according to his/her population percentage"? If this criterion is to be applied uniformly, then we will have to worry about overrepresentation as well as underrepresentation. I notice, using the data provided with the article, that the group Asian/Pac. Isl. is currently earning more than four times as many doctorates as its population would warrant. Should we then hope that by 2025 the percentage of doctorates earned by this group will be reduced by a factor of four? I doubt that Professor Medina would be in favor of such a resolution, but it's an unfortunate theorem that if we are to eliminate underrepresentation we will have to suppress overrepresentation. (Unfortunately we can't be like the students in the schools of Lake Woebegone who are all above average).

I have seen the criterion of population proportionality applied in other cases, notably that of gender, and I object to it there for the same reasons. The fact is, some groups in our society—whether cultural, ethnic, or even religious—have put more emphasis on mathematical achievement than

others. It seems to me that to try to level everyone by imposing the arbitrary criterion of population percentage would be a serious mistake.

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Homework and Google

For decades calculus texts included answers to their odd-numbered exercises in an appendix; answers to the even-numbered exercises were available only in limited circulation instructor's answer manuals. This pedagogical arrangement was undercut here in Chapel Hill a decade ago when a teaching assistant put the answer manual on reserve in the library: Not only did his 35 students receive access to solutions for all of their assigned even-numbered problems, but so did some of the 700-odd students in the other 20-odd sections of that course that semester (namely, those students who happened upon the manual or who learned about it from a friend).

After I turned in my undergraduate group theory grades in May, one student told me that he had been able to find solutions for some standard problems via Web searches when he got completely stuck. Later I tried typing the following into Google:

Solution Prove that the composition of two injective functions is injective

Some of the top ten hits gave solutions for related problems, but none included a solution for this problem. Recently I reported this failed effort to the student while a friend of his was present. She opined that her peers were far more adept at Googling than faculty and suggested artfully placing quotes while using her favorite search engine. So I then typed

Solution "Prove that" "the composition of" "injective functions is" "injective" into `http://www.dogpile.com`. One of the top five hits (which was not in North America) provided a solution to this problem (in English). My brief sojourns indicated that the instructors who posted solutions were providing them to their students *after the fact* (following a quiz or an assignment). Postings that are not removed then provide solutions to students elsewhere *before the fact*.

The first student also told me that he was also able to find many posted solutions for our current discrete mathematics text at one site by Googling a combination of the author's name and some other suggestive words.

Not everyone will agree that posting solutions on the Web nets out negatively. We're supposed to be in the business of disseminating mathematical knowledge! All along, many of us have accepted solutions from students industrious enough to comb library books. However, the excessive convenience of the Web (copy and paste!) could soon remove many nice/classic problems from being eligible for graded homework assignments. If you are concerned about the potential effects of your actions on the pedagogical approaches of fellow instructors (even if this does not affect your own teaching techniques), you may wish to consider posting your solutions in a secure area requiring a password for access. If this approach is not readily available, some low-tech strategies could be considered: Avoid including the name of the text's author in your website. Do not include the statement of the problem. Deliberately misspell the key words. (This could be fun: "Let f and g be infective functions…"!) Remove your posting as soon as possible.

Some solutions will always be available on the Web. Proofs/solutions of all standard statements/problems will eventually appear on the Web, perhaps in a central eBourbaki repository. But in the meantime: If most instructors post their solutions below the search engines' radars, this could lower the Googling success ratio sufficiently so that the less industrious students won't bother.

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Review of *A Handbook of Mathematical Discourse*

Steven G. Krantz's review of my book, *A Handbook of Mathematical Discourse*, in the August 2004, issue of *Notices of the American Mathematical Society* shows that he misunderstood the purpose of the book. Its focus is on mathematical discourse, that is, the way math is communicated in speech and writing. It is not primarily about math itself.

Students may deal dysfunctionally with quantifiers, definitions, the use of English words such as *if* with meanings different from those of ordinary discourse, and other aspects of mathematical discourse. Much of the book is devoted to discussing these difficulties, which inhibit understanding of many mathematical ideas. No attempt was made to cover all the important specific mathematical concepts.

The *Handbook* has entries about certain ideas from linguistics, rhetoric, cognition theory, and research in mathematical education and argues that, if teachers and students are aware of these ideas from outside mathematics, they will improve their chances of communicating successfully. These ideas may not belong in a book about math, but they do belong in a book about mathematical discourse.

As the introduction states, the book is incomplete in meeting its own goals, never mind the goals Professor Krantz expected it to have. Much more work in this field is necessary.

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