How Mathematicians Can Contribute to K–12 Education

As the so-called "Math Wars" calm down, many mathematicians and mathematics educators are now working together in order to, if you will, "win the peace". But as our political leaders have discovered in Afghanistan and Iraq, this is not as easy as one might wish it were. On the ground one finds complications that are often not noticed from a distance. These include state-adopted standards for K–12 education, pressure from the new federal law known as "No Child Left Behind" (NCLB), and high-stakes tests on which students' graduation and teachers' jobs depend.

Over the last several years I have reviewed many of the new state standards for school mathematics and have participated in numerous discussions about the changing landscape of standards and high-stakes testing. Based on this experience, I offer four suggestions for how mathematicians can constructively contribute to the improvement of K–12 mathematics education.

- Notwithstanding serious problems of clarity in many state standards—problems that are being addressed in most states—the variation in mathematics expectations among state standards is far less than the variation in mathematical preparation of teachers (or subsequently in the mathematical proficiency of their students). Not surprisingly, the mathematical proficiency of teachers is more important than the wording of standards in determining what students learn. *Thus, mathematicians should focus first on the mathematical preparation of teachers (which is, after all, one of higher education's most important obligations).*
- Increasingly, high-stakes exams conflict with high performance standards. When teachers' jobs or students' graduation are on the line as the result of scores on a single test, it becomes very difficult to maintain political consensus on the value of high performance standards. One state after another is postponing, reducing, or evading the consequences of students' failure (or sometimes merely the prediction of failure) to meet the ideals of high standards. *To preserve momentum for high standards, mathematicians should advocate policies that judge students, teachers, and schools using multiple criteria rather than single high-stakes tests.*
- Overly specific standards obscure the rich internal connections of mathematics and lead to an atomized "check-off" approach to pedagogy and assessment. In sharp contrast, the 2001 National Research Council study *Adding It Up* stresses that mathematical proficiency consists of several "interwoven and interdependent" strands: namely, conceptual understanding, adaptive reasoning, procedural fluency, productive disposition, and strategic competence. These same five elements (albeit in different words) anchor the discussion of mathematical understanding in *Standards for Success*, a recent consensus report on what students need to know and be able to do to succeed in entry level university courses. *Thus in any discussion of K–12 standards and assessment, mathematicians should advocate balance not only of content but also among these interwoven strands of mathematical proficiency.*
- Many state mathematics standards have been rightly criticized for including statements that are unclear, incorrect, or meaningless. But without well-chosen sample problems, even carefully crafted standards cannot accurately convey the degree of cognitive sophistication desired to achieve appropriate mathematical proficiency. Rhetoric alone, even when clear and correct, cannot communicate mathematical understanding. For example, "solve systems of 2 x 2 linear equations" is perfectly clear but omits any sense of complexity concerning the nature of variables or coefficients. Adding restrictions such as "integer coefficients" adds clarity but, by limiting expectations, contributes to the atomization of learning. *To help students experience the power of mathematical thinking, mathematicians should create and contribute exemplary problems that convey important aspects of mathematical thinking.*

During the last two decades the standards movement has achieved many good results, not least public recognition of the increasing breadth and utility of mathematics, the importance of mathematics in the education of all students, and the value of striving for high aspirations. But the thin, repetitive, and disconnected rhetoric of standards too often undercuts their potential; they become just a list of bullets arranged in some partially arbitrary linear order. By its very nature—its *essential* nature—mathematics is interconnected and multidimensional, where distant parts link through logic and common structure. Rich problems can convey the distinctive cohesiveness of mathematics in a way that narrative standards never can.

In short, mathematicians can best contribute to the K–12 standards movement by offering problems—problems that provoke, problems that surprise, problems that expand minds. What problems would attract and stretch fifth-grade students? What problems would entice eighth-grade students to continued study of mathematics? What problems must a high-school graduate be able to do in order to succeed in today's high-performance workplace? What problems will make teachers (or prospective teachers) into better teachers?

These are not the problems normally found on standardized tests or college placement exams. Neither are they examples designed to illustrate one or another standard, nor are they the template questions favored by test item writers. We really do not need more of these kinds of problems. Both to prepare teachers to think mathematically and to broaden students' mathematical experience, we need a rich collection of thought-provoking problems that will exercise the interconnected skills of conceptual understanding, adaptive reasoning, procedural fluency, productive disposition, and strategic competence. To adapt a trite expression, one problem that makes students think mathematically is worth a hundred standards that just tell students what they should think.

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This article is adapted from remarks from a panel discussion at the January 2004 Joint Mathematics Meetings in Phoenix sponsored by the AMS Committee on Education on the topic "The evaluation of state mathematics standards: How can mathematicians contribute?".

Letters to the Editor

Alternative Freshman Mathematics

I would like to offer a suggestion for improving mathematics education at colleges in the U.S. I wish that freshman mathematics education did not always begin with calculus. For many people (particularly, I think, those of a philosophical or poetic turn of mind), calculus is a terrible place to start.

I took calculus in high school and found it so unsatisfactory that I quit studying math for ten years. (Mathematics is important to me, and I was severely depressed during this decade without mathematics.) I made very good grades, so no one listened to me when I complained that calculus made no sense to me. When I returned to the study of mathematics, I learned real analysis. After this, calculus made sense.

I recently read a book (*Everything and More: A Compact History of Infinity,* by David Foster Wallace) which recalled to me all my frustrations with calculus. The author, Mr. Wallace, states that he has hated and done badly in every mathematics class he has ever taken. Both this book and the novel *Infinite Jest* make clear that Mr. Wallace is fascinated by the theorems of calculus but sometimes misinterprets them. I do not think that he has ever taken a course in pure mathematics. *Everything and More* contains many errors and misleading statements. But what I notice most is that Mr. Wallace, a very intelligent person, has sought mathematical knowledge in several college math classes and has been shamefully shortchanged. I fear that this happens to many students who are not majoring in engineering or physics.

I propose that there be two standard mathematical curricula in college rather than just one. One track would begin with calculus in what is now the usual way. The other would begin with pure mathematics. Calculus would follow real analysis, not the other way around.

Currently I teach a course called Discrete Mathematics, which is actually an introduction to set theory, logic, and mathematical proof. The official prerequisite for this course is two semesters of calculus. I really don't think this makes sense. Some freshman have no trouble with calculus. For others, the standard calculus class is profoundly unsatisfying. Some people need to learn analysis before they can be comfortable with calculus. If we could offer two standard tracks—one that begins with calculus and one that begins with pure mathematics—I think that more people in this country would be able to understand and enjoy mathematics.

It would be easy to make this change. Please, let's try it.

—Amy Babich
Austin, Texas

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Role of Mathematics, As They Think of It

PISA (Programme for International Student Assessment) is a major international programme for investigation of the quality of school education. It estimates knowledge and skills of 15-year-old school students in three domains: reading literacy, mathematical literacy, and scientific literacy. Its main proclaimed aim is to test how well the students are prepared for solving problems of everyday adult life. So, looking at the mathematical part of its tests, we have a chance to learn finally how our science can be used in this life.

I quote in italics PISA's program document available via `www.pisa.oecd.org/Docs/Download/PISAFrameworkEng.pdf`. *Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen.*

In testing mathematical literacy, *questions are organised in terms of three "competency classes" defining the type of thinking skill needed. Class 1: reproduction, definitions, and computations. Class 2: connections and integration for problem solving.* (Well, I use connections and integration to solve mathematical problems, so probably I am in the second class.) *Class 3: mathematical thinking, generalisation and insight.*

Example of problems from Class 1: *Solve the equation* $7x - 3 = 13x + 15$. From Class 2: *Mary lives two kilometers from school, Martin five. How far do Mary and Martin live from each other?* (No, I am not in this class...)

And finally Class 3, the top level of mathematics applications in the adult life of a "constructive, concerned and reflective citizen".

In a certain country, the national defense budget is \$30 million for 1980. The total budget for that year is \$500 million. The following year the budget is \$35 million, while the total budget is \$605 million. Inflation during the period covered by the two budgets amounted to 10 per cent.

a) You are invited to give a lecture for a pacifist society. You intend to explain that the defense budget decreased over this period. Explain how you would do it.

b) You are invited to lecture to a military academy. You intend to explain that the defense budget increased over this period. Explain how you would do this.

In this problem, mathematics is considered just as a tool for political prostitution and dirty manipulation with data and uncertain notions. To get a maximal mark for it, one should have no idea that in solving a problem, one should first decide *for himself* what is the truth in this problem, after which the suggestion that in one case he "intends to explain" something opposite is absolutely insulting. (A soft discussion of ethic aspects is given in the book where this problem occurred first, but no trace of it is reproduced in this fundamental document.)

This document was issued in 1999 and is acknowledged by national educational organizations of thirty-two countries conducting this assessment. All these years, nobody cried out with horror, as I do now; doesn't it mean that all its readers and implementors (mainly school teachers and other educators) agree with its "identification and understanding of the role that mathematics plays in the world"? What have we done to deserve this shame?

The same document quotes a list of mathematical "big ideas", including chance, change and growth, space and shape, etc. However, it misses (and undermines) the basic idea of science: that of Objective Truth, which roughly is as follows. Everybody solving a problem should

(1) acknowledge the existence of objective truth, independent of our desires, presumptions, or abilities to describe it;

(2) try to discover it, being ready to accept it in any shape and form in which it will occur to us; and

(3) be ready to defend it after that in all legal ways, respecting one's right to have a different opinion—but not two at the same time!

A fundamental role of science is the dissemination and strengthening of this boring truism in our quite deceitful world.

And the last question: How should one rely on this investigation? What is *its* subtask b)?

—V. A. Vassiliev
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Alternative Journal Pricing

Gerard van der Geer's article on *Compositio* in the May issue of the *Notices* shows a line of action which editors can take if they own the name of a journal and are worried about the price. In most cases, however, the publishers own the title.

The editors of *Topology*, in discussion with the publishers, came up with another route a few years ago. There is now an alternative subscription which offers immediate electronic access to the journal with paper copies at the end of the year for half the price of the standard subscription (which incidentally gives a figure less than *Compositio*'s new price). Since the driver for much of the current discussion on open access is the immediate availability of online versions, this in principle offers what many consumers want.

What the future holds is anybody's guess, but we are nowadays used to the fact that there is no single price for an airline ticket or a cellular phone contract. Everything depends on a balance of delivery methods, forward planning, and volume. Maybe that is what we should expect in scientific publishing.

—Nigel Hitchin
Editor, Topology

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Coding Theory and the Genetic Code

It might be said that the mathematical community today has three wishes: a wish for greater public recognition, a wish to attract talented men and women into the field, and a wish to increase funding levels. What can be done to make these wishes come true? No single achievement would do as much to improve the public image of mathematics as a forceful application of the theory of codes to the structure of DNA sequences. Imagine the result if a single child's life could be saved in this way: mathematicians would appear on the cover of *Time*.

An open mind about possible models will be crucial to any attempt to apply coding theory to the genetic code. To hint at the possibilities, I will sketch a newly noticed parallel between DNA sequences which code for proteins and arguments in Aristotelian logic.

For logical purposes Aristotle classified sentences into four types: A (universal affirmative), I (particular affirmative), E (universal negative), and O (particular negative). For Aristotle, the basic kind of argument is the syllogism. A syllogism is a sequence of three sentences and so is coded by a triplet in the AEIO code. For example "All men are mortal; Socrates is a man; Socrates is mortal" has the code AAA. A proof in Aristotelian logic is a sequence of syllogisms and so has a sequence of triplets in the AEIO code as its code.

Proteins are coded for by a sequence of triplets in the TAGC code, each of which codes for an amino acid, where T = Thymine, A = Adenine, G = Guanine, and C = Cytosine. For example, the triplet GCA codes for Alanine, and CAACAC codes for Glutamine, followed by Histidine. Chargaff's Rules $T \leftrightarrow A$ and $G \leftrightarrow C$ are analogous to the Medieval Square of Opposition $A \leftrightarrow O$ and $E \leftrightarrow I$. There are twenty-one amino acids and three stop codons, for a total of twenty-four. There are fifteen universally valid forms of the syllogism and nine conditionally valid forms, for a total of twenty-four.

When will coding theory be applied to the genetic code?

—Sherwood Washburn
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Wu's Comment on Roitman Letter

Professor Roitman's remarks about the fostering of geometric intuition are well taken. I was at fault for not being sufficiently precise: one must spend time to build up prospective teachers' geometric intuition, but most of the class time must still be devoted to the mathematics of the school classroom. I sincerely hope that the lonely reference to the work by B. Braxton and myself would be interpreted more as an implicit criticism of the vacuum that at present exists in the literature than as an attempt at self-advertisement, Professor Roitman's recommendations notwithstanding.

—H. Wu
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Correction

The August 2004 issue of the *Notices* carried my article "Has the Women-in-Mathematics Problem Been Solved?". On page 778, the text of the article says that 46 of the Ph.D.'s granted between 1995 and 2003 in the Department of Computational and Applied Mathematics at Rice University were granted to women. A percent sign was inadvertently dropped; the correct figure is 46%, not 46.

—Allyn Jackson