

Mathematics of Computation

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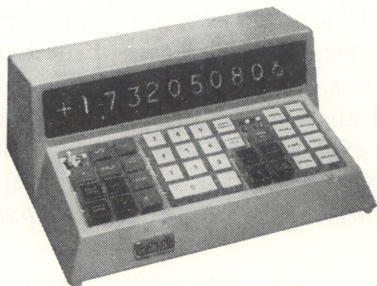
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Index of Government-Sponsored Computer Projects

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As stored on computer tape, the index will be available for such representative machine searches as the listing of all projects for which a particular investigator has responsibility, or the determination of the amount of money spent by a certain agency on all projects in the computer sciences. Also, the index offers a large corpus for experiments in machine indexing.

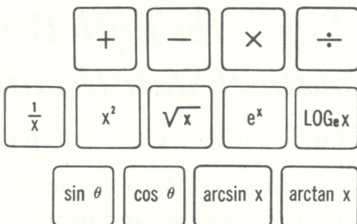
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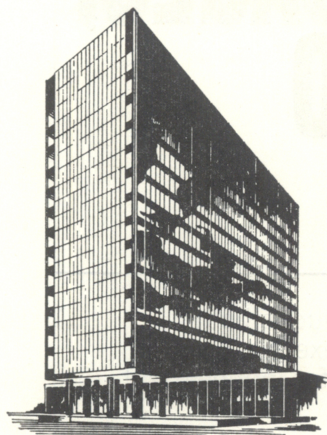
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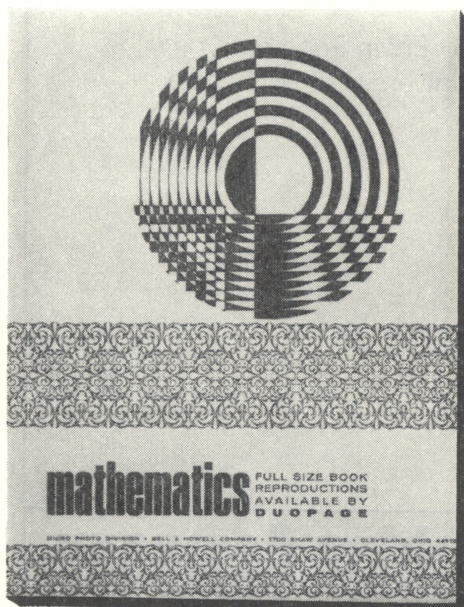
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