Book Review

Flatterland: Like Flatland, Only More So

Reviewed by Jody Trout

Geometry, the Final Frontier. These are the mathematical voyages of Vikki Line of Flatland….Wait a minute. Does this sound like a review of a mathematics book or a science fiction novel? Mathematics and science fiction? For several generations, the reading public has assumed that the main focus of science fiction (SF) was mainly, well, science and its technological toys. Lasers, spaceships, robots, time machines, atomic reactors, intelligent computers, warp drives, and genetic engineering are just some of the familiar literary devices of mainstream SF. But, why not curved surfaces, hyperspheres, fractals, Hamming metrics, projective lines, and non-Euclidean geometries? Couldn't mathematics also be the queen and servant of science fiction, to corrupt that famous saying?

Of course, as many sci-fi fans and readers of the *Notices* know, mathematical concepts have appeared in several science fiction stories over the past century or so. The main examples, from a literary viewpoint, are the fourth dimension of time in the classic novel *The Time Machine* (1895) by H. G. Wells and a hypercubical home in Robert A. Heinlein's timeless tale "—And He Built a Crooked House" (1940).

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Because of its special relation to Einstein's theories of relativity, the geometry of the fourth dimension—along with its resident hypercubes and such twisted topological beasties as the Klein bottle and the Möbius strip—has provided the most popular mathematical morsel. But, with apologies to Euclid, no element of geometric literature could be more famous or enjoyable than that satirical Victorian romance of many dimensions, *Flatland*.

Written in 1884 by the school headmaster, clergyman, Shakespearean (and decidedly non-mathematical) scholar Edwin Abbott Abbott, that delightful little book has charmed generations of readers and tempted many of them to become mathematicians, including me. Many of you already know the plot by heart. *Flatland* tells the tale of how the lowly A. Square, a four-sided inhabitant of a two-dimensional Euclidean universe, receives heretical knowledge of higher dimensions from a visit by that most symmetric of Solids, The Sphere. Armed with the Theory of the Third Dimension, our planar hero sets out on a crusade to convert the narrow-minded and sexist polygonal citizens of Flatland to a more enlightened higher-dimensional view of the

mysteries of space and time. However, like Galileo, A. Square discovers the timeless truth that those who put the prevailing cosmic paradigm on trial are all too often the subject of a trial themselves.

Since it first appeared, *Flatland* has been in continuous print in numerous editions and in many foreign languages. And, as many good books do, it has spawned several sequels. The main examples are the story *An Episode of Flatland* (1907) written by the colorful logician Charles Howard Hinton, the novel exposition of curved spaces *Sphereland* (1965) crafted by the Dutch physicist Dionys Berger, and *The Planiverse* (1984) by the computer scientist A. K. Dewdney, which develops the physics, astronomy, and biology of a 2D universe in a more rigorous and consistent manner. By the way, it is rumored that C. H. Hinton is the person to whom Abbott obliquely refers in the dedication of *Flatland* when he writes, "To the Inhabitants of Space In General And H. C. In Particular…." Hinton was influential in getting the public at the turn of the twentieth century interested in the fourth dimension by writing popular science articles and books on the mysterious topic. (He even claimed he could see four-dimensionally and, by the way, also invented the baseball throwing machine!)

There have also been several short stories involving Flatland or discussing it in some detail, such as a Flatland spoof by A. G. Birch called "An Adventure in the Fourth Dimension", which appeared in the October 1923 edition of the famous pulpzine *Weird Tales*. Nelson Bond wrote "The Monster from Nowhere" (1974), a creepy story about a 4D being captured by a human, and there is Rudy Rucker's dark tale "Message Found in a Copy of Flatland", which appeared in *Mathenauts* (1997), the anthology he edited (but which, sadly, is now out of print). In this story, Rucker places the physical location of Flatland in the basement of a questionable Indian restaurant in the city of London!

In fact, over the past several decades there have been so many science fiction tales involving ideas streaming from *Flatland*, the fourth dimension, and mathematics in general, that "mathematical science fiction" should be treated now as its own subgenre of SF. Indeed, three years ago I collaborated with my Dartmouth colleague Laurence Davies, who is in the comparative literature and English departments, in designing a course to study this emerging literary form. The course, which we called *Mathematics and SF: The Fire in the Equations*, was developed under the auspices of the Mathematics Across the Curriculum (MATC) project. The MATC grant was part of a multi-institutional effort by the National Science Foundation to foster interdisciplinary courses involving mathematics. We taught the course for the second time during the spring 2001 term and concentrated mainly on sci-fi stories involving geometric ideas, such as the fourth dimension, relativistic spacetime, parallel and curved universes, projective geometry, other non-Euclidean geometries, and topology. And, of course, *Flatland* was the perfect starting point for the course, as well as a useful source of metaphors and analogies for more advanced geometric concepts.

Had Ian Stewart's novel been published sooner, we would have surely considered it for our course syllabus since it discusses all of the topics we covered and then some! *Flatterland: Like Flatland, Only More So* is the latest sequel to the tri-dimensional journey of A. Square. Stewart is a mathematician at the University of Warwick, where he also directs the Mathematics Awareness Center. A well-known popular writer about science and mathematics and the author of over sixty books, Stewart was awarded the prestigious Michael Faraday Medal from the Royal Society for his contributions to furthering the public's understanding of science and mathematics. In 1999 he received the Communications Award from the Joint Policy Board for Mathematics. Stewart has also written the "Mathematical Recreations" column in *Scientific American*.

Flatterland begins with the discovery of an old family copy of A. Square's original testimony one hundred years later by his lineal great-great-granddaughter Victoria Line. Upset by her father's pigheaded insistence to make sure that the embarrassing memories of the imprisonment of crazy old Albert and the suppression of his subversive 3D Theory no longer cause the family any shame, the precocious Vikki secretly scans the ancient scroll into her personal computer before handing it over to her father to be burned. Studying the files, she finds a secret message from her ancestor on how to contact Those from the Third Dimension.

Vikki is soon visited by the Space Hopper, a tame horned sphere homeomorphic to the original spatial sage. (Alexander's wild horned sphere makes a brief appearance later when they visit Topologica.) The Space Hopper promises to take her on a fantastic voyage of the Mathiverse (short for the Mathematical Universe), that Platonic realm where mathematical objects, geometries, and spaces have their own Alice-in-Wonderland existence. To help his one-dimensional charge understand and visualize the multidimensional marvels of the Mathiverse, the Space Hopper equips the "Flatty" Vikki with a Virtual Unreality Engine (VUE) and then whisks her away from her planar home, without her even saying goodbye.

First they explore Spaceland, the idealized three-dimensional world of Euclid, which is separate from the "Planiturthian" Universe of the earth-bound humans, whose mathematical mindsets give rise to the quasi-independent existence of the Mathiverse. Running throughout the story is a philosophical chicken-and-egg conundrum about the exact nature of the relationship between mathematics and the

For more information concerning Edwin Abbott and *Flatland*, as well as the mathematics and applications of higher dimensions, visit the "Math Spans All Dimensions" website

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http://mam2000.mathforum.com/765/index.html
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designed by Thomas Banchoff of Brown University and David Cervone of Union College. A math-sf bibliography and related links can be found at our course website

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http://math.dartmouth.edu/~c18s01/.
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Other useful bibliographic references are Alex Kasman's "Mathematical Fiction" website

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http://math.cofc.edu/faculty/MATHFICT/
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(discussed in the August 2000 issue of the *Notices)* and Appendix B of Clifford Pickover's delightfully X-File-ish book *Surfing through Hyperspace.*

—*J. T.*

true geometry and structure of the universe. Is the universe constructed out of mathematics or is mathematics the construct of human minds, which are part of the universe? Stay tuned.

After stacking circles and spheres to highlight the differences between 2D and 3D, the duo then proceeds to a brief tour of higher dimensions. Various uses of "dimension" are investigated, such as treating time as a dimension and using parametric dimensions, for example, when they visit the discrete geometry of the Double- and Triple-Digit Districts, where Vikki learns about error-correcting codes and the Hamming metric. She then has a weird dream about painting herself into the interior of a closed hole by painting the outside of a ball, thus discovering the boundaryless nature of a 3-sphere. There then ensues a discussion of the kissing number for hyperspheres. (It is mentioned that the kissing numbers are known only in dimensions 1, 2, 3, 8, and 24, but they are known now also in dimensions 5, 6, and 7.) They then finish with how dimensions affect knots: There are no knots in 2D, there are oodles in 3D, and every knot can be undone in 4D.

Interspersed throughout her travels are disconnected vignettes of the plight of her Flatland family at the sudden disappearance of their beloved yet high-strung daughter and Vikki's feeble whining to her dear diary about how much she supposedly misses her family. These scenes are a little flat (pardon the pun) and the tension they are supposed to create is not very believable.

Through the next several chapters, Vikki and her smiling horned companion take a fantastic voyage through various geometries and spaces of the Mathiverse. They take a hike through the Fractal Forest where they meet the infinitely-crinkly Helge the Snowflake on their way to the infinitely-paved Quadratic City, which has streets and avenues for each (x, y) in the plane. A simple recursive rule for taxi drivers leads to mass confusion if one tries to leave this complex city! So, of course, the perfect fractal for the job of Taxi Controller is none other than the "Mandelblot" himself! (Groan...) Next, they warp themselves to the continuously deforming landscape of Topologica, the Rubber-Sheet Continent, where they meet such Carrollian characters as the Doughmouse, who can turn himself into a saucer, and Moobius the Cow, that half-twisted strip of beef that keeps her milk in...you guessed it...a Klein bottle. (Double groan...The word play is cute at first but then becomes a bit tiresome.) Afterwards, they take a safari out to infinity in the Projective Plain to capture Projective Lions, which are always polite when meeting each other and never run in parallel paths. Taking a much-needed wine break (not suitable for underage readers) at the Running Turtle Bar, where they are served by the Chicken Mock Nugget, the tired dimensional explorers help the bartender solve his vineyard planting problem using the geometry of finite projective planes.

With all these conflicting notions of what constitutes a "geometry", the Space Hopper then helps Vikki understand that geometry is nothing more than a space equipped with a transformation group of symmetries. This helps her to digest the distastefully non-Euclidean geometry of Platterland, the politely curved cousin of her own 2D world where curves are lines and lines are curves and the circle at infinity is always just beyond one's next ever-shrinking step. Thus ends the purely mathematical voyage of Victoria Line two-thirds of the way into the tale.

The rest of the story concentrates on Vikki's trying to understand the geometry and structure of the Planiturthian Universe. The best way is to start out small—really, really small—by going into the subatomic litter box of Cat Country, which is the domain of Superpaws, that mortally confused pet of the master of quantum superposition himself, Erwinschrödinger. (No, I did not make a typo. For some obscure reason, Ianstewart concatenates all—and only—human names in his story. I am sure Edwinabbottabbott would not have been amused.) By studying the photoelectric effect, Vikki learns of the slippery dual nature of subatomic quanta that can discreetly change their clothes from solid particles to pastel waves. (What Stewart actually describes here is not the photoelectric effect, where high frequency photons kick off electrons from certain reactive metals, but rather the rarer inverse process of electron-impact photon emission.)

Next, they enter the cosmological arena of Alberteinstein where they meet the likes of the Paradox Twins: Twindledumb and Twindledumber. Guess who made the wrong travel plans and ended up forty years older? To spice things up a bit, they then make the acquaintance of the very causal Space Girls: Curvy, Bendy, Pushy, Squarey, and Minny Space. (Outer Space must have left the band before the book was written....) Minny instructs Vikki in the subtle nature of relativity using her light-coney diagram by Hermannminkowski. Then, they dash off to the spatial engineering domain of the capitalist Hawk King. Vikki desperately wants his majesty to grant them a temporal trip through one of his exotic matter wormholes, but they are instead duped into falling through the looking glass event horizon of a common black hole. Thanks to space-time gymnastics that would challenge even Mr. Spock's temporal lobes and make the Cheshire cat frown, they arrange to bootstrap themselves out of the hungry maw of the naked singularity.

To finish up her Mathiversian instruction, Vikki is led to the forefront of mathematical physics: cosmic strings, supersymmetry, quantum gravity, p-branes, and M-theory. And, according to her diary logs, she experiences the wondrous totality of her entire voyage through the Mathiverse in only a semester's worth of time! (If only our students could absorb even a fraction of this material in the same amount of time.) But, alas, she finally misses her family too much, so her horned conductor takes Vikki back to her pentagonal flat just in time to have leftover Crisp Moose dinner and celebrate New Year's with her grateful family.

However, using her enhanced VUE of things, she discovers a secret that will set her on a revolutionary social crusade in the spirit of her rectangular ancestor. The one-dimensional women of polygonal-dominated Flatland are, in fact, the intersection of Flatland with orthogonal polygons when viewed from the larger, supersymmetric world of Shadow Matter! Vikki then begins to spread her mathematical feminism through the cyberworld of the Flatland Interline. Grrl-power goes interdimensional.

Personally, I enjoyed reading this book. It presents advanced mathematical and physical topics in a fun and whimsical manner. But, of course, with so many topics, several are not discussed in any significant detail. And, regrettably, hypercubes were shorted the most, despite being the higher-dimensional topic that is the most fascinating and accessible to the general public. Some may complain that Stewart attempted to cover too much mathematics in one book; after all, analogies and metaphors can only go so far before the need for rigor sets in. Some might also find the book too long, especially when compared to the original tale, which gives a brief exposition of just one topic. *Flatterland*

can be far too cute at times, and, as already mentioned, the word play can grate on the ears. Also, some of the British in-jokes, relating to railways and the tawdry soap Eastenders, will not tickle American readers. My biggest complaint has to do with, of all things, the title! There is so little to do with Flatland itself, except for the handful of miniscenes where Vikki's family mopes about her absence, and Vikki so quickly begins to talk and act like a 3D human (seeming to know several obscure facts of human science and history) that there really was no need to set it in Flatland to begin with. One might be confused at times as to whether *Flatterland* is supposed to be a sequel to Abbott's book or to Lewis Carroll's *Through the Looking Glass*, and in fact I think it would have been better to make Vikki a descendant of Alice rather than of Albert.

Acknowledgments

References

[1] NELSON BOND, The monster from nowhere, *As Tomorrow Becomes Today* (Charles W. Sullivan, ed.), New York: Prentice-Hall, 1974.
[2] CLIFFORD PICKOVER, *Surfing through Hyperspace: Understanding Higher Universes in Six Easy Lessons*, Oxford University Press, 1999.