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Notices

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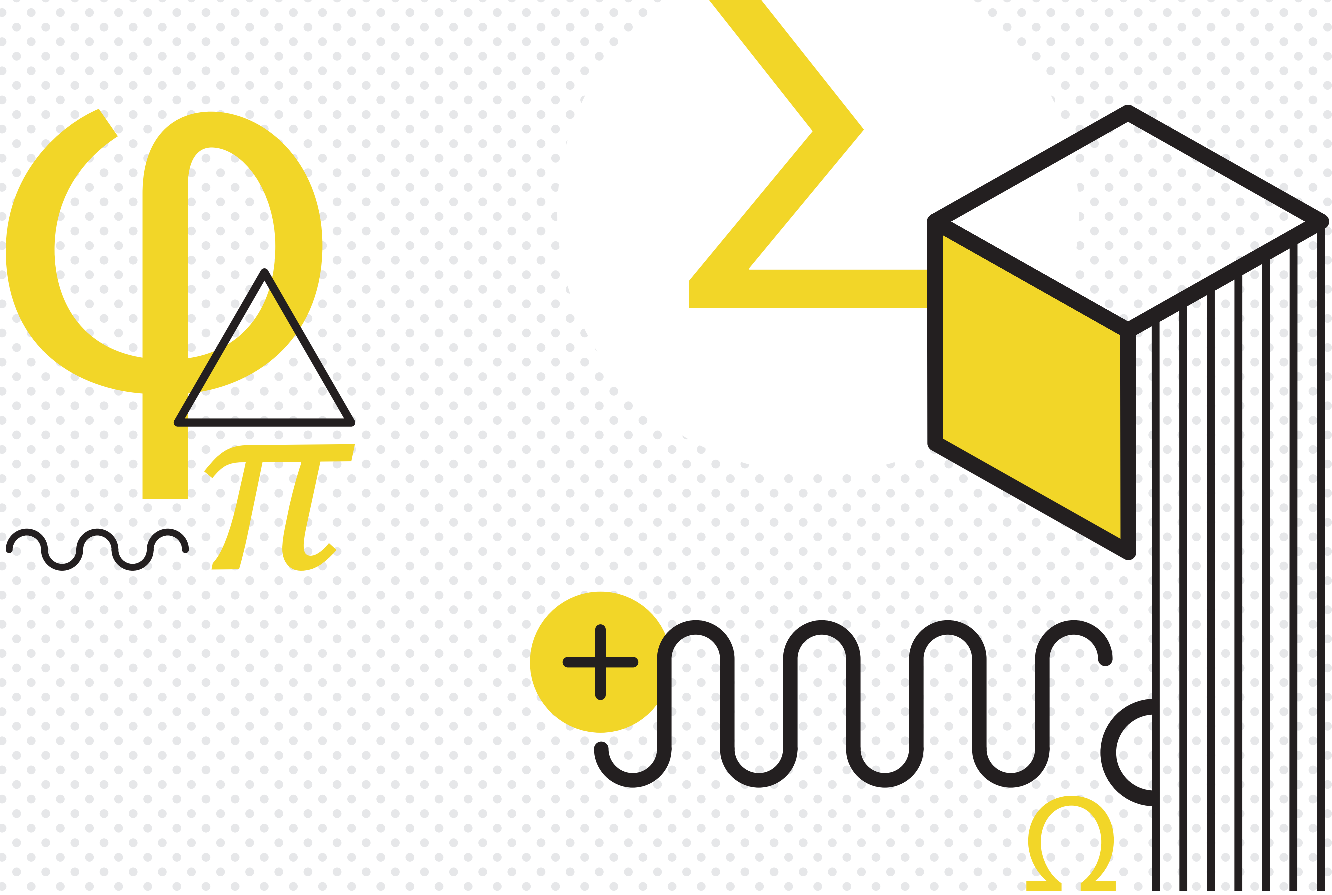

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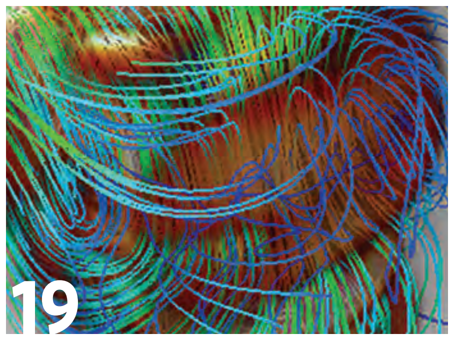

All of us, wherever we are, can celebrate together here in this issue of *Notices* the San Diego Joint Mathematics Meetings. Our lecture sampler includes for the first time the AMS-MAA-SIAM Hrabowski-Gates-Tapia-McBay Lecture, this year by Talithia Williams on the new PBS series *NOVA Wonders*. After the sampler, other articles describe modeling the heart, Dürer's unfolding problem (which remains open), gerrymandering after the fall Supreme Court decision, a story for Congress about how geometry has advanced MRI, "My Father André Weil" (2018 is the 20th anniversary of his death), and a profile on Donald Knuth and native script by former *Notices* Senior Writer and Deputy Editor Allyn Jackson. It is with some sadness and much appreciation that I report that Allyn, after over 30 years at the AMS, is moving on to other activities. The editors have found it an honor and privilege to work with her and wish her every happiness in whatever lies ahead; don't miss the appreciation on page 58. —***Frank Morgan, Editor-in-Chief***

ALSO IN THIS ISSUE

COMMENTARY

Notices
of the American Mathematical Society

FROM THE AMS SECRETARY

CURRENT INVITATIONS FROM THE AMS

IN EVERY ISSUE

Corrigendum: In the Journals Backlog Report from the November 2017 issue of *Notices*, the incorrect median time from submission to final acceptance was reported for *Geometriae Dedicata*. The correct time (in months) is 4.4.

CALL FOR NOMINATIONS

D. R. FULKERSON PRIZE

The Fulkerson Prize Committee invites nominations for the Delbert Ray Fulkerson Prize, sponsored jointly by the Mathematical Optimization Society (MOS) and the American Mathematical Society. Up to three awards of $1,500 each are presented at each (triennial) International Symposium of the MOS. The Fulkerson Prize is for outstanding papers in the area of discrete mathematics. The prize will be awarded at the 23rd International Symposium on Mathematical Programming to be held in Bordeaux, France on July 1–6, 2018.

Eligible papers should represent the final publication of the main result(s) and should have been published in a recognized journal or in a comparable, well-refereed volume intended to publish final publications only, during the six calendar years preceding the year of the Symposium (thus, from January 2012 through December 2017). The prizes will be given for single papers, not series of papers or books, and in the event of joint authorship the prize will be divided.

The term “discrete mathematics” is interpreted broadly and is intended to include graph theory, networks, mathematical programming, applied combinatorics, applications of discrete mathematics to computer science, and related subjects. While research work in these areas is usually not far removed from practical applications, the judging of papers will only be based on their mathematical quality and significance. Further information about the Fulkerson Prize can be found at **www.mathopt.org/?nav=fulkerson** and at **www.ams.org/prizes/fulkerson-prize.html**.

The Fulkerson Prize Committee consists of Maria Chudnovsky (Princeton University), Friedrich Eisenbrand (EPFL), Chair, and Martin Grötschel (Berlin-Brandenburgische Akademie der Wissenschaften). Please send your nominations (including reference to the nominated article and an evaluation of the work) by **February 15, 2018**, to the chair of the committee. Electronic submissions to Professor Eisenbrand at **Friedrich.Eisenbrand@epfl.ch** are preferred.

Friedrich Eisenbrand
EPFL, Station 8
CH-1015 Lausanne
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Friedrich.Eisenbrand@epfl.ch

Notices
of the American Mathematical Society

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EVENTS THAT YOU WON'T WANT TO MISS:

WEDNESDAY, JANUARY 10

AMS Logo Reveal • **12:30 pm**

Learn more about the future of the AMS brand as we unveil our new logo, and enjoy some light refreshments.

MAA Press Announcement • **12:45 pm**

The high-quality mathematics titles and textbooks of the MAA Press will now be published as an imprint of the AMS Book Program. Join us to celebrate and learn more about this exciting news.

MathSciNet® Demonstration • **2:15 pm**

Enter to win a $100 American Express gift card!
As a thank you for attending JMM, you will receive free access to MathSciNet for the duration of the meeting.

Grand Opening Raffle • **Winner drawn at 5:00 pm**

THURSDAY, JANUARY 11

Professional Photographer for AMS Members • **9:30 am–12:30 pm**

Schedule your appointment at: amermathsoc.simplybook.me

MathSciNet® Demonstration • **2:15 pm**

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While you're in the booth, make sure to update your MathSciNet author profile.

FRIDAY, JANUARY 12

MathSciNet® Demonstrations • **2:15 pm**

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CHECK THE JMM DAILY NEWSLETTER FOR DATE/TIME OF THE FOLLOWING:

AMS Prize-Winning Authors Meet & Greet

Meet this year's prize-winning authors and enjoy some light snacks.

AMS American Mathematical Society

JMM 2018 LECTURE SAMPLER

From left to right: Talithia Williams, Gunnar Carlsson, Jill C. Pipher, Ruth Charney, Federico Ardila, Erica Walker, Dana Randall, André Neves, Ronald E. Mickens.

Some of the Joint Mathematics Meetings invited speakers have kindly provided these introductions to their lectures in order to entice meeting attendees and to include non attendees in the excitement.

—Frank Morgan

DOI: http://dx.doi.org/10.1090/noti1614

Talithia Williams

Williams is a host of PBS's new *NOVA Wonders* series, premiering this spring, Tuesdays at 10 pm.

Mathematics for the Masses (AMS-MAA-SIAM Hrabowski-Gates-Tapia-McBay Lecture)

In recent months, we've witnessed Americans grapple with the significance of science, technology, engineering, and mathematics (STEM) through events ranging from the Paris Agreement to the nationwide March for Science, where people marched to defend the role of science in society. In the wake of a renewed excitement for STEM, I'm thrilled to be joining the Public Broadcasting System (PBS) family as one of the hosts of a new *NOVA* series called *NOVA Wonders*, premiering this spring. *NOVA* is the most-watched prime-time science series on television, reaching an average of five million viewers weekly.

NOVA Wonders is a six-part series that will journey to the frontiers of science, where researchers are tackling some of the most intriguing questions about life and the cosmos. My goal in hosting the show is to open individuals to the power of mathematics and data to pursue answers to questions in a clear and purposeful way. Each hour explores a different big question, from the mysteries of astrophysics to the challenges of inventing technologies that could rival the abilities of the human mind. During this talk, I plan to share with you an early clip from *NOVA Wonders*, which I have the pleasure of hosting along with neurobiologist André Fenton and computer scientist Rana El-Kaliouby. I'll also discuss ways that we can take our mathematics to the masses and share techniques that have been successful in my environment. We all have a responsibility to inspire a new generation in STEM and nurture the dreams of future mathematical leaders.

Talithia Williams is associate dean for Research and Experiential Learning and associate professor of mathematics at Harvey Mudd College. Her email address is twilliams@hmc.edu.

DOI: http://dx.doi.org/10.1090/noti1616

Image Credits

Photo from *NOVA Wonders* ©2017 WGBH.
Author headshot ©Harvey Mudd College.

ABOUT THE AUTHOR

Talithia Williams researches the spatial and temporal structure of data and environmental applications. She has partnered with the World Health Organization on a cataract surgical rate model for Africa. Her TED talk, "Own Your Body's Data," has over a million views. She is a guest *Notices* editor for the upcoming February issue for Black History Month.

Talithia Williams

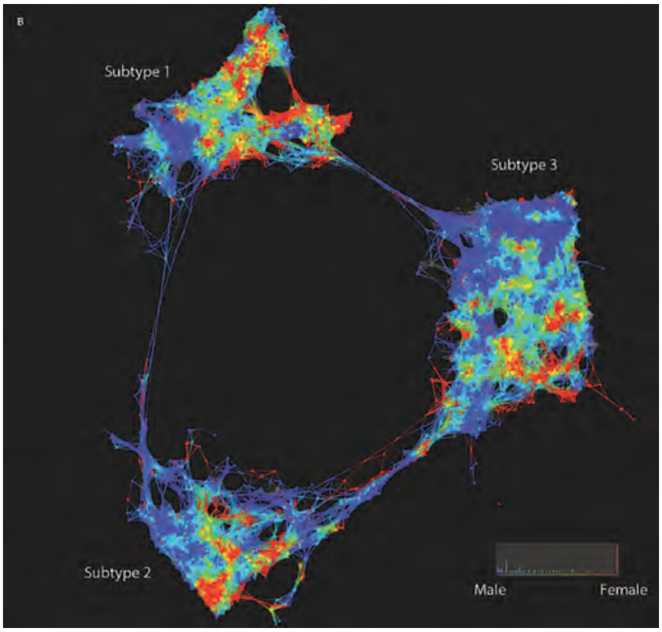

Figure 1. A network model for diabetics shows three distinct types, one heavily correlated with cancer.

Gunnar Carlsson

Topological Modeling of Complex Data

In the talk, we will be discussing the modeling of complex data sets by networks. Here are a couple of examples of applications of this idea. The first was a model for a data set of type 2 diabetics constructed by a group at the Mt. Sinai School of Medicine. The data set included genomic data as well as information from electronic medical records. Figure 1 shows a layout of the network that was obtained.

You can see that the network contains three large groups connected by "thin wires." The conclusion the Mt. Sinai researchers were able to draw from this is that type 2 diabetes, rather than being a single disease, is actually made up of three distinct types, and they found that one of these groups was heavily correlated with cancer. The finding that there are three distinct diseases will clearly have implications for treatment of the disease and constitute a contribution to "precision medicine."

A second example comes from the laboratory of David Schneider, a microbiologist at Stanford University. He studies the progression of infectious diseases. He has constructed a number of data sets using both physiological as well as genomic measurements on the subjects. Figure 2 shows an image of a collection of network models he constructed for mice and humans infected by flu and malaria.

You will notice that the models are both loops. This reflects the fact that they represent a phenomenon that begins at the healthy state, proceeds through gradual development of the disease until the immune response becomes strong, and then returns to the healthy state.

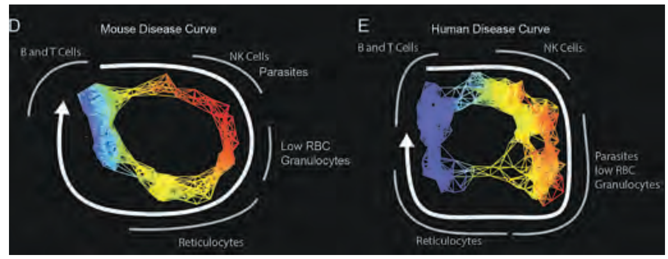

Figure 2. Mouse and human disease network models each form a loop through disease and recovery.

These models provide a time-independent model for the actual state of the subject. It is important to have such a model since (a) the progression of disease occurs at different rates for different subjects and (b) we don't generally have knowledge of the time the infection occurred. This should also be viewed as a contribution to precision medicine.

In the lecture we will discuss these and numerous other examples.

Image Credits

Figure 1 courtesy of "From identification of type 2 diabetes subgroups through topological analysis of patient similarity," L. Li, W-Y. Cheng , B. Glicksberg, O. Gottesman, R. Tamler, R. Chen, E. Bottinger, and J. Dudley *Science Translational Medicine* 28 Oct 2015: Vol. 7, Issue 311, pp. 311ra174. DOI:10.1126/scitranslmed.aaa9364.

Figure 2 courtesy of From Tracking Resilience to Infections by Mapping Disease Space, B.Y. Torres, J.H.M. Oliveira, Tate A. Thomas, P. Rath, K. Cumnock, and D.S. Schneider, *PLoS Biol.* **14**(4): e1002436. DOI:10.1371/journal.pbio.1002436.

Author photo courtesy of Gunnar Carlsson.

Gunnar Carlsson

ABOUT THE AUTHOR

Gunnar Carlsson spent most of his career working in the pure aspects of topology, but has turned to applications of the subject in the last fifteen years.

Gunnar Carlsson is professor of mathematics at Stanford University. His email address is `carlsson@stanford.edu`.

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Jill C. Pipher

Nonsmooth Boundary Value Problems (AWM–AMS Noether Lecture)

The regularity properties of solutions to linear PDEs in domains in $\mathbb{R}^n$ depend on the structure of the equation, the degree of smoothness of the coefficients of the equation, and of the boundary of the domain. Quantifying this dependence is a classical problem, and modern techniques can answer some of these questions with remarkable precision. For both physical and theoretical reasons, it is important to consider partial differential equations with nonsmooth coefficients. We'll discuss how some classical tools in harmonic and complex analysis have played a central role in answering these questions.

In two papers, in 1958 and in 1962, L. Carleson gave a new solution to the problem of interpolation of bounded analytic functions and solved a related problem about the spectrum of bounded holomorphic functions in the disk (the "corona" theorem). The interpolation problem is about finding a bounded analytic function that takes on prescribed values w_j at prescribed locations z_j. Carleson's dual formulation of the interpolation problem led him to the following question:

Let $d\mu$ be a measure in the unit disk. Suppose G is an analytic function in the unit disk with the norm, for $p \geq 1$,

$$\|G\|_p = \lim_{r\to 1}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}|G(re^{i\theta}|^p d\theta\right)^{1/p}.$$

Then, under what conditions on $d\mu$ do we have the following inequality:

$$\int |G(z)|^p d\mu(z) \leq C\|G\|_p^p$$

Carleson's 1958 paper answered this question for certain point masses, and his 1962 paper gave the general solution. Measures with this property became known as *Carleson measures* and found their way into many different areas of analysis, including operator theory, the Caldéron-Zygmund theory of singularity integrals, and geometric measure theory, to name a few. The main thrust of this lecture is the crucial role such measures have played in the development of the theory of elliptic and parabolic equations.

Indeed, in exactly the same timeframe of 1958–1962, there were some groundbreaking developments in the theory of divergence form elliptic and parabolic equations, known as the De Giorgi–Nash–Moser theory. The objects of study here are solutions to elliptic equations or parabolic equations with nonsmooth coefficients, broad generalizations of two classical operators, Laplace's equation and the heat equation. Nash's 1958 paper begins by describing his motivation for the consideration of nonsmooth coefficients. He wanted to understand the behavior of solutions to the nonlinear parabolic equations of flow for a heat-conducting fluid. De Giorgi was led to the same problem of regularity of solutions to elliptic equations with nonsmoooth coefficients in an effort to resolve Hilbert's nineteenth problem, which asks if "the solutions of regular problems in the calculus of variations [are] always necessarily analytic."

The elliptic equations considered by De Giorgi and Nash are in divergence form: $L = \operatorname{div} A(x)\nabla$, where the matrix $A(x)$ has bounded measurable coefficients $a_{i,j}(x)$ and is strongly elliptic. The ellipticity condition means that the spectrum of eigenvalues of $A(x)$ lies between two fixed positive parameters. Observe that when A is the identity matrix, L is just the classical Laplacian, whose solutions are the harmonic functions.

What does it mean to have a solution to such an equation, $\operatorname{div} A(x)\nabla u = 0$, when we can't differentiate the coefficients $a_{i,j}(x)$? Putting this aside for the moment, we can instead assume that the coefficients $a_{i,j}(x)$ are smooth and ask what estimates on solutions can be obtained that are *independent of any quantitative measure of that smoothness*. De Giorgi and Nash proved independently that solutions to such divergence form elliptic equations possess a degree of continuity in the interior of the domain that can be measured in terms of the ellipticity parameters alone. This degree (Hölder) of continuity is best possible in such generality. This contrasts sharply with harmonic functions, which are real analytic by virtue of solving Laplace's equation.

In 1961 Moser took a different approach to proving continuity of solutions via a powerful iterative technique. His proof relied on a property of functions of *bounded mean oscillation* (BMO) that had just been discovered by John and Nirenberg. (In a striking coincidence, their seminal paper on this subject was published in the same issue of the CPAM journal as Moser's paper.) In Moser's use of the John-Nirenberg theory of BMO functions, one finds a surprising connection to Carleson's measures, for it turns out, as C. Fefferman showed ten years later, that BMO functions and Carleson measures are intimately related: the harmonic extension of a bounded or BMO function on $\mathbb{R}^{n-1}$ gives rise to an expression that is a Carleson measure in the upper half-space $\mathbb{R}^n_+$. This discovery about harmonic functions has found far-reaching extensions in the elliptic theory.

I hope to give the flavor of some results and open problems in this subject at the interface of harmonic analysis and PDE and describe some joint work in recent years with M. Dindos, S. Hofmann, C. Kenig, S. Mayboroda, S. Petermichl, D. Rule, T. Toro, and others.

Jill C. Pipher is Elisha Benjamin Andrews Professor of Mathematics, Brown University. Her email address is `Jill_Pipher@brown.edu`.

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Image Credit

Author photo courtesy of Brown University.

Jill C. Pipher

ABOUT THE AUTHOR

Jill C. Pipher was formerly president of the Association for Women in Mathematics and is founding director of the Institute for Computational and Experimental Research in Mathematics. She is currently the American Mathematical Society president-elect.

Ruth Charney

Searching for Hyperbolicity

As students, we first encounter groups as algebraic objects, but groups can also be viewed as symmetries of geometric objects. This viewpoint gives rise to powerful tools for studying infinite groups. The work of Max Dehn in the early 1900s on groups acting on the hyperbolic plane was an early indication of this phenomenon. Dehn's ideas were vastly generalized in the 1980s by Cannon and Gromov to a large class of groups, known as Gromov hyperbolic groups. In recent years there has been an effort to push these ideas even further. If a group fails to be Gromov hyperbolic, might it still display some hyperbolic behavior? Might some of the techniques used in hyperbolic geometry still apply?

This talk will begin with an introduction to some basic ideas in geometric group theory, including Gromov's notion of hyperbolicity, and conclude with a discussion of recent work on finding and encoding hyperbolic behavior in more general groups. A version of this talk was given at the AWM Research Symposium 2017, and a more extended abstract can be found in the "Lecture Sampler" in the April 2017 *Notices*.[1]

Image Credit

Author photo courtesy of Michael Lovett.

Ruth Charney

ABOUT THE AUTHOR

Ruth Charney's research focuses on the interplay between groups and geometry. She has also worked extensively with professional organizations, serving as vice-president and trustee of AMS and as president of AWM.

Ruth Charney is Theodore and Evelyn Berenson Professor of Mathematics at Brandeis University. Her email address is charney@brandeis.edu.

[1] www.ams.org/publications/journals/notices/201704/rnoti-p341.pdf

DOI: http://dx.doi.org/10.1090/noti1623

Federico Ardila

Algebraic Structures on Polytopes

AUTHOR'S NOTE. My talk will discuss the algebraic and combinatorial structure of a beautiful family of polytopes. It is based on joint work with Marcelo Aguiar. It will be accessible to undergraduates and will not assume any previous knowledge of these topics.

Two Classical Problems: Inverting Power Series

Multiplicative Inversion

Let us consider two power series $A(x) = 1 + \sum_{n\geq 1} a_n \frac{x^n}{n!}$ and $B(x) = 1 + \sum_{n\geq 1} b_n \frac{x^n}{n!}$ that are multiplicative inverses; i.e., $A(x)B(x) = 1$. The first few coefficients of $B(x) = 1/A(x)$ are

$$\begin{aligned} b_1 &= -a_1, \\ b_2 &= -a_2 + 2a_1^2, \\ b_3 &= -a_3 + 6a_2a_1 - 6a_1^3, \\ b_4 &= -a_4 + 8a_3a_1 + 6a_2^2 - 36a_2a_1^2 + 24a_1^4. \end{aligned}$$

Compositional Inversion

Consider two power series $C(x) = x + \sum_{n\geq 2} c_{n-1}x^n$, and $D(x) = x + \sum_{n\geq 2} d_{n-1}x^n$ that are compositional inverses; i.e., $C(D(x)) = x$. The first few coefficients of $D(x) = C(x)^{\langle -1 \rangle}$ are

$$\begin{aligned} d_1 &= -c_1, \\ d_2 &= -c_2 + 2c_1^2, \\ d_3 &= -c_3 + 5c_2c_1 - 5c_1^3, \\ d_4 &= -c_4 + 6c_3c_1 + 3c_2^2 - 21c_2c_1^2 + 14c_1^4. \end{aligned}$$

It is natural to ask: What do these coefficients count? Two families of polytopes hold the answers.

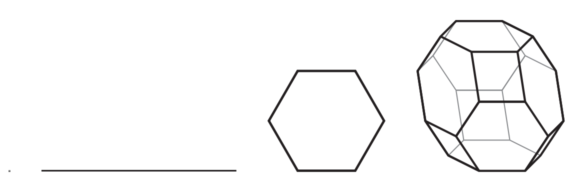

Figure 1. The permutahedra $\pi_1, \pi_2, \pi_3, \pi_4$ whose face structures tell us how to compute the multiplicative inverse of a series.

Federico Ardila is professor of mathematics, San Francisco State University, and Simons Professor, Mathematical Sciences Research Institute. His email address is `federico@sfsu.edu`.

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The *permutahedron* π_n (see Figure 1) is the polytope in $\mathbb{R}^n$ whose vertices are the $n!$ permutations of $\{1, 2, \dots, n\}$, regarded as vectors. Every face of a permutahedron is a product of permutahedra of lower dimensions.

The face structure of permutahedra tells us how to compute $B(x) = 1/A(x)$. For example, the formula for b_4 shown above is determined by the faces of π_4:

- 1 truncated octahedron π_4,
- 8 hexagons $\pi_3 \times \pi_1$ and 6 squares $\pi_2 \times \pi_2$,
- 36 segments $\pi_2 \times \pi_1 \times \pi_1$, and
- 24 points $\pi_1 \times \pi_1 \times \pi_1 \times \pi_1$.

The signs are given by the dimensions of the faces.

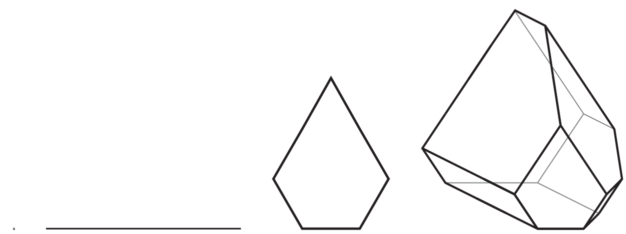

Figure 2. The associahedra $\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3, \mathfrak{a}_4$ whose face structures tell us how to compute the compositional inverse of a series.

The *associahedron* $\mathfrak{a}_n$ (see Figure 2) is a polytope in $\mathbb{R}^n$ whose vertices correspond to the $C_n = \frac{1}{n+1}\binom{2n}{n}$ associations of a product $x_1 \cdots x_n$ into binary products. For example, when $n = 5$, one such association is $(x_1x_2)((x_3x_4)x_5)$. Every face of an associahedron is a product of associahedra of smaller dimensions.

The face structure of permutahedra tells us how to compute $D(x) = C(x)^{\langle -1 \rangle}$. For instance, the formula for d_4 shown above comes from the faces of $\mathfrak{a}_4$:

- 1 three-dimensional associahedron $\mathfrak{a}_4$,
- 6 pentagons $\mathfrak{a}_3 \times \mathfrak{a}_1$ and 3 squares $\mathfrak{a}_2 \times \mathfrak{a}_2$,
- 21 segments $\mathfrak{a}_2 \times \mathfrak{a}_1 \times \mathfrak{a}_1$, and
- 14 points $\mathfrak{a}_1 \times \mathfrak{a}_1 \times \mathfrak{a}_1 \times \mathfrak{a}_1$.

Again, the signs come from the face dimensions.

We discovered these results unexpectedly when studying a more general family of polytopes.

Hopf Monoids and Generalized Permutahedra

Edmonds, Stanley, and others taught us that to study combinatorial objects, it is often helpful to build polyhedral models for them. *Generalized permutahedra* (or equivalently submodular functions) are a particularly useful family of polyhedra. They are deformations of the permutahedron, obtained by moving the faces while preserving their directions. Two important families are the permutahedra and associahedra, but there are many more. Figure 3 shows a few three-dimensional examples.

Joni and Rota, Joyal, Stanley, and others showed that to study combinatorial objects, it is often helpful to endow them with algebraic structures. In their book *Monoidal Functors, Species and Hopf Algebras* (2010), Aguiar and Mahajan provide a particularly useful framework: *Hopf*

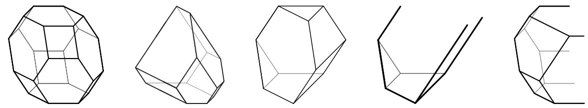

Figure 3. Five generalized permutahedra.

monoids in species. This framework applies to families of combinatorial objects having natural operations of *merging* two disjoint objects into one and *breaking* an object into two disjoint parts, subject to various axioms.

Many families of combinatorial objects carry such polyhedral and algebraic structures: graphs, matroids, posets, set partitions, and simplicial complexes, to name just a few. The main idea of this project is to bring together these two points of view:

Theorem 1. *Generalized permutahedra form a Hopf monoid in species* GP. *In fact, they are the universal family of polyhedra with this algebraic structure. The Hopf monoid* GP *contains or projects to the Hopf monoids* $\mathrm{G}, \mathrm{M}, \mathrm{P}, \Pi, \mathrm{SC}$ *of graphs, matroids, posets, set partitions, and simplicial complexes.*

Combinatorial Hopf Monoids and Antipodes

A key component of a Hopf monoid is its *antipode map* s, which is analogous to the inverse map in a group. The antipode is given by a very large alternating sum, generally involving lots of cancellation. A fundamental and often difficult question is to compute this antipode. Figure 4 shows the antipode of a graph and of a poset.

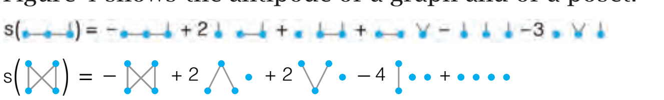

Figure 4. The antipode of a graph and of a poset.

These formulas result from simplifying alternating sums of 13 and 75 terms, respectively. How can we systematically explain the extensive cancellation that is taking place?

This is a rather subtle question. For instance, although most of the Hopf algebraic structures $\mathrm{G}, \mathrm{M}, \mathrm{P}, \Pi, \mathrm{SC}$ have been known for decades, optimal formulas for their antipodes were not known until very recently.

Fortunately, generalized permutahedra provide a geometric setting where the cancellation may be completely understood topologically. This allows us to obtain the optimal formula for the antipode of generalized permutahedra.

Theorem 2. *The antipode of the Hopf monoid* GP *is given by the following* ***cancellation-free*** *and* ***grouping-free*** *formula: If* $\wp$ *is a generalized permutahedron in* $\mathbb{R}^n$*, then*

$$\mathrm{s}(\wp) = \sum_{\mathfrak{q} \text{ face of } \wp} (-1)^{n-\dim \mathfrak{q}} \mathfrak{q}.$$

This automatically yields the optimal formulas for the antipodes of $\mathrm{G}, \mathrm{M}, \mathrm{P}, \Pi,$ *and* SC.

Inverting Formal Power Series, Revisited

Once we discovered these results, we asked: What happens when we apply the general theory of the Hopf monoid *GP* to the special families of polytopes $\pi_1, \pi_2, \pi_3, \ldots$ and $\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3, \ldots$? The answer was very surprising to me: Theorem 2 implies that the multiplicative and compositional inverses of power series are given by the alternating sum of the faces of permutahedra and associahedra, as explained above.

Image Credits

Figures courtesy of Federico Ardila.
Author photo courtesy of May-Li Khoe.

Federico Ardila

ABOUT THE AUTHOR

Federico Ardila works in combinatorics and its connections to other areas of mathematics and applications. He also strives to build joyful and empowering mathematical spaces. He has advised forty thesis students in the US and Colombia; among his US students, more than half are members of underrepresented groups and more than half are women. Outside of work, he is often playing fútbol or DJing with his wife, May-Li, and Colectivo La Pelanga.

Erica Walker

Hidden in Plain Sight: Mathematics Teaching and Learning through a Storytelling Lens

Our stories about mathematical excellence are too often hidden. The recent movie *Hidden Figures*, surrounding Katherine Johnson and her colleagues for their work at NASA, is the kind of story about mathematics that can inform us how to make mathematics learning experiences—in and out of school—more diverse, rewarding, engaging, transformative, rigorous, and commonplace. Such stories can help develop cadres of young people with strong and positive mathematics identities who are excited about mathematics; who see themselves and are seen as talented, knowledgeable doers and users of mathematics; and who are leaders of robust learning communities. Our work as mathematicians and educators must include uncovering the hidden and making known the unknown. How can we use narratives of excellence to address the many leaky junctures within the school mathematics pipeline: more negative attitudes towards math the longer students are in school; disparities in course-taking critical for college access and completion; fewer and fewer students interested in pursuing mathematics related careers? And how can we learn from the use of narratives in other disciplines to promote mathematics engagement and competence in the public sphere? My research suggests that there is much to be learned from the stories that mathematicians tell about the people and places that were critical to their mathematics development. And these stories can inspire teachers and students and generate public interest in mathematics and mathematicians. The narratives and themes shared in this talk will reveal successful strategies using hidden stories in the service of the human and mathematical development of all.

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Author photo courtesy of Bruce Gilbert.

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Erica Walker's research explores sociological and cultural factors influencing participation and performance in mathematics. She has published in many journals and is the author of two books, one of which is a study of mathematicians in the United States (*Beyond Banneker: Black Mathematicians and the Paths to Excellence*, published by SUNY Press). A museum explorer and inveterate stroller, she is fond of finding mathematics in everyday and unusual spaces.

Erica Walker

Dana Randall

Emergent Phenomena in Random Structures and Algorithms

Monte Carlo methods have become ubiquitous across science and engineering to model dynamics and explore large sets of configurations. The idea is to perform a random walk among the allowable configurations so that even though only a very small part of the space is visited, samples will be drawn from close to a desirable distribution, enabling statistical inferences. Over the last twenty-five years there have been tremendous advances in the design and analysis of Markov chains for this purpose, building on insights from physics, discrete probability, and theoretical computer science. The main objectives include designing provably efficient sampling algorithms, finding computational problems amenable to this approach, and developing new mathematical tools for bounding convergence times for these stochastic processes.

One of the striking discoveries has been the realization that many natural Markov chains undergo a *phase transition*, whereby they abruptly change from being efficient (and usable) to being inefficient (and thereby impractical for large problems) as some parameter of the system is varied. Figure 1 shows an example in which the particles align only above a critical density. Another example is the following chain that samples from the set of independent sets of a graph, known in the statistical physics community as the *hard-core lattice gas model.* We are given a graph G and a parameter $\lambda > 0$, known as the *activity* (or *fugacity*). The state space Ω is the set of independent sets of G and our goal is to sample from the *Gibbs* (or *Boltzmann*) *distribution*

$$\pi(I) = \lambda^{|I|}/Z,$$

where $I \in \Omega$ is an independent set, $|I|$ is its size, and $Z = \sum_{J\in\Omega} \lambda^{|J|}$ is the normalizing constant known as the *partition function*. When $\lambda > 1$ we are favoring dense independent sets, and when $\lambda < 1$ we are favoring sparse ones. Local dynamics can be defined so that we can move between pairs of configurations with Hamming distance one, i.e., independent sets that differ by the addition or deletion of a single vertex, or we can stay where we are. The celebrated Metropolis algorithm tells us the probabilities with which to implement these transitions so that iterating the moves for sufficiently long will generate samples from close to the target distribution π. Starting at any configuration $\sigma \in \Omega$, say the empty independent set (with no vertices), we repeat the following: choose a vertex v at random; if v is in the current independent set, remove it with probability $\min(1, \lambda^{-1})/2$; if it is not in the independent set, add it with probability $\min(1, \lambda)/2$ if doing so does not violate the independence requirement; in all other cases, keep the independent set unchanged. It is simple to show that this chain connects the state space and converges to the unique stationary distribution π, so our goal is to determine whether it converges quickly enough to be practical.

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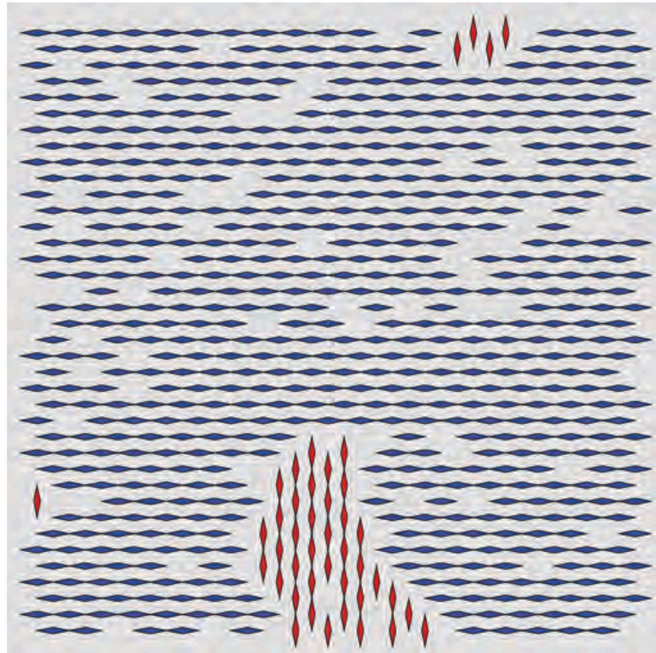

Figure 1. A discrete model for programmable active matter where particles align only above a critical high density.

It turns out that for small values of λ, the Metropolis chain on independent sets converges quickly to stationarity and provides an efficient way to sample, while for large values of λ it is prohibitively slow. To see why, imagine the underlying graph G is an $n \times n$ region of $\mathbb{Z}^2$. Dense independent sets dominate the stationary distribution π when λ is large, and it will take exponential time (in n) to move from an independent set that lies mostly on the odd sublattice to one that is mostly even.

This type of dichotomy is well known in the statistical physics community, where many models have been shown to abruptly transition from a disordered state to a predominantly ordered one, characterized by some *emergent phenomenon*. Physicists observe phase transitions when determining whether there is a unique limiting Gibbs measure on the infinite lattice, known as a *Gibbs state*. For the hard-core model on $\mathbb{Z}^2$, it is believed that there exists a critical value λ_c such that for $\lambda < \lambda_c$ there is a unique Gibbs state, while for $\lambda > \lambda_c$ there are multiple Gibbs

Figure 2. Randall working with former PhD students Amanda Pascoe Streib and Sarah Miracle.

states. This has been verified for small and large values of λ bounded away from the conjectured critical point $\lambda_c \approx 3.79$ in both the computational and the physics settings [1], [2].

We will explore how phase transitions in random structures and algorithms can provide valuable insights in three contexts. First, they allow us to understand the efficacy and limitations of certain classes of sampling algorithms, potentially leading to faster alternative approaches. Second, they reveal statistical properties of stationary distributions, giving insight into various interacting models from across the sciences. Examples include colloids or binary mixtures of molecules in suspension; segregation models, where individuals are more likely to move when they are dissatisfied with their local demographics; and interacting particle systems from statistical physics. Last, we can harness emergent phenomena due to phase transitions as a new algorithmic tool in order to coordinate behavior in various asynchronous distributed systems. Examples include models of programmable active matter and swarm robotics. We will see how these three research threads are closely interrelated, informing one another in surprising ways.

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Figure 1 courtesy of Shengkai Li, School of Physics, Georgia Tech.
Figure 2 courtesy of Georgia Tech College of Computing.
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Dana Randall

Dana Randall studies randomized algorithms at the interface between theoretical computer science, discrete probability, and statistical physics. Last year she became co-executive director of the Institute for Data Engineering and Science, an interdisciplinary research institute at Georgia Tech that she co-founded. She also serves as the ADVANCE Professor of Computing as an advocate for equity and inclusion.

André Neves

Minimal Surfaces, Volume Spectrum, and Morse Index

A recurrent question in mathematics consists in finding critical points for a given functional defined on some space. Typical questions ask to

(i) characterize the most stable critical points, i.e., those that for all but a small number of perturbations still persist;
(ii) count the number of critical points;
(iii) in case there are infinitely many critical points, study their asymptotic behavior as they become increasingly more unstable.

To give a concrete example, one can consider the space of all closed hypersurfaces in a Riemannian $n+1$-manifold M and the functional that computes the area of these hypersurfaces. Such spaces are well studied in geometric measure theory, a field pioneered by Federer and Fleming in the 1960s. The critical points in this case are called minimal surfaces and are ubiquitous not only in geometry but in may other branches of mathematics as well.

For this particular case, there have been new and exciting developments regarding the questions outlined above, namely:

(i) jointly with Fernando Marqués, the most stable minimal surfaces in the round 3-sphere were classified and used to solve the Willmore Conjecture;

(ii) infinitely many minimal surfaces Σ_k have been found for a large class of Riemannian manifolds [3], [5] (those with positive Ricci curvature), and their area plus their degrees of instability have been shown to be increasingly large [4];

(iii) the area of these infinitely many minimal surfaces obey a Weyl asymptotic law [2]. More precisely, the sequence $\frac{\text{area}(\Sigma_k)}{k^{1/(n+1)}}$ converges, as k tends to infinity, to a universal constant (depending only on n) multiplied by $(\text{Vol}\, M)^{n/(n+1)}$.

We end this brief introduction by noting that the sequence $\text{area}(\Sigma_k)$ is called the volume spectrum, and a better understanding of its properties would provide answers to other open problems in geometry. For instance, Irie, Marqués, and myself [1], used the Weyl Law to show that for a generic metric, not only are there infinitely many minimal hypersurfaces, but they are dense. In particular this answered a long-standing conjecture of Yau.

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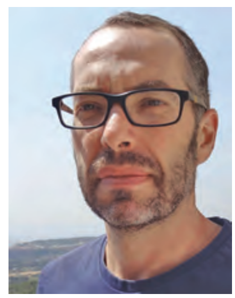

André Neves

ABOUT THE AUTHOR

André Neves is a geometer, whose goal is to understand which restrictions curvature imposes on space or on its submanifolds. With Fernando Marqués, he proved that the least bended torus in space is the Clifford torus and, in doing so, solved the Willmore conjecture.

Ronald E. Mickens

Nonstandard Finite Difference Schemes

Prelude

Nonstandard finite difference (NSFD) schemes[1] are an alternative methodology for constructing discretizations of differential equations for the purpose of calculating numerical solutions. This technique arose from earlier attempts to formulate for classical mechanics a structure based on discrete time rather than the assumption of a continuous time-independent variable [1]. This formalism is consistent with the fact that most differential equations are models for realizable physical phenomena, and the data on these systems are obtained at discrete times. However, since these differential equations cannot be solved in terms of a finite combination of elementary functions, there is a fundamental need to determine numerical approximations to their solutions.

Standard methods based on finite difference discretizations are generally based on mathematical considerations of consistency, convergence, and stability. These issues arise to help in the prevention of numerical instabilities (NI), i.e., the existence of solutions to the finite difference equations which do not correspond to any solutions of the differential equations.

The NSFD methodology is a "physical" based procedure, in the sense that it incorporates into the discretizations many of the important features of the differential equations and/or their solutions.

Exact Schemes

The genesis of our NSFD methodology is based on the realization of the existence of exact finite difference representations for certain classes of differential equations. To see this, consider the scalar ordinary differential equation

(1) $$\frac{dx}{dt} = f(x, t, \lambda), \quad x(t_0) = x_0,$$

where $f(\cdots)$ has properties such that a unique solution exists and λ denotes the set of parameters characterizing the system modeled. Let F denote this solution of Eq. (1), i.e.,

(2) $$x(t) = F(x_0, t_0, t, \lambda).$$

Now let the following be a finite difference discretization of Eq. (1) etc.:

(3) $$x_{k+1} = G(x_0, k, h, \lambda),$$

where $h = \Delta t$, $t_k = hk$, $x_k = x(t_k)$. The expression in Eq. (3) is an exact finite difference scheme if

(4) $$x_k = x(t_k), \quad h > 0.$$

For this to occur, we must have

(5) $$G(x_0, k, h, \lambda) = F(x_k, t_k, t_{k+1}, \lambda);$$

i.e., the exact finite difference scheme for Eq. (1) is

(6) $$x_{k+1} = F(x_k, t_k, t_{k+1}, \lambda).$$

Our work on the construction of the NSFD methodology was motivated by Eq. (6); i.e., if the general solution to Eq. (1) is known, then from it, its exact finite difference scheme can be constructed.

Examples of Exact Scheme

The following three elementary equations and their exact schemes clearly illustrate the nonstandard features of these constructions in comparison with the standard discretizations:

$$\frac{dx}{dt} = \lambda_1 x - \lambda_2 x^2 \rightarrow \frac{x_{k+1} - x_k}{(e^{\lambda_1 h} - 1)/\lambda_1} = \lambda_1 x_k - \lambda_2 x_{k+1} x_k,$$

$$\frac{d^2x}{dt^2} + \Omega^2 x = 0 \rightarrow \frac{x_{k+1} - 2x_k + x_{k-1}}{\left(\frac{4}{\Omega^2}\right)\left[\sin\left(\frac{\Omega h}{2}\right)\right]^2} + \Omega^2 x_k = 0,$$

$$\frac{d^2x}{dt^2} = \lambda\frac{dx}{dt} \rightarrow \frac{x_{k+1} - 2x_k + x_{k-1}}{(e^{\lambda h} - 1)h/\lambda} = \lambda\left(\frac{x_k - x_{k-1}}{h}\right).$$

NSFD Methodology

In the early 1970s we began to analytically derive exact discretization schemes for hundreds of both linear and nonlinear differential equations. From analyses of these results, the general rules for the NSFD methodology emerged. The two most significant rules are: (i) the requirement of more complex mathematical structures for the discrete representations of derivatives and (ii) the nonlocal representation of functions.

In more detail, we have

(7) $$\frac{dx}{dt} \rightarrow \frac{x_{k+1} - x_k}{\phi}, \quad \phi = h + O(h^2),$$

where ϕ is called a denominator function and, in general, is not the simple h used in most of the standard procedures. For particular classes of differential equations, there exists a procedure for calculating ϕ. Note that ϕ depends not only on the step-size, $h = \Delta t$, but also on one or more parameters appearing in the differential equations.

The nonlocal representation of functions means that these terms have, in general, discretizations spread over more than one grid point. The following are some

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[1] *unrelated to nonstandard analysis*

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examples of these possibilities:

(8a)

$$x = 2x - x \;\rightarrow\; 2x_k - x_{k+1},\ \text{1st-order ODE},$$

(8b)
$$x^2 \;\rightarrow\; \begin{cases} x_{k+1}x_k, & \text{1st-order ODE}, \\ \left(\frac{x_{k+1}+x_k+x_{k-1}}{3}\right)x_k, & \text{2nd-order ODE}, \end{cases}$$

(8c)
$$x^3 \;\rightarrow\; \left(\frac{x_{k+1}+x_{k-1}}{2}\right)x_k^2,\ \text{2nd-order ODE}.$$

The two major techniques to implement the NSFD methodology as applied to particular differential equations are (a) the principle of dynamic consistency (PDC) and (b) the method of partial equations (MPE).

PDC centers on incorporating some property satisfied by the differential equations and/or their solutions into the NSFD scheme. Such properties may include a positivity condition, special solutions such as fixed-points, particular asymptotic properties, etc.

MPE breaks a differential equation into three or more parts of subequations, determines the exact finite difference scheme for each subequation, and then combines them into an overall consistent, single discretization for the original differential equation.

Examples of NSFD Schemes

The initial value problem

(9)
$$\frac{dx}{dt} = -\lambda\sqrt{x}, \quad x(0) = x_0 > 0$$

can be discretized via the NSFD scheme

(10)
$$\frac{x_{k+1}-x_k}{h} = -\lambda\left(\frac{x_{k+1}}{\sqrt{x_k}}\right) \text{ or } \frac{x_{k+1}-x_k}{h} = -\lambda\left(\frac{\sqrt{x_{k+1}}+\sqrt{x_k}}{2}\right).$$

Similarly, the equation for a simple combustion model

(11)
$$u_t = u_{xx} + u^2(1-u), \quad 0 \le u(x,0) \le 1,$$

has an NSFD scheme

$$\frac{u_m^{k+1}-u_m^k}{\Delta t} = \left(u_{m+1}^k\right)^2 + \left(u_{m-1}^k\right)^2 - \left(\frac{u_{m+1}^k+u_{m-1}^k}{2}\right)u_m^{k+1} - \left[\frac{(u_{m+1}^k)^2+(u_{m-1}^k)^2}{2}\right]u_m^{k+1} + \frac{u_{m+1}^k - 2u_m^k + u_{m-1}^k}{(\Delta x)^2},$$

where $t_k = (\Delta t)k)$ $x_m = (\Delta x)m$, and $u_m^k \approx u(x_m, t_k)$.

Comments

The NSFD methodology has been applied to a broad range of differential equations modeling a diverse set of physical phenomena which include:

- interacting populations,
- nonlinear heat transport,
- impulsive systems,
- delay equations,
- equations having fractional derivatives,
- computational electromagnetics,
- singular perturbation problems, and
- simulation of robotic systems.

A review paper summarizing such applications is the recent publication by Patidar [3].

An important insight coming from an analysis of the NSFD methdology is that a proper or valid discretization of a differential equation involves a deep understanding of the whole equation. This is in opposition to standard discretization methods which are based on constructing discrete models of the individual terms appearing in the differential equation and then adding them together to form the final full discretization. One consequence of this fact is that black-box, "plug-in algorithms," which are implemented for standard numerical integration techniques, do not exist for the NSFD methodology.

We welcome you to come to our talk, where a more thorough and deeper discussion of the above issues will be given. This will also allow you to ask interesting and thoughtful questions related to the construction and use of the NSFD philosophy to discretize differential equations.

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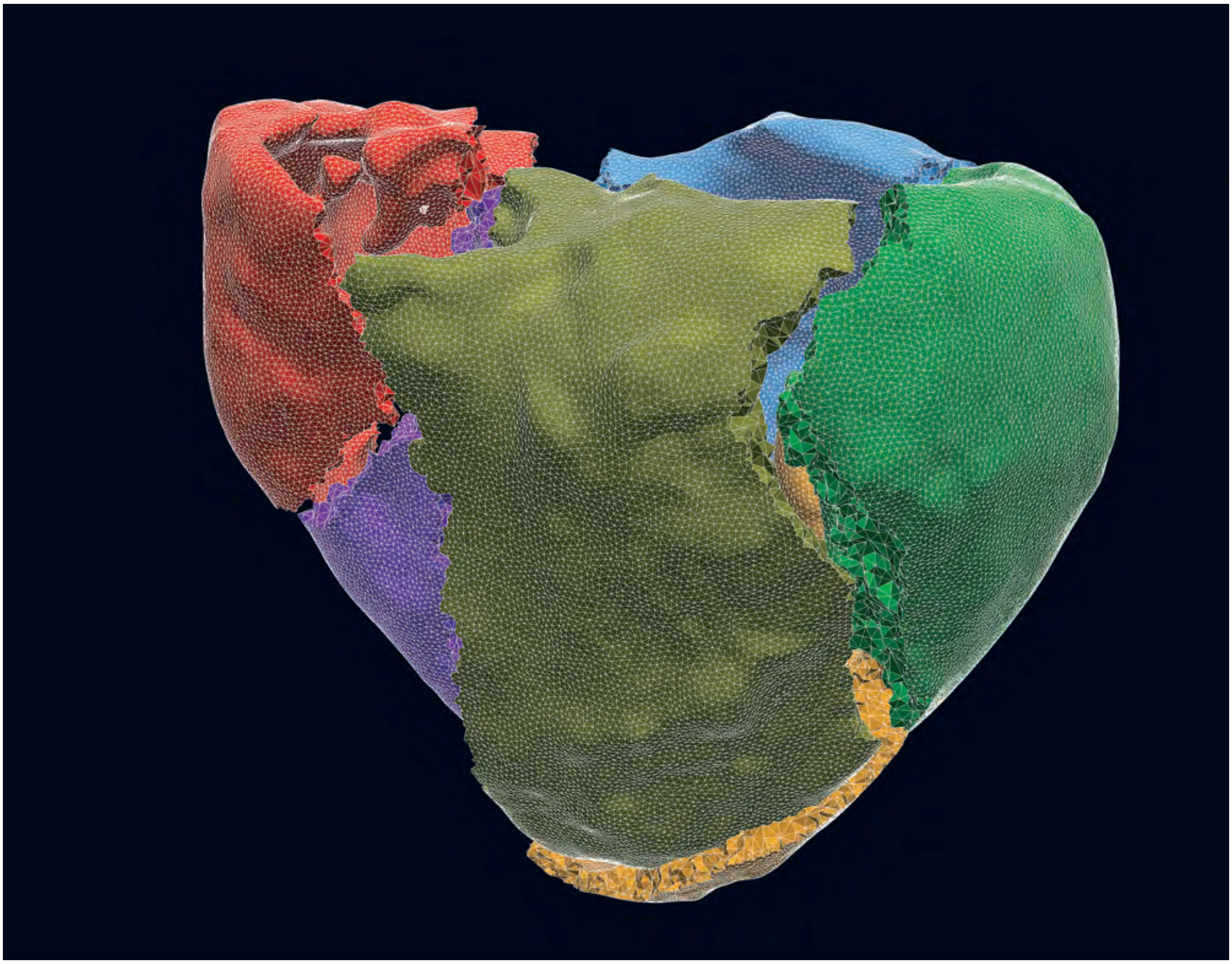
Ronald E. Mickens

ABOUT THE AUTHOR

Ronald E. Mickens's research interests include asymptotics of difference equations, numerical analysis, mathematical modeling of interacting populations, nonlinear oscillations, and the history and philosophy of science. He is also involved in a variety of mentoring activities for students and junior faculty.

Taking Mathematics to Heart



Alfio Quarteroni

Introduction

One day, a virtual version of your own heart may help doctors diagnose heart disease and determine the best treatment for you, without the need for unnecessary invasive clinical practices. The five-year ERC Advanced Grant iHEART (*an integrated heart model for the simulation of the cardiac function*) of the European Research Council has precisely the ambition to make a significant step forward in the direction of constructing a mathematical virtual heart. Its ultimate goal is to simulate heart function with increasing accuracy and to be personalized to your heart based on medical imaging (CT scans, MRI, etc.). This would hopefully help to prevent or treat cardiovascular disease by providing a personalized virtual heart to patients, essentially a detailed mathematical description of a patient's heart and how it functions—or malfunctions.

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The Human Heart

The human heart is an extraordinarily complex organ that pumps an estimated 180 million liters of blood in one's lifetime, which would fill more than seven Olympic-sized

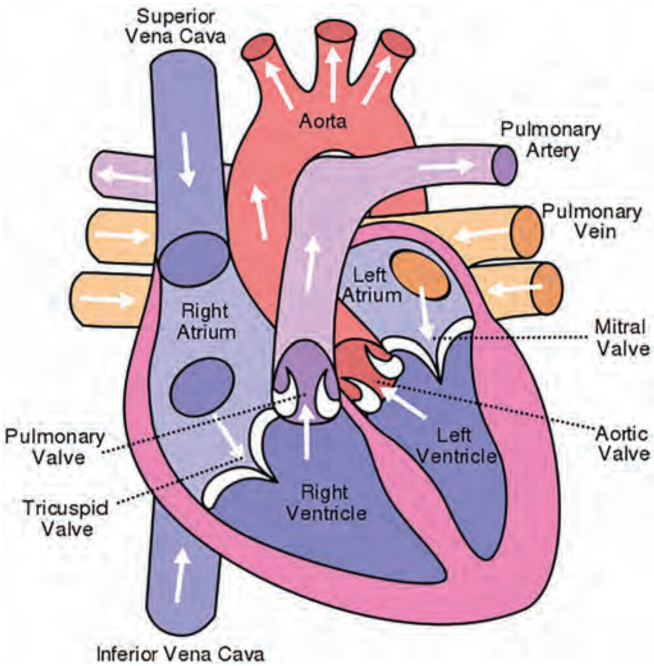

Figure 1. The heart has many working parts, but it is their synchronization that ensures the heartbeat efficiency.

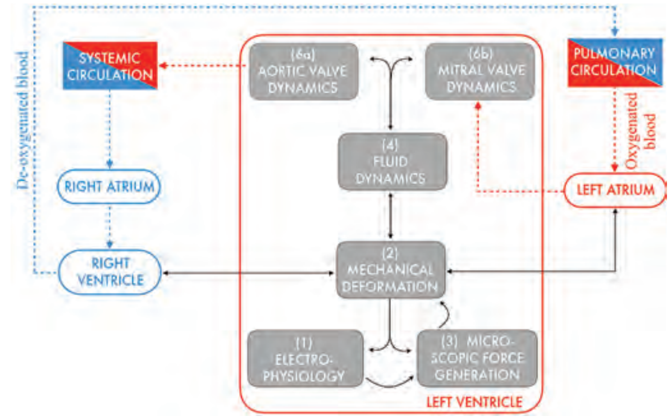

Figure 2. Sketch of the mathematical model with detail provided for the left ventricle. A mathematical model for the left ventricle requires several systems of differential equations. Each grey-shaded block represents a core cardiac model; the corresponding equations and coupling conditions are described in the text. Similar models describe the other cardiac chambers: their connection with the left ventricle is indicated by arrows; red and blue indicate oxygenated and de-oxygenated blood. Their ensemble identifies the integrated heart model (IHM).

swimming pools, thereby ensuring that oxygenated blood reaches the entire body.

Such an endeavor requires incredible strength and resistance: the work done by the heart to ensure every single heartbeat is comparable to that needed to tightly squeeze a tennis ball. As unreal as it appears, the heart is also the casket that guards our emotions (as already conjectured by the Greeks). The so-called "broken heart" carries some truth: after experiencing an emotionally traumatic event, our body releases stress hormones into the blood stream that can temporarily "shock" the heart and even mimic heart attack symptoms.

Cardiovascular problems may lead to malfunction, disease, or death. Heart disease causes 40 percent of deaths in the EU and costs an estimated 200 billion Euros (over 200 billion US dollars) a year, yet 80 percent of acquired heart diseases and stroke episodes are preventable. Mathematics can play a great role in improving the current available knowledge of heart function and malfunction.

Building a realistic virtual heart is becoming a reality.

The human heart (Figure 1) is made of two atria and two ventricles, a cardiac muscle (the myocardium, surrounded by two layers, the endocardium, and the pericardium), and four valves, which undergo electrical, fluid, and mechanical processes (functions). Modeling the heart cannot be limited to the mere (albeit tremendously difficult) construction of individual core models that describe the single cardiac function, as it is the beautiful harmony of their synchronization that ensures the heartbeat efficiency.

On the other side, every person's heart is unique. Correctly modeling the intricacies of each individual heart therefore requires a customizable mathematical description of both its geometry and its dynamics. But doing so in a mathematically sound way is no easy task; it requires patient-specific data and computational power to solve complex equations. Thanks to increasingly powerful computers, building a realistic virtual heart is becoming a reality.

An Integrated Heart Model (IHM)

The first *cardiac models* date back to the early 1970s with the seminal work of C. S. Peskin (Courant Institute) to simulate ventricular blood flow using the celebrated immersed boundary method [3]. Software development has also progressed quite rapidly in the last few years. As for today, the *UT heart simulator* from the group of T. Hisada (Tokyo University) [5] has provided perhaps the most comprehensive simulations of cardiac function.

Despite the scientific advances and impressive results achieved in the past forty years, a mathematically driven, *fully integrated model for the entire heart function* is still missing. The goal of iHEART is indeed the construction, mathematical investigation, numerical analysis, solution, data analysis, implementation into a comprehensive software library for large-scale scientific computing, model personalization, and validation on clinically relevant subject-specific cases, of an Integrated Heart Model.

Figure 3. Wave pattern generated by the solution of the electrophysiology problem on a two-dimensional muscle specimen affected by arrhythmia.

Modeling of the Core Cardiac Components

The individual *core cardiac models* are represented by the electrophysiology, the passive and active mechanics of the cardiac muscle, the microscopic force generation in sarcomeres (the basic contractile units of the cardiomyocytes), the blood flow in the heart chambers, and the valve dynamics. Figure 2 shows a sketch of the models for the left ventricle and the connections with the other three heart cavities and the external circulation. Electrophysiology is the process that drives the rhythm of the heart. In the cardiac tissue, the problem at the macroscopic level is typically described by the so-called *bidomain* model:

$$(1)\qquad \begin{cases} c_m \partial_t v + I_{ion}(v, \mathbf{w}, \mathbf{c}) - \nabla \cdot (D_i \nabla (v + u_e)) = 0 \\ -\nabla \cdot (D_i \nabla v) - \nabla \cdot ((D_i + D_e)\nabla u_e) = I^e_{app} \\ \dot{\mathbf{w}} = \mathbf{R_w}(v, \mathbf{w}, \mathbf{c}) \qquad \dot{\mathbf{c}} = \mathbf{R_c}(v, \mathbf{w}, \mathbf{c}) \end{cases}$$

This is a system of nonlinear parabolic-elliptic PDEs coupled with a system of ODEs representing the evolution of the *transmembrane potential* $v = u_i - u_e$ between the intra-u_i and extra-u_e cellular potentials, the intracellular *concentrations* $c \in \mathbb{R}^P$ of different ionic species (in particular calcium), and the *gating variables* w regulating transmembrane currents [1]. See Figure 3 for an example of complex wave patterns for the transmembrane potential.

Passive and active solid mechanics models provide the *displacement* d_S of the cardiac muscle under the action of external forces (exerted by blood flow and external tissues) and internal forces (triggered by the electrical activity:

$$(2)\qquad \begin{cases} \rho_S \partial_{tt} \mathbf{d}_S - \nabla \cdot \sigma_S(\mathbf{d}_S) = \mathbf{0} \\ \sigma_S(\mathbf{d}_S) = \sigma_S(\mathbf{F}(\mathbf{d}_S), \mathbf{F}_A(\gamma_f)) \end{cases}$$

where ρ_s denotes the density and σ_s denotes the stress tensor, which depends on deformation gradient tensors F in turn depending on d_S and γ_f, the *macroscopic active deformation* along the fibers direction caused by the electrical activity. The passive response of the heart muscle is that of a nearly-incompressible hyperelastic transversally isotropic material comprised of mutually orthogonal fibers and collagen sheets. The orthotropic Holzapfel-Ogden constitutive law accounts for stiffening effects along the directions of fibers and sheets [4]. To model internal microscopic forces, the so-called *active stress* and *active strain* approaches are used [4]. The former considers part of the stress tensor as the active component, while the latter expresses the total solid deformation as $F = F_E F_A$, where F_E is the deformation gradient of the passive elastic response and F_A *depends* on the active deformation γ_F.

icroscopic force generation is the complex mechanism behind the excitation-contraction coupling between electrophysiology and active deformation, and characterizes the evolution of γ_f in terms of intracellular calcium concentration (a component of c) and the muscle deformation $\mathbf{d}_S$, which reads:

$$(3)\qquad \dot{\gamma}_f = g_A(\gamma_f, \mathbf{c}, \mathbf{d}_S)$$

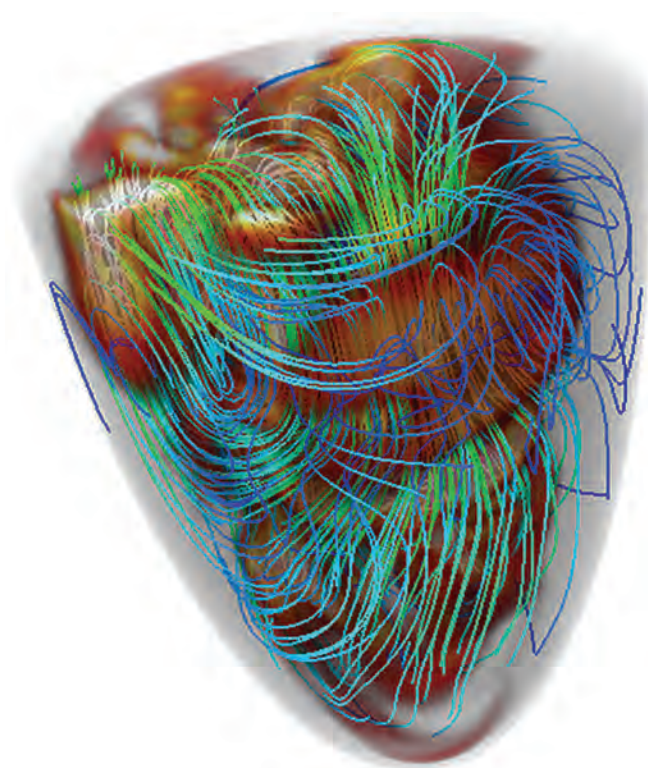

Figure 4. Velocity streamlines and vortex structures in a left ventricle.

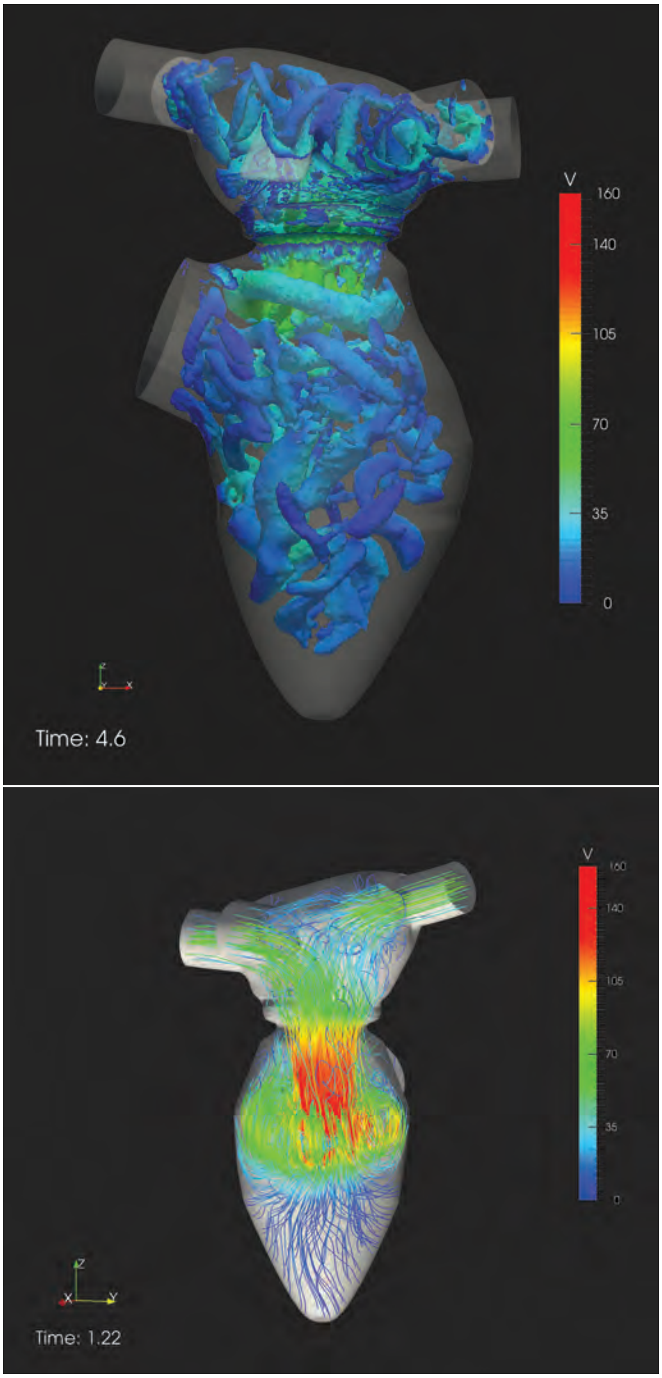

Figure 5. Computational fluid dynamics in the left heart: vortex structure and velocity streamlines in the left atrium and ventricle.

Blood flow dynamics in the heart is typically modeled by the incompressible Navier–Stokes equations. In normal conditions, the blood flow regime varies from laminar to transitional up to nearly turbulent during the heartbeat. Since each chamber undergoes large deformations, the Navier-Stokes equations can be set in an arbitrary Lagrangian-Eulerian framework. Boundary conditions must accommodate the interaction between blood flow and the surrounding tissue (the endocardium, including the valves), yielding a fluid-structure interaction (FSI) problem involving kinematic and dynamic fluid-solid interface conditions. See some numerical examples in Figures 4 and 5.

Valve dynamics is one of the most challenging components of heart modeling, due to their fast and complex motion. Valve response has often been represented by simple algebraic equations, ODEs in terms of leaflet displacement, 2D shells (involving the valve Cauchy stress), or as boundary conditions of the Navier–Stokes equations modeling blood flow. Moving toward a 3D valve description in the fluid domain, the immersed boundary

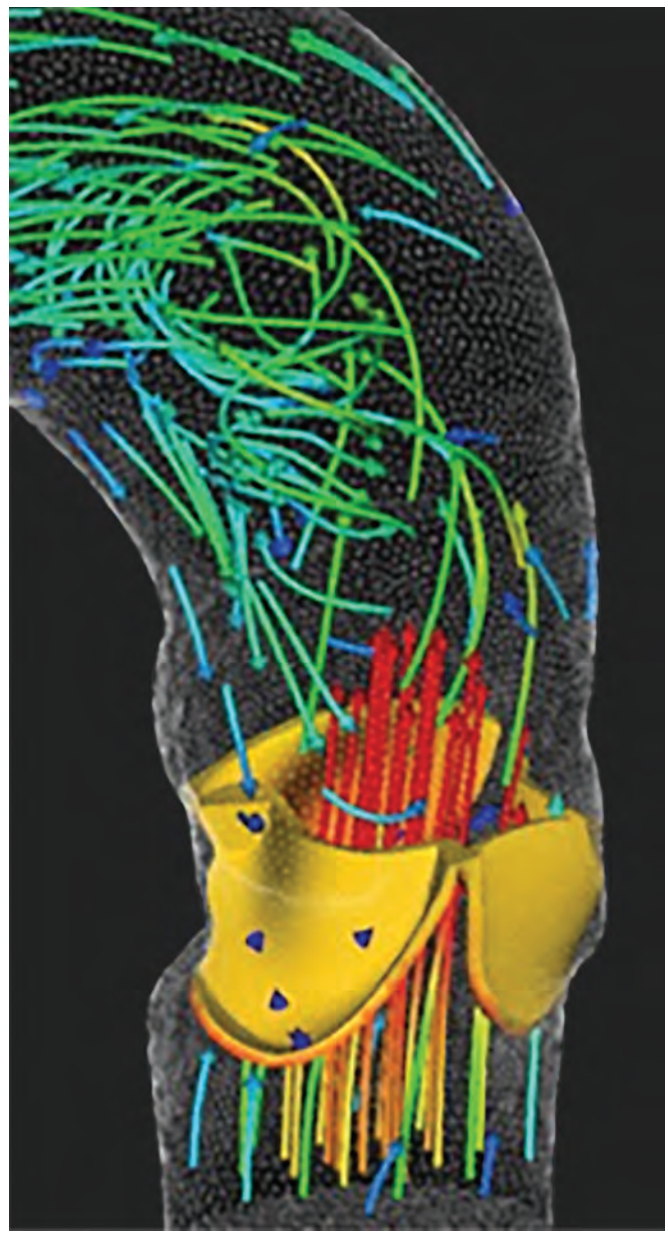

Figure 6. Velocity streamlines in the aortic root past the tricuspid aortic valve.

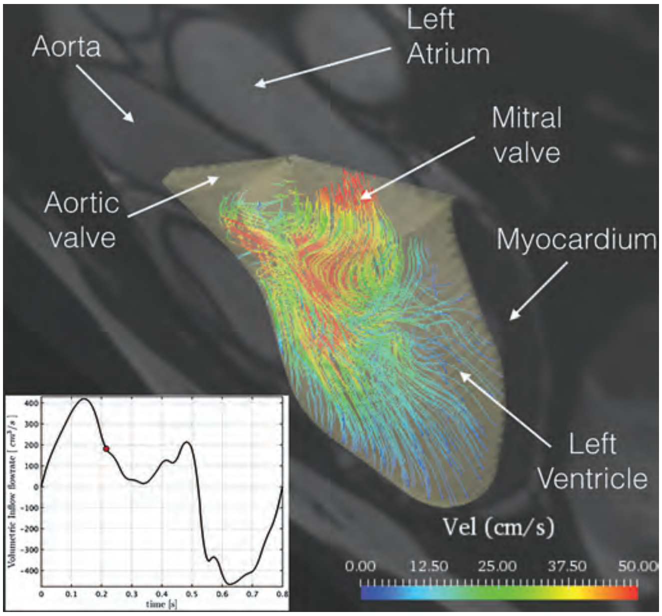

Figure 7. Flow simulation in a patient-specific left ventricle. Aortic and mitral valves are modeled through appropriate nonlinear adaptive Robin conditions.

method [3] and resistive methods [2], [4] introduce extra terms in the Navier–Stokes equations depending on Dirac masses concentrated on the valve leaflets. The position of the latter is determined by the solution of an additional mechanical problem, yielding an FSI problem. See Figures 6 and 7 for numerical simulations of blood flow in the presence of cardiac valves.

Numerical Approximation of the IHM

Regarding the numerical approximation of the IHM, properties like *stability, accuracy*, and *efficiency* of numerical methods approximating the core models are not automatically inherited by the IHM. This is even more true when using *nonconforming* (independent) approximations for different core components, as well as *geometric multiscale* strategies to accommodate for the coexistence of submodels featuring different spatial dimensions.

Coupled multiphysics problems can be approximated by either *monolithic* or *staggered* methods. With the former, all the equations are solved simultaneously; with the latter, they are solved separately with distinct solvers, often applied sequentially, a treatment that sometimes may indeed manifest time splitting errors and severe stability restrictions to the time step. Examples of numerical results obtained for the coupled electromechanics are shown in Figure 8.

Monolithic methods often employ Newton linearization, block preconditioners, and so-called Krylov iterations. The construction of efficient preconditioners is tremendously challenging and should ideally feature the properties of *optimality, scalability* with respect to the number of processors (for the parallel solution of large-scale problems as shown in Figure 9), and *robustness*

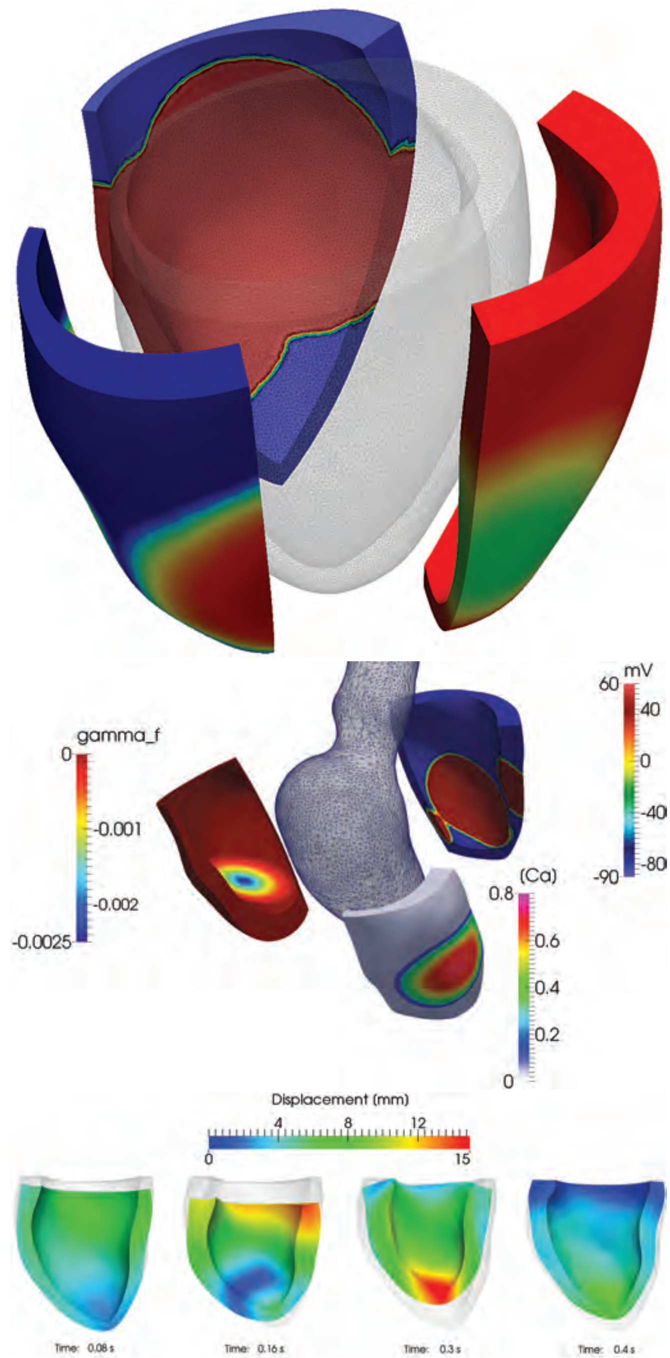

Figure 8. Numerical simulations for cardiac electromechanics in a patient-specific left ventricle; colors refer to the magnitude of the tissue displacement.

(stability with respect to the problem's parameters) in order to afford realistic and physically meaningful simulations.

Addressing Variability and Uncertainty

Patient-specific data and parameters are obtained by measurement and medical images such as magnetic resonance

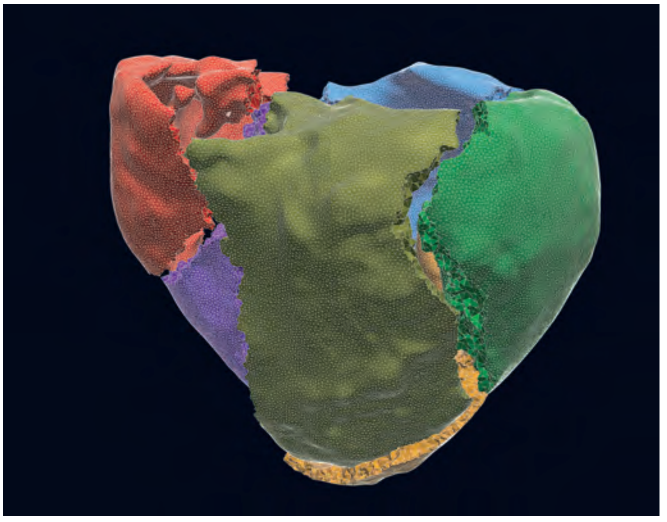

Figure 9. Multidomain partition of a 3D heart geometrical model for parallel computing. Different colors indicate different subdomains; the finite element mesh is overlaid.

imaging (MRI), computed tomography (CT), ultrasound, angiography, or Doppler echocardiography. They provide information on tissue and blood properties, pressure and flow rates determining boundary conditions, and, primarily, the shape of the heart and its chambers after image enhancement and segmentation. Data and parameters are inputs of the core cardiac models and yield *parametrized* systems of *PDEs*, whose numerical approximation for *many* instances of input parameters can be dramatically accelerated using *reduced basis methods*. Such methods exploit a suitably constructed, small dimensional linear subspace of the whole solution manifold.

Data may be insufficient, incomplete, and inaccurate. This source of *uncertainty* is often treated probabilistically and represented in terms of random fields to describe inter-patient variability, then propagated by a forward problem relying, e.g., on sampling techniques and multiple queries to the PDE model. Another challenging problem is that of *parameter estimation*, where some input quantities, not directly observable, can be identified thanks to additional clinical measurements. A possible road to parameter identification is to solve inverse problems, e.g., in a Bayesian framework, by sequential Monte Carlo methods or (extended/ensemble) Kalman filters [4].

A strict collaboration with cardiologists, cardiac surgeons, and radiologists is of paramount importance for providing a personalized virtual heart to patients, improving the understanding of heart function and dysfunction, benefiting *diagnostics, decision-making for treatment, surgical planning*, and *design* of prosthetic devices.

Acknowledgments

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Author photo courtesy of Alfio Quarteroni.

Alfio Quarteroni

ABOUT THE AUTHOR

Alfio Quarteroni, Euler Vorlesung lecturer in 2017 and winner of the International Galileo Galilei prize for Sciences in 2015, is author of twenty-two books and more than 350 papers in numerical modeling of fluid mechanics, geophysics, medicine, and sports. His group has carried out the mathematical simulation for the *Alinghi* yacht, winner of two editions of the America's Cup.

Dürer's Unfolding Problem for Convex Polyhedra

Mohammad Ghomi

Convex polyhedra are among the oldest mathematical objects. Indeed the five platonic solids, which constitute the climax of Euclid's books, were already known to the ancient people of Scotland some 4,000 years ago; see Figure 1. During the Renaissance, polyhedra were once again objects of fascination while painters were discovering the rules of perspective and laying the foundations of projective geometry. This remarkable confluence of art and mathematics was personified in a number of highly creative individuals including the German painter Albrecht Dürer, who was based in Nüremberg at the dawn of the sixteenth century and is credited with ushering in the Renaissance in Northern Europe. During extended trips over the Alps, Dürer learned the rules of perspective from his Italian contemporaries, and he subsequently described them in his influential book, *The Painter's Manual*. Aside from being the first geometry text published in German, this work is remarkable for containing the first recorded examples of unfoldings of polyhedra; see Figure 2.

remarkable confluence of art and mathematics

An (edge) unfolding of a polyhedron P is the process of cutting it along a collection of its edges, without disconnecting it, so that the resulting surface may be developed isometrically into the plane. Many school children are familiar with the process of cutting out a template from craft books, and folding the paper along dotted lines to form simple polyhedra such as a tetrahedron or a cube; an unfolding is the reverse process. Note that the cuts are made along a connected subset

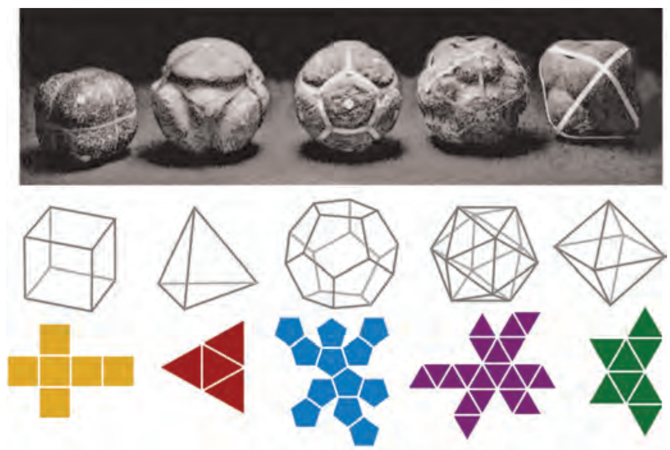

Figure 1. First Row: Neolithic carved stones from 2000 BC discovered in Scotland.[1] [1]
Second Row: The familiar representations of platonic solids studied in Euclid's *Elements*.
Third Row: Examples of unfoldings of the platonic solids.

Figure 2. A self-portrait of Dürer completed in the year 1500 at the age of twenty-eight, together with some illustrations from his book, *The Painter's Manual*, including the first recorded examples of unfoldings.

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[1] *For more information see* Time Stands Still: New Light on Megalithic Science *by Keith Critchlow.*

of the edges of P which contains each vertex of P and no closed paths. In other words, the cut set forms a *spanning tree* of the edge graph of P, and thus a convex polyhedron admits many different unfoldings depending on the choice of this tree. Furthermore, it is not the case that every unfolding of every polyhedron is simple or nonoverlapping. For instance there are even some (nonregular) tetrahedra which admit some unfoldings that overlap themselves. On the other hand, all the examples of unfoldings which Dürer constructed were simple, and in the intervening five centuries no one has yet discovered a convex polyhedron which does not admit some simple unfolding.

The problem of existence of simple unfoldings for convex polyhedra was explicitly posed in the 1970s by Shephard, and the assertion that a solution can always be found, or that every convex polyhedron is unfoldable (in one-to-one fashion) has been dubbed Dürer's conjecture. There is, however, substantial empirical evidence both for and against this supposition. On the one hand, computers have found simple unfoldings for countless convex polyhedra through an exhaustive search of their spanning edge trees. On the other hand, there is still no algorithm for finding the right tree, and computer experiments [4] suggest that the probability that a random edge unfolding of a generic polyhedron overlaps itself approaches 1 as the number of vertices grows. To date the problem remains wide open, and it is not even known whether simple classes of polyhedra such as prismatoids (polyhedra generated by the convex hull of a pair of convex polygons in parallel planes) are unfoldable.

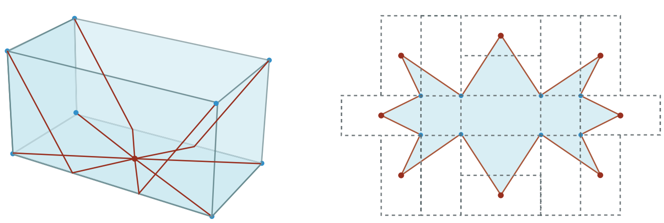

Figure 3. A. D. Alexandrov observed that one may develop a convex polyhedron injectively into the plane by cutting along geodesics which connect the vertices to a generic point.

If one is not confined to cut only along the edges, then it is quite easy to develop a polyhedron into the plane in one-to-one fashion, as had been observed by the influential geometer A. D. Alexandrov in his seminal work *Convex Polyhedra*; see Figure 3. Why then is Dürer's problem so difficult? Perhaps because the edges of convex polyhedra are not well understood, in the sense that there is no known procedure or simple algorithm for detecting an edge by means of intrinsic measurements within the surface. Indeed, Alexandrov's embedding theorem for convex surfaces—which states that any locally convex polyhedral metric on the sphere $\mathbf{S}^2$ may be realized as a convex polyhedron in Euclidean space $\mathbf{R}^3$—is not constructive and gives no hint as to which geodesics between a pair of vertices are realized as edges. In 2008, a more constructive proof was given by Bobenko and Izmestiev; however, this proof does not specify the location of the edges either.

The edge graph of a convex polyhedron is not the unique 3-connected embedded graph in the polyhedron whose vertices coincide with those of the polyhedron, whose edges are distance minimizing geodesics, and whose faces are convex. It seems reasonable to expect that Dürer's conjecture should be true if and only if it holds for this wider class of *pseudo-edge graphs*. This approach was studied by Alexey Tarasov in 2008, and has been further investigated by the author and Nicholas Barvinok very recently [2]. We claim to have constructed a convex polyhedron with 176 vertices and a pseudo-edge graph which does not admit any nonoverlapping unfolding. Thus one may say that Dürer's conjecture does not hold in a purely intrinsic sense.

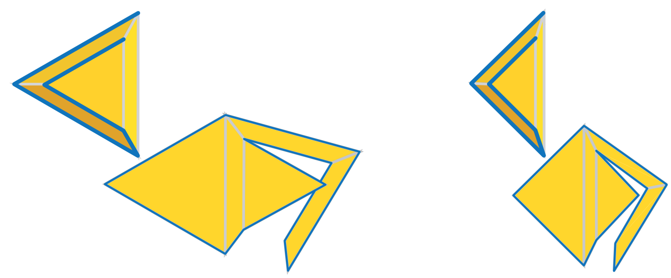

Figure 4. The left side shows a truncated tetrahedron (viewed from above) together with an overlapping unfolding of it generated by a monotone edge tree. As we see on the right side, however, the same edge tree generates a simple unfolding once the polyhedron has been stretched.

On the other hand, in 2014 the author [5] had obtained a positive result in this area by solving a weaker form of Dürer's problem posed by Croft, Falconer, and Guy [3, B21]: is every convex polyhedron combinatorially equivalent to an unfoldable one? It turns out that the answer is yes, and therefore there exists no combinatorial obstruction to a positive resolution of Dürer's problem. What the author shows is that every convex polyhedron becomes unfoldable after an affine (or linear) transformation. More explicitly, suppose that a convex polyhedron P is in general position in $\mathbf{R}^3$, i.e., no two of its vertices are at the same height. Then it is easy to construct a spanning tree T of P which is *monotone*, i.e., if T is rooted at the lowest vertex r of P, then the branches of T which connect its leaves to r have strictly decreasing heights or z-coordinates. Now stretch P via a rescaling along the z-axis. Then the corresponding unfolding eventually becomes simple, as illustrated in Figure 4. The proof that this stretching procedure works is by induction on the number of leaves (or branches of T which connect each leaf to the root r). The first step, i.e., when T consists of only one branch, is relatively simple to prove and follows from a topological characterization for embeddings among immersed disks in the plane. The inductive step is more technical.

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ABOUT THE AUTHOR

Mohammad Ghomi works primarily on classical problems involving curves and surfaces in Euclidean space, ranging from differential geometry and topology to real algebraic geometry and combinatorics.

Mohammad Ghomi

Sharon Arroyo Interview

Conducted by Melinda Lanius

Communicated by Alexander Diaz-Lopez

Sharon Arroyo is Technical Fellow in the Applied Mathematics Organization at The Boeing Company. Sharon obtained her PhD in complexity theory in linear optimization from Cornell University. She is a member of the Steering Committee of the BIG Math Network. Her email is `sharon.f.arroyo@boeing.com`.

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***Lanius:** How did you know you wanted to be a mathematician?*

Arroyo: I have always loved solving math problems! Math was always my favorite subject in school starting in first grade. I knew I wanted to focus on math as a career the day I learned about vector spaces in a linear algebra class when I was an undergraduate. I thought vector spaces were so cool!

***Lanius:** Who encouraged or inspired you?*

Arroyo: I am thankful to have been encouraged by many people. My family has always been encouraging me to fulfill my dreams. Many teachers along the way and Professor Ralph Cohen, my undergraduate advisor at Stanford University, and Professor Jim Renegar, my PhD advisor at Cornell University, taught me not only a lot about math but also instilled me with confidence to continue to the next level.

***Lanius:** How would you describe your work to a graduate student?*

Arroyo: I am a member of the Applied Mathematics Group within Boeing Research and Technology. I spend a lot of my time partnering with internal Boeing engineers to develop mathematical algorithms and tools that are used to reduce costs and improve product designs. For example, I recently have been working with Boeing's Supplier Management organization to develop optimization algorithms and tools that are used to reduce contracting costs. I also spend time developing algorithms and tools that can be used across Boeing. For example, I developed a non-smooth convex optimization algorithm that has been used in a number of application areas from air traffic management to multi-sensor multi-source data fusion. I also spend time leading projects and establishing technical direction for Boeing's applied research in operations research. Finally, I spend time mentoring and teaching to expand the Boeing use and value of operations research.

When developing algorithms and tools with internal Boeing customers, I get involved in all aspects of the mathematical problem solving process from defining problem requirements, developing mathematical problem formulations, constructing and implementing algorithms, delivering tools, and supporting analysis.

Lanius: *Do you have a favorite past project you could share with us?*

Arroyo: I am fortunate to have been involved in a lot of exciting projects over the years developing algorithms and tools to solve problems across Boeing in areas including airline scheduling, supply chain, transportation scheduling, production scheduling, communication networks, sensor scheduling, and aerial refueling tanker scheduling.

A recent favorite project of mine is my work with Supplier Management organizations at Boeing. We have teamed together to develop optimization algorithms and methods that have been used and are being used to reduce Boeing's contracting costs for items including raw materials and airplane fasteners. I have enjoyed working closely with the Supplier Management team to understand the details of the problems to determine the appropriate models and algorithms to develop to reduce costs. Together we have changed the way Supplier Management approaches contracting and other analytical problems. Seeing mathematics provide value to Boeing is extremely exciting to me!

Lanius: *What is a typical workday like?*

Arroyo: My days vary quite a bit, depending on my current projects and their states. For example, if I am in requirements and problem formulation stages or tool delivery and analysis stages of an internal customer project, I will tend to have a few more meetings with my Boeing customers than usual. During algorithm development and implementation phases of projects, I will have fewer meetings and will spend more time working on project aspects including algorithm and code development. Overall, I spend a fair amount of my time working individually and guiding and brainstorming with teammates to develop solutions to technical problems.

Lanius: *What is the work culture like at Boeing?*

Arroyo: The Applied Mathematics Group has 43 members across Seattle, Washington; Huntsville, Alabama; and Charleston, South Carolina, with over 60% having a PhD in a mathematics related field (e.g., applied and computational mathematics, operations research, statistics).

Projects assignments are made to maximize the value of the work being done by the Applied Mathematics Group. However, individual career goals and technical and application interests are also taken into account when possible. Overall, there is flexibility in choosing the projects on which we work when possible, especially when you have led in establishing them.

Many of our customer projects result from extension of existing projects. New projects often result from organizations hearing about the successful work of the Applied Mathematics Group and from our internal marketing. We also have a Math Hotline where Boeing employees can get a few hours of mathematical consulting. We often are able to solve the problems within a few hours, and many times the connection leads to a longer-term project.

Lanius: *How do you balance career and outside interests?*

Arroyo: This is an on-going question that I hope someone will answer for me! Spending time on a career typically means less time than desired with outside interests. Overall, I make sure I am focused while working to maximize my productivity and, thus, time available for outside interests.

Lanius: *What advice do you have for graduate students?*

Arroyo: While determining your career goals, be open and learn as much as you can about industrial career options as early in your studies as possible. Talk with faculty working with industry to learn about their experiences and ideas for other avenues to learn more. Utilize information about industrial careers on professional society web sites including SIAM, INFORMS, and The BIG Math Network[1]. In addition, even if you are fairly sure you would like a career in academia, try to get an industrial internship early in your program, ideally at a company involved in applied mathematical-related research. The experience could expose you to new research areas of interest and future collaborations in your career in academia and might even cause you to choose an industrial career!

> *Be open and learn as much as you can about industrial career options.*

Lanius: *Any final comments or advice?*

Arroyo: Be true to yourself when establishing your career goals. Be flexible in how you achieve them and engage in continuous learning along the way. Finally, enjoy the journey!

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Melinda Lanius

ABOUT THE INTERVIEWER

Melinda Lanius, a Wellesley College graduate, is currently earning her PhD in mathematics at the University of Illinois at Urbana–Champaign.

[1] bigmathnetwork.wordpress.com/industry-and-government-careers-links-for-graduate-students/

WHAT IS…

an Acylindrical Group Action?

Thomas Koberda

Communicated by Cesar Silva

The group $\mathbb{Z}$ acts on the real line $\mathbb{R}$ by translation. It is difficult to find a nontrivial group action which is easier to understand: the orbit of every point moves off to infinity at a steady and predictable rate, and the group action preserves the usual Euclidean metric on $\mathbb{R}$. Of course, this action is a covering space action, and the quotient space of the action is the circle, which is completely free of any topological pathologies.

Regular covering spaces in algebraic topology give rise to prototypically nice group actions. Among the most important features of a deck group action on a covering space is that it is free (i.e. no nontrivial element of the deck group has a fixed point) and properly discontinuous (i.e. for every compact subset K of the cover, there are at most finitely many deck group elements g such that $g \cdot K \cap K \neq \emptyset$, at least in the case where the base space is locally compact).

One can relax the freeness of an action without introducing insurmountable difficulties.

For certain purposes in topology and geometry, one can relax the freeness of an action while keeping discreteness, without introducing insurmountable difficulties. Discrete groups of isometries of Euclidean and hyperbolic space, for example, oftentimes contain torsion elements such as rotations, and finite order isometries of Euclidean or hyperbolic spaces always have a fixed point. By considering quotients of Euclidean and hyperbolic spaces by discrete groups of isometries, one naturally obtains the class of Euclidean and hyperbolic orbifolds, thus enlarging the class of Euclidean and hyperbolic manifolds. Orbifolds enjoy many of the salient features of manifolds, so that a mild relaxation of freeness of group actions still allows for reasonable geometry to persist.

Relaxing proper discontinuity can lead to some pathological phenomena, for instance quotient maps whose quotient topologies fail to be Hausdorff or even fail to have any nontrivial open sets. Consider, for example, a group of rotations of the circle generated by an irrational multiple of π. Since the circle is compact, this action of $\mathbb{Z}$ cannot be properly discontinuous—indeed, every orbit is countably infinite and dense. Hence, the quotient is an uncountable space with no open sets except the empty set and the whole space. Nevertheless, group actions which are not properly discontinuous abound in mathematics and have led to the development of entire fields, such as noncommutative geometry in the sense of A. Connes. Group actions which are not properly discontinuous are also important and common in geometric group theory, with the following example being of central importance:

Let S be an orientable surface and let $\gamma \subset S$ be a *simple closed curve*, as illustrated in Figure 1 or in Figure 2. A curve is *essential* if it is not contractible to a point, and *nonperipheral* if it is not homotopic to a puncture or boundary component of S. The *curve graph* of S, denoted $\mathcal{C}(S)$, is the graph whose vertices are nontrivial homotopy classes of essential, nonperipheral, simple closed curves, and whose edge relation is given by disjoint realization. That is, γ_1 and γ_2 are adjacent if they admit representatives which are disjoint. Thus, the

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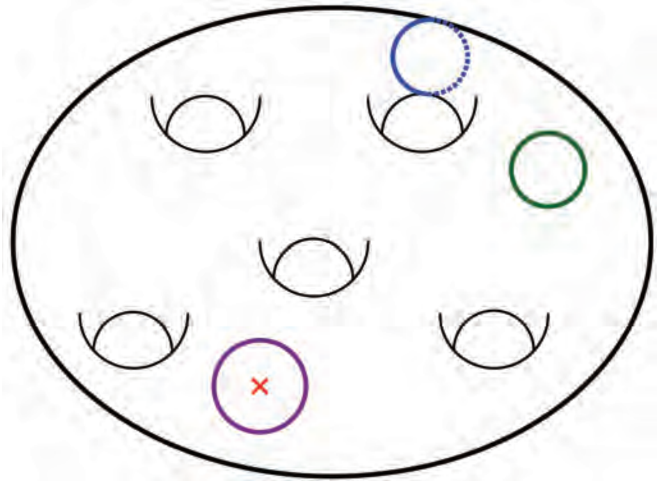

Figure 1. A surface of genus 5. The blue curve is essential and nonseparating. The green curve is inessential. The purple curve bounds a puncture or boundary component, denoted by a red x, and is therefore peripheral.

curve graph encodes the combinatorial topology of one-dimensional submanifolds of S. Note that for relatively simple surfaces, $C(S)$ may be empty or may fail to have any edges as they are defined here. For sufficiently complicated surfaces however, $C(S)$ has a very intricate and interesting structure.

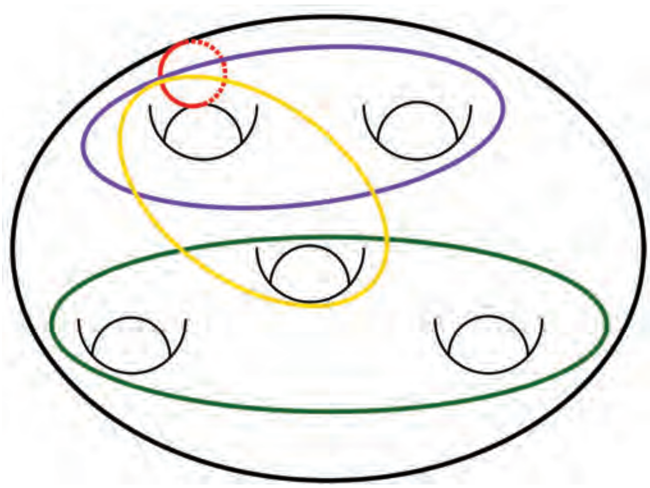

Figure 2. A surface of genus 5 with four essential curves drawn. The subgraph of $C(S)$ spanned by them is given in Figure 3.

Whereas the curve graph as defined here is a manifestly combinatorial object, it is also a geometric object with the metric being given by the graph metric.

It is an interesting exercise for the reader to prove that if $C(S)$ admits at least one edge, then $C(S)$ is connected, is locally infinite, and has infinite diameter.

The *mapping class group* of S is the group of homotopy classes of orientation preserving homeomorphisms

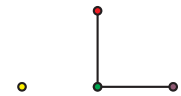

Figure 3. The subgraph of $C(S)$ spanned by the curves in Figure 2. The colored curves are represented by vertices of the corresponding color. The graph metric distance from the purple curve to the red curve in $C(S)$ is exactly two.

of S, and is written $\mathrm{Mod}(S)$. Mapping class groups are of central interest to geometric group theorists, as well as of significant interest to algebraic geometers, topologists, and homotopy theorists. From the point of view of geometric group theory, mapping class groups are studied via the geometric objects on which they act. Homeomorphisms of S act on the set of embedded loops on S, and similarly homotopy classes of homeomorphisms act on homotopy classes of embedded loops on S, and hence of simple closed curves. Since the adjacency relation in $C(S)$ is a topological property, $\mathrm{Mod}(S)$ acts by graph automorphisms and hence by graph metric isometries on $C(S)$.

As natural as the action of $\mathrm{Mod}(S)$ on $C(S)$ is, its geometry is extremely complicated. For one, the quotient $C(S)/\mathrm{Mod}(S)$ is finite, since two simple closed curves γ_1 and γ_2 are in the same mapping class group orbit if and only if $S\setminus\gamma_1$ and $S\setminus\gamma_2$ are homeomorphic to each other, as follows easily from the classification of surfaces. Thus, the action of $\mathrm{Mod}(S)$ on $C(S)$ is highly transitive. This is in spite of the fact that $C(S)$ is locally infinite, as mentioned above: if $C(S)$ has at least one edge, then each vertex of $C(S)$ has infinite degree. Thus, the action of $\mathrm{Mod}(S)$ on $C(S)$ is far from properly discontinuous.

Note that proper discontinuity (as we have defined it at least) is perhaps not the best property to require from the action, since $C(S)$ is not locally compact (by virtue of being a locally infinite graph). A better notion which is meaningful for actions on spaces like $C(S)$ is *properness*. If G is a group generated by a finite set S, then G can be viewed as a metric space by declaring g and h to have distance one if $g = h \cdot s$ for some $s \in S$, and in general defining the distance between g and h to be the least n such that $g = h \cdot s_1 \cdots s_n$ for elements $\{s_1, \ldots, s_n\} \subset S$. The reader may recognize this as the graph metric on the (right) Cayley graph of G with respect to S. If G acts on a metric space X, the action is *proper* if (roughly) for all $x \in X$, the orbit map $G \to X$ given by $g \mapsto g \cdot x$ is a proper map of metric spaces. The first example considered in this article, i.e. the translation action of $\mathbb{Z}$ on $\mathbb{R}$, is a proper action. Note that we can build another action of $\mathbb{Z}$ on $\mathbb{R}$, where a generator of $\mathbb{Z}$ acts by multiplication by 2. This action of $\mathbb{Z}$ on $\mathbb{R}$ is not proper. Returning to the situation at hand, since vertices of $C(S)$ have infinite stabilizers in $\mathrm{Mod}(S)$, the action of $\mathrm{Mod}(S)$ on $C(S)$ is not proper.

One way to see this is to observe the following: let $\gamma \subset S$ be an essential, nonperipheral, simple closed curve as in Figure 4. The surface $S\setminus\gamma$ is a surface with boundary,

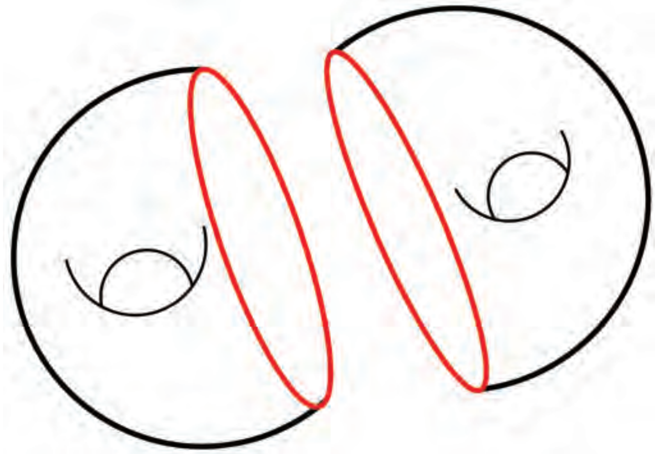

Figure 4. By considering an essential, nonperipheral, simple closed curve in red on this surface of genus 2, we can see that the action of $\mathrm{Mod}(S)$ on $C(S)$ cannot be proper. Fortunately, it satisfies a weaker property: it is acylindrical.

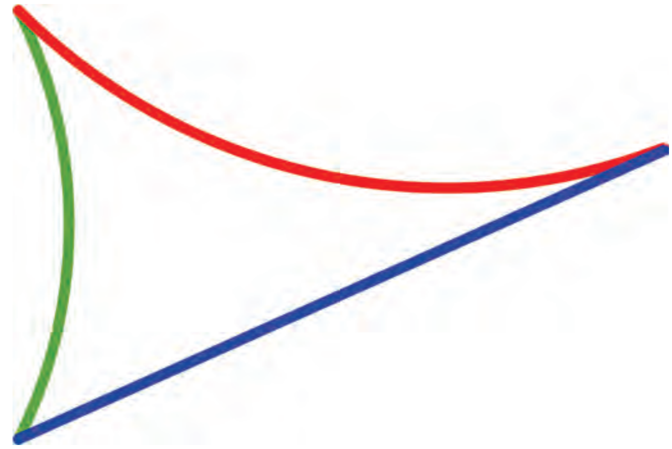

Figure 5. A path-metric space (such as a connected graph) is hyperbolic if there exists a $\delta \geq 0$ such that for every geodesic triangle, a δ-neighborhood of two sides contains the third side.

albeit simpler (in the sense that the Euler characteristic is strictly larger).

The surface $S \setminus \gamma$ generally admits many homotopically nontrivial homeomorphisms which act by the identity near γ, which therefore extend to homeomorphisms of S which fix γ. Moreover, one can build the *Dehn twist* about γ, which is given by cutting S open along γ and regluing with a full twist. These (homotopy classes of) homeomorphisms taken together form an infinite subgroup of $\mathrm{Mod}(S)$ which fixes the vertex γ of $C(S)$, whence it is clear that the action of $\mathrm{Mod}(S)$ on $C(S)$ cannot be proper.

The next best thing after properness.

How badly behaved is the action of $\mathrm{Mod}(S)$ on $C(S)$? Can something be said about it which is not a general statement about isometric group actions on graphs? It turns out that yes, indeed one can. The action is *acylindrical*, which is in some sense the next best thing after properness.

Acylindrical actions were first generally defined by Bowditch [1] in 2008. Let G be a group acting by isometries on a path-metric space X. The action of G on X is *acylindrical* if for all $r \geq 0$, there exist constants $R, N \geq 0$ such that for any pair $a, b \in X$ with $d(a,b) \geq R$, we have

$$|\{g \in G \mid d(g \cdot a, a) \leq r \text{ and } d(g \cdot b, b) \leq r\}| \leq N.$$

The set of elements

$$\{g \in G \mid d(g \cdot a, a) \leq r\}$$

is not quite the stabilizer of a, but rather the *r-quasi-stabilizer* of a. Acylindricity can be summed up as saying that "for all r, simultaneous r-quasi-stabilizers of sufficiently distant points are uniformly small." In more informal terms, an acylindrical action is "uniformly proper on sufficiently distant pairs of points." Dropping the uniformity condition (i.e. replacing the uniform constant N by a requirement that the relevant subset of G is finite), one gets the closely related notion of a *weakly properly discontinuous* action. This latter notion appears in a 2002 paper of M. Bestvina and K. Fujiwara.

Observe that, like many concepts in geometric group theory and coarse geometry, acylindricity is blind to phenomena on a bounded scale. For instance, a group action on a bounded metric space is always acylindrical: just let R be greater than the diameter of X.

Bowditch proved the following fundamental result:

Theorem 1. *The action of* $\mathrm{Mod}(S)$ *on* $C(S)$ *is acylindrical.*

The usefulness of acylindricity is perhaps not immediately clear. The most productive setting for studying acylindricity is in the case where X is a *hyperbolic graph*. This means that X is a graph equipped with the graph metric, and the graph metric is *(Gromov) hyperbolic*. That is to say, there is a constant $\delta \geq 0$ such that for any triple $x, y, z \in V(X)$ of vertices and geodesic segments $[x,y], [y,z], [x,z] \subset X$, we have

$$[x,z] \subset N_\delta([x,y] \cup [y,z]).$$

In other words, every geodesic triangle is δ–thin in the sense that a δ–neighborhood of two sides contains the third side (see Figure 5).

Note that the definition of hyperbolicity makes sense for any geodesic metric space, and indeed this is the definition of a *hyperbolic (metric) space* which is not necessarily a graph. It is highly nonobvious though true that $C(S)$ is a hyperbolic graph, by a deep result of H. Masur and Y. Minsky from 1999.

In the case of an action of a group G on a hyperbolic graph X, acylindricity of the G-action gives a tractable geometric shadow of G in X, given by considering the orbit of an arbitrary vertex $v \in V(X)$. To make sense of this notion, we define the *translation length* of an element

$g \in G$, a definition which makes sense for any isometric action of G on X. We write

$$\tau(g) = \lim_{n\to\infty} \frac{d(g^n \cdot x, x)}{n},$$

a limit which always exists and which is independent of the choice of x.

The translation length of g is either positive or zero. In the former case, the element g is called *loxodromic*. An example of a loxodromic isometry is a homothetic expansion of the upper half-space model for hyperbolic space. One way that $\tau(g)$ can be zero is if some (or indeed every) g-orbit has finite diameter, in which case g is called *elliptic*. An example of this latter case is rotation of hyperbolic space. General actions can have elements such that $\{g^n \cdot x\}_{n\in\mathbb{Z}}$ is unbounded but where $d(g^n \cdot x, x)$ grows strictly sublinearly as a function of n, in which case g is *parabolic*. The map $(x, y) \mapsto (x+1, y)$ of the upper half-plane is an example of a parabolic isometry of the upper half-space model of hyperbolic space. Parabolic isometries can imbue group actions with significant complexity, as in the case of lattices acting on symmetric spaces. In many higher rank situations, parabolic elements can generate lattices which can often be shown to never admit interesting acylindrical actions.

For acylindrical actions on hyperbolic graphs, Bowditch proved the following general fact which simplifies the picture somewhat:

Theorem 2. *Let G be a group acting acylindrically on a hyperbolic graph X. Then every nontrivial $g \in G$ is either loxodromic or elliptic. Moreover, there is a constant $\epsilon > 0$ depending only on the acylindricity and hyperbolicity constants such that if g is loxodromic then $\tau(g) \geq \epsilon$.*

In terms of terminology, acylindrical actions are either *elementary* or *nonelementary*. An action is elementary if it is purely elliptic or if there is (essentially) only one cyclic subgroup consisting of loxodromic elements. Nonelementary acylindrical actions are the only interesting ones. As might be expected, the $\mathrm{Mod}(S)$ action on $\mathcal{C}(S)$ is nonelementary.

It is possible to show that loxodromic elements are exactly the mapping classes such that no power fixes a simple closed curve, as was done by Thurston. Such mapping classes are called *pseudo-Anosov*, and are arguably the most interesting mapping classes. Thurston's beautiful 1988 article in the *Bulletin of the American Mathematical Society* provides an accessible introduction. The literature on the coarse geometry of subgroup of $\mathrm{Mod}(S)$ which consist entirely of pseudo-Anosov elements (i.e. *purely pseudo-Anosov subgroups*) is vast, and is the domain of *convex cocompact subgroups* of $\mathrm{Mod}(S)$. Convex cocompact subgroups, which we will not define precisely here, are of central importance in the geometry of the moduli space of curves, hyperbolic group extensions, and the algebraic and geometric structure of mapping class groups. Note that it is not obvious *a priori* that there exist noncyclic purely pseudo-Anosov subgroups of $\mathrm{Mod}(S)$. They can be produced directly with some work, but the acylindricity of the $\mathrm{Mod}(S)$ action on $\mathcal{C}(S)$ again intercedes to furnish a profusion of them, by the following recent result of F. Dahmani, V. Guirardel, and D. Osin:

Theorem 3 (Dahmani–Guirardel–Osin (2017))**.** *Let G be a group acting acylindrically on a hyperbolic space X. Then there exists a natural number $N > 0$ such that for every loxodromic $g \in G$, the normal closure $\langle\langle g^N \rangle\rangle$ is free and purely loxodromic.*

In recent years, there has been an explosion of results by many authors on acylindrical actions of various groups on hyperbolic spaces. In addition to mapping class groups, examples of groups admitting nonelementary acylindrical actions on hyperbolic spaces include nonelementary hyperbolic groups, groups which are nonelementary hyperbolic relative to proper subgroups, outer automorphism groups of free groups, Cremona groups, nonvirtually nilpotent groups acting properly on a hyperbolic space of uniformly bounded geometry, right-angled Artin groups, and compact 3-manifold groups which are not Seifert fibered. A lot of these examples have been organized and their properties developed recently by the work of J. Behrstock, M. Hagen, and A. Sisto, to form the class of hierarchically hyperbolic groups.

Acylindricity has been an extremely productive and pervasive concept in geometric group theory, and has led to fast paced and dramatic advances. Undoubtedly, it will continue to do so for some time.

References

1. Brian H. Bowditch, Tight geodesics in the curve complex, *Invent. Math.* **171** (2008), no. 2, 281–300. MR 2367021 (2008m:57040)

Image Credits

Figures courtesy of Thomas Koberda.

Photo of Thomas Koberda courtesy of Angelo Mao.

Thomas Koberda

ABOUT THE AUTHOR

In addition to doing mathematics, Thomas enjoys reading, foreign languages, cooking, and running.

AMS Blogs
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Graduate Student Blog
by and for math graduate students

The AMS Graduate Student Blog, by and for math graduate students, includes puzzles and a variety of interesting columns. `blogs.ams.org/mathgradblog`.

Daily Quizzes: the…Bad and the Ugly

by Sarah Salmon

Quite some time ago,[1] I tried to convince you that giving your students a one- or two-question quiz every single day had a myriad of good aspects. ...Now, we're going to discuss the bad (easily fixable) and ugly (not so easily fixable) issues which I ran into that semester….

The Bad:

- Keeping track of the papers was a nightmare….
- …The weeks when I got behind on grading were horrible….

The Ugly:

- …My students felt unnecessarily pressured to master material quickly….
- Giving a quiz every day took class time….

In trying to hold on to the benefits of daily quizzes while addressing some of the issues, I tried something different last spring. Instead of a quiz at the start of each day, I would write the same question that I would have given as a quiz on the board for the students to work on as they came to class, and we would discuss paths to solutions together. On Friday, I would give a quiz consisting of questions nearly identical to those we had worked on for the past week. There was incentive to arrive on time since the students got a preview for the quiz, but I didn't have to keep track of so many papers. The students didn't have a strong reason to study ahead of time (since no part of this was graded) but we struggled through the problems together, resulting in some really good conversations about the previous material….

Photo Credit

Photo of Sarah K. Salmon courtesy of Devin Fox.

[1] *"Daily Quizzes: the Good, the Bad, and the Ugly—Part 1,"* `blogs.ams.org/mathgradblog/2016/05/30/daily-quizzes-good-bad-ugly-part-1/`

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Sarah K. Salmon

Can Math Help the Supreme Court Free Congress from Gerrymanders?

By Karen Saxe

In October the Supreme Court heard its first partisan gerrymandering case (Gill v. Whitford[1]) in more than a decade. Gerrymandering is the intentional manipulation of territory toward some desired electoral outcome. In 2012, Republicans won 60 out of 99 seats in the Wisconsin State Assembly while receiving only 48.6 percent of the statewide vote. In 2014 and 2016, Republicans extended their advantage as in Figure 1. Partisanship will be more carefully watched in the next round of redistricting, which will take place after the 2020 census.

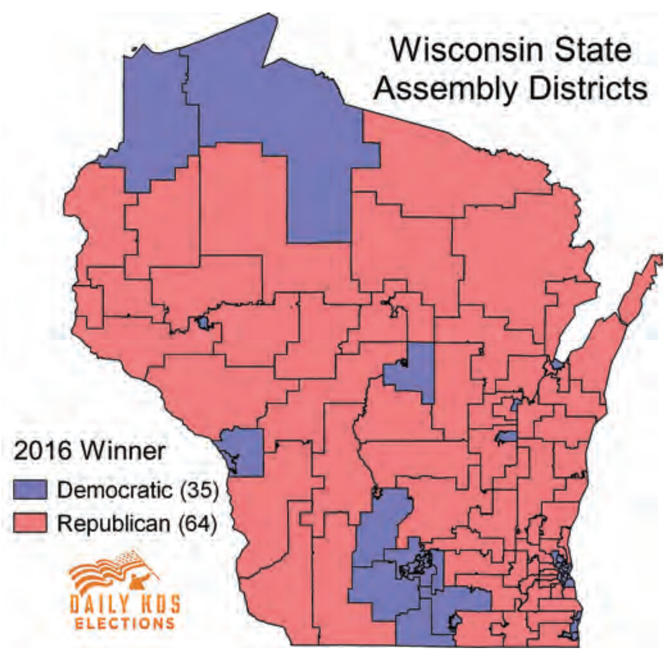

Figure 1. In 2016, Republicans won 64 of the 99 seats in the Wisconsin State Assembly while receiving only 53 percent of the statewide vote.

A state legislature typically divides the state into election districts, one for each member of Congress. Map drawers usually build congressional districts using census blocks as the base unit. In Minnesota, for example, there are 259,777 census blocks, which need to be assembled into eight congressional districts. Minnesota will likely lose a seat after the 2020 census, which will make the upcoming redistricting even more difficult and contentious. With the current eight districts, there are approximately 5 x 10^{38} possible maps.

Measures often compare the district to some ideal geometric object such as a circle, a polygon, or the convex hull of the district.

Of course, there are many constraints on *how* the 259,777 blocks are put into the eight districts. Each district must contain the same number of people (and census blocks vary in population size), and each district must be "contiguous" (path connected). These two constraints alone reduce the number substantially, but to what sort of magnitude?

The pivotal Justice Kennedy has called for a "manageable standard" that can be used to detect whether or not a specific map is a partisan gerrymander. In a 1983 case (Karcher v. Daggett[2]) involving New Jersey's districting, Justice Stevens had written that "Substantial divergences from a mathematical standard of compactness may be symptoms of illegitimate gerrymandering." What exactly

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[1] `www.scotusblog.com/case-files/cases/gill-v-whitford/`

[2] `bit.ly/2wvK401`

this means is an interesting question, but you can bet it has nothing to do with open covers. Measures often compare the district to some ideal geometric object such as a circle, a polygon, or the convex hull of the district, or require that the perimeter be relatively small compared to the area (a legislated isoperimetric inequality!). There are further guidelines arising from the Voting Rights Act. And there are other districting aspirations such as preserving communities of interest, incumbent protection, and creating competitive districts.[3] How can we detect whether there has been partisan intent or a partisan effect in the map-drawing process? There are social science measures that aim to identify partisan bias in districts, including the *efficiency gap*,[4] which is fairly new, central to the Wisconsin case, and getting a lot of attention from the press.

Take the line drawing out of the hands of those with ulterior motives.

While automated redistricting algorithms have been around for half a century, computational limitations have been a barrier. Due to vastly increased computational power, together with theoretical headway, we are currently seeing significant advances being made in the way we can utilize computer simulations to assess maps and help identify highly biased maps. In short, computers can generate a very large number of maps and for each of these maps partisan bias is measured using, for example, the efficiency gap. If the proposed map is an outlier in its partisan bias, as compared to the sample's distribution of the bias measure, then the proposed map can be considered biased. A new algorithm based on viewing redistricting as a graph-cut problem shows real promise; this algorithm generates maps that are contiguous and compact (as determined by a specified compactness measure) and further constraints (such as status quo bias) can be built in. This algorithm produces a representative sample from the space of all possible maps, subject to the constraints.

The power of such algorithms shows us that we are moving a long way toward being able to identify partisan gerrymanders. Unfortunately, sophisticated map-drawers with intention will be able to use the technology to draw maps that are biased but still legal. It seems that the only way out of this quagmire is to take the line drawing out of the hands of those with ulterior motives.

While there are federal laws, centrally the Voting Rights Act, that govern redistricting, the states[5] have substantial autonomy in how they draw their lines. And, for most states, this means that the maps are drawn by the party currently in power in the state. Some argue that change needs to happen up front, that redistricting should be taken out of the hands of state legislators who are—quite understandably—interested in seeing their own party stay in power. Americans across the political spectrum agree by wide margins that gerrymandering is bad. In 2013, a Harris poll[6] found that seven in ten Americans agreed that those who stand to benefit from drawing electoral lines should not have a say in the way those lines are drawn. This view cut across partisan lines, with 74 percent of Republicans, 73 percent of Democrats, and 71 percent of independents in agreement.

There are many civil rights organizations working on this. What about in Congress? In the current Congressional session there have been two bills introduced that, if adopted as law, would require non-partisan commissions in every state to draw congressional districts. Representative Don Beyer (Virginia) has introduced the Fair Representation Act, which includes an independent commission as part of a larger package of congressional election reforms. Representative Zoe Lofgren's (California) Redistricting Reform Act of 2017 also requires each state to establish an independent redistricting commission.

potential to positively impact the redistricting process

This past summer I participated in the Geometry of Redistricting Workshop. The workshop participants included many mathematicians, and also lawyers, political scientists, and computer scientists. The first three days were open to the public and consisted of talks preparing us in all relevant areas of math, computation, political science, and legal history. For the last two days small groups convened in three training tracks—information/technology, expert witness, and teaching. The teaching track introduced high-school and college teachers to the mathematical discipline of voting theory and offered concrete tools for incorporating mathematical topics related to voting, gerrymandering, and civil rights into their teaching. Information track folks engaged in a code-a-thon of sorts, creating databases, visualizations, apps, and other open-source tools for redistricting.

In my track, the expert witness track, we pored through witness testimonies and deposition transcripts, we talked about what we might ask a lawyer when initially contacted to potentially serve as a witness, and we discussed writing *amici curiae*. It was intense, and I know I speak for many in my track when I say that I now feel *much* more comfortable with the idea of being contacted to serve as an expert witness in a redistricting case.

We are two years away from our nation's next census and, following it, our next round of reapportionment of Congress and redistricting of the nation. Mathematicians

[3] *For more on constraints in redistricting by the states, see* `redistricting.lls.edu/where-tablefed.php`

[4] `papers.ssrn.com/sol3/papers.cfm?abstract_id=2457468`

[5] `redistricting.lls.edu/who.php`

[6] `bit.ly/2rh3eWb`

and statisticians have the potential to positively impact the redistricting process, and our community is increasingly ready to do so.

EDITOR'S NOTE. See Saxe's blog *Capital Currents* `blogs.ams.org/capitalcurrents/` and "A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap" in the October 2017 *Notices.*

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Karen Saxe

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Karen Saxe recently became one of the four experts comprising the Common Cause Minnesota Redistricting Leadership Circle.

From Blackboard to Bedside: High-dimensional Geometry is Transforming the MRI Industry

Aide Mémoire for a Congressional Briefing

David Donoho

Twice per year, the American Mathematical Society (AMS) and the Mathematical Sciences Research Institute (MSRI) jointly sponsor a Congressional briefing.[1]

These briefings provide an opportunity for communicating information to policymakers and, in particular, for the mathematics community to tell compelling stories of how our federal investment in basic research in mathematics and the sciences pays off for American taxpayers and helps our nation maintain its place as the world leader in innovation. Attendees on 28 June 2017 included Congressional staffers and representatives from the NSF Division of Mathematical Sciences, and Senate Minority Leader Charles Schumer and House Minority Leader Nancy Pelosi dropped in. David Eisenbud (MSRI), Karen Saxe (AMS), Anita Benjamin (AMS), and Kirsten Bohl (MSRI) organized this event. Presenter David Donoho has kindly shared these notes for his Congressional briefing. It is notable that the mathematicians had little interest in applications, yet the math inspired original MRI research and applications.

—Karen Saxe, Director, AMS Washington Office, and David Eisenbud, Director, MSRI

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1. Background

MRI scans are crucial tools in modern medicine: 40 million scans are performed yearly in the US. Individual 2-D MR images portray soft tissues that X-ray CT can't resolve, without the radiation damage that X-rays produce. MRIs are essential in some fields, for example to neurologists seeking to pinpoint brain tumors or study demyelinating diseases and dementia.

MRI technology is remarkably flexible, constantly spawning new applications. Dynamic MRIs allow cardiologists to view movies showing muscular contractions of the beating heart. 3-D head MRIs allow neurosurgeons to meticulously plan life-and-death brain surgeries, in effect to conduct virtual fly-throughs ahead of time.

Traditionally MRIs required lengthy patient immobilization—in some cases hours. Long scan times limit the number of patients who can benefit from MRI, and increase the cost of individual MRIs. Long scan times also make it difficult to serve fidgety children. Ambitious variations of MR imaging—such as dynamic cardiac imaging and 3-D MRI—require far longer scan times than simple 2-D imaging; such long scan times have typically been awkward or even prohibitive. Yet patients with arrhythmias and afib could get better treatment based on dynamic cardiac imaging; patients with aggressive prostate cancer could get much more accurate biopsies under 3-D MRI guidance; and many neurosurgeries could be much safer and more effective if surgeons could plan surgeries with 3-D MRI-based fly-throughs beforehand.

Help is on the way: inspired by federally funded mathematical sciences research, patients everywhere will soon

[1] *See the first item of "Inside the AMS" in the September 2017* Notices.

complete traditional clinical MRI scans much more rapidly. Ambitious but previously rarely available MRI applications will go mainstream.

2. Accelerated Imaging by Compressed Sensing

In 2017, the FDA approved two new MRI devices, which dramatically speed up important MRI applications from 8x to 16x. Siemens' technology (CS Cardiac Cine) allows movies of the beating heart; GE's technology (HyperSense) allows rapid 3-D imaging, for example of the brain.

Both manufacturers say their product is using *Compressed Sensing (CS),* a technology that first arose in the academic literature of applied mathematics and information theory.

The underlying mathematics was solid, reliable, and applicable.

Roughly ten years ago, mathematicians, including Emmanuel Candès, now at Stanford, Terence Tao of UCLA, and me, put forward theorems showing that compressed sensing could reduce the number of measurements needed to reconstruct certain signals and images. Partly inspired by these new mathematical results, MRI researchers such as Michael Lustig (now at Berkeley) and collaborators in John Pauly's lab at Stanford began to work intensively on new scanning protocols and algorithms. Compressed-sensing-inspired imaging became in the next few years a leading theme at international conferences of MRI researchers. Convincing clinical evidence soon emerged. Shreyas Vasanawala at Lucille Packard Children's Hospital (Stanford) and co-authors (including Lustig and Pauly) showed in 2009 that pediatric MRI scan times could be reduced in representative tasks from 8 minutes to 70 seconds, while preserving the diagnostic quality of images. Fidgety children could thus be imaged successfully and comfortably with far less frequent use of sedation. Other researchers demonstrated impressive speedups in dynamic heart imaging in the clinical setting. Manufacturers ultimately became convinced; serious commercial development followed, leading to 2017's FDA bioequivalence approvals.

Researchers at GE and Siemens tell me that accelerated imaging can be expected to spread broadly to many other MRI settings. The industry welcomes the new approach to accelerate MRI scans, and is seeking to deploy it where possible.

Transforming the MR industry takes time. In each proposed application, the FDA must first certify that accelerated imaging is *bioequivalent* to traditional imaging: that it really can produce diagnostic quality images in less scan time. Seeking FDA approval demands a rigorous multi-step evaluation taking years. From this viewpoint, it seems stunning that compressed-sensing inspired products are now on the market, only about a decade after the initial academic journal articles. This is certainly a testament to the energy and talents of MRI researchers and developers, and also a sign that the underlying mathematics was solid, reliable, and applicable.

The technology remains to be deployed into hospitals and clinics. More than 5 billion USD in MR scanners are sold annually; service and maintenance costs add billions more. There are tens of thousands of MRI scanners installed in the US currently, and many more worldwide. Improving MRI, by whatever means—mathematical ideas like compressed sensing, or physical ingredients like more powerful magnets—is always a gradual process of getting new equipment into the marketplace or retrofitting existing scanners.

For patients, such improvements can't come soon enough. My own son is a neurosurgery resident at a large county hospital that serves many indigent and uninsured patients. He treats dramatic injuries to the head, from gunshot wounds, blunt force trauma, and car wrecks; but also less dramatic yet serious problems like aneurysms and brain tumors. In his hospital, which does not yet have the accelerated MRI technology I've been speaking about, lengthy delays for MRIs are common; sometimes he must open the patient's cranium without any MRI at all. In other cases, he only can inspect a few 2-D slices, rather than a full 3-D image.

My son's comment about accelerating MRIs: "Faster, please."

3. The Role of Mathematics Research

NSF-funded Mathematical Sciences research, and NIH funding of cognate disciplines played a key role in these developments.

First: how can mathematics be helpful? Don't the MRI researchers have all the equipment they need to simply do whatever they need, and then see experimentally if it works?

The answer is interesting. Mathematicians build formal models of systems and then derive logical consequences of those formal models. Whatever mathematicians discover about such models is *logically certain.* In everyday experience, almost anything can be doubted, nothing is as it seems, whatever can go wrong will go wrong. In mathematics, a true statement is true, full stop. If a mathematical theorem says something surprising or impressive, you don't waste time doubting the messenger—you just read the proof and (if you have the background) you will see why the theorem is true.

In mathematics, a true statement is true, full stop.

Mathematical results can be transformative when applied to a phenomenon in the real world that is poorly understood and as a result controversial. And in the case of accelerated scanning, that was the case. Prior to the mathematical work I mentioned above, isolated projects had observed experimentally that in special circumstances, researchers obtained impressive speed improvements over traditional MRI scanning. However, the MRI community as a whole was uncertain about the scope of such isolated results, and the impact of the published experimental evidence was therefore limited.

Mathematical research can go farther and deeper than experiments ever will go. It can give guarantees that certain outcomes will *always* result or that certain outcomes will never result. When you first hear of such a guarantee, it can rock your world.

Early mathematical results about compressed sensing *guaranteed* imaging speedups under certain conditions. This got the attention of MRI researchers and inspired some, such as Michael Lustig, to go all in on compressed sensing. Eventually, the mathematical certainty offered by the guarantees and their breadth broke the apparent 'logjam' of hesitation and suspicion that would have otherwise greeted MRI. Compressed sensing became a hot topic in MRI research.

guarantees that certain outcomes will always result

Mathematics can also be a floodlight illuminating clearly the poorly understood path ahead. MRI researchers have always wanted to accelerate MRI; they just didn't see how to do it. Mathematicians proposed new algorithms and gave persuasive guarantees based on illuminating principles.

4. Some Mathematics

I'll mention very briefly some mathematical ideas that were mobilized to study compressed sensing.

At its heart, compressed sensing (CS) proposes that we take seemingly too few measurements of an object—so from the given data there are many possible reconstructions. CS selects from among the *many* possible candidates the one minimizing the so-called Manhattan distance[2]. Mathematics guarantees that the optimizing reconstruction (under conditions) is **exactly** the one we want.

The bridge between imaging and mathematics is produced by mathematical analysis of convex optimization algorithms that shows that the following statements are equivalent.

- Consider 100,000 random measurements of a 1000 by 1000 image. Suppose the underlying image has only 10,000 nonzero wavelet coefficients. With *overwhelming probability* this image can be reconstructed by minimizing the Manhattan distance of its wavelet coefficients.
- Consider a random 900,000-dimensional linear subspace of 1,000,000-dimensional Euclidean space. The chance that this slices into a certain regular simplicial cone with 10,000-dimensional apex is *negligible.*

The mathematical heart of the matter is thus the following *geometric probability* problem. We are in an *N*-dimensional Euclidean space, *N* large. We consider a convex cone *K* with its apex at zero, and sample an *M*-dimensional *random* linear subspace *L*. What is the probability that *L* intersects *K*? (See Figure 1.)

Figure 1. The underlying mathematics asks for the probability that a high-dimensional cone intersects a high-dimensional plane.

As it turns out, this probability depends on the cone *K*, and on the dimensions *M* and *N*—let's denote it P(*K; M,N*). The central surprise of compressed sensing is that, for the cones *K* we are interested in, we can have P(*K; M,N*) essentially zero, even when $M << N$. This mathematical fact is equivalent to saying that compressed sensing works—reconstructs the object of interest from undersampled measurements.

To get an idea how P(*K; M,N*) behaves, we have to become comfortable with probabilities of intersections of cones and subspaces.

It is helpful to first assume the subspace *L* is of dimension M=1. Then we are asking: *if we draw a random line through the origin, what is the chance that this line intersects the cone K?*

Such questions have fascinated curious people for centuries. In dimension N=3, the probability that *L* intersects *K* is the same as the probability that a random point on the sphere *S* lands in a certain region, *R*, on the sphere. This is just the surface area of the region *R*. (See Figure 2.)

In order to compute P(*K; M,N*) in general, one must generalize Gauss's formula to dimension M>1, for the regions in question.

In the 1960s two papers by federally-funded US university professors invented key tools to attack our problem. Harold Ruben, then at Columbia, generalized Gauss's formula for spherical triangles in dimension 3 to all higher dimensions, obtaining general formulas for the spherical volume of high-dimensional spherical simplices. Branko Grünbaum at University of Washington formalized the

[2] *Here we really mean l_1 norm. I figured that since Senator Charles Schumer would be present, we really should have a way to connect the proceedings to New York State!*

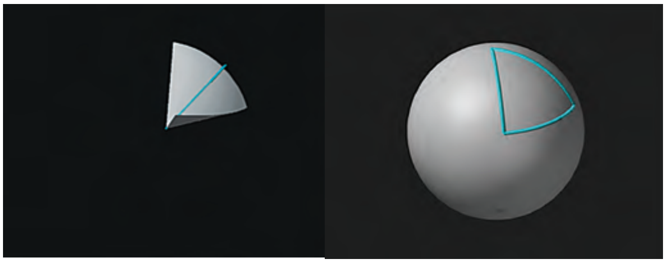

Figure 2. The probability that a line intersects a cone is given by the area of the corresponding region on the surface of the sphere. When *R* is a spherical triangle, the great mathematician Gauss found a beautiful formula for area: simply the sum of the three internal angles, minus π. This formula is easy to understand but conceals a surprising wealth of intellectual meaning.

cone intersection probability P(*K; M,N*) in the case *K*>1 and developed some fundamental formulas for P(*K; N,M*).

In the 1990s, German geometric probabilist Rolf Schneider and Russian probabilist Anatoly Vershik showed how to use Ruben's and Grünbaum's formulas to compute P(*K'; M,N*) for some interesting cones *K'*. In effect, they showed that (Ruben's generalization of) Gauss's charming formula was the heart of the matter; everything reduced to the computation of volumes of spherical simplices.

By the mid 2000s several approaches were developed to show that the central miracle of compressed sensing extended over a wide range of combinations of *M* and *N*; Candès and Tao (mentioned above) had developed by other means upper bounds on P(*K; M,N*), establishing that there was a useful such range. NSF-funded postdoc Jared Tanner (now at Oxford) worked with me to apply geometric tools of Schneider, Vershik, and Ruben to the cones *K* of interest. We developed precise formulas revealing the precise number of measurements needed for exact reconstruction.

Since then, slick machinery was developed by two teams at Caltech, Oymak/Hassibi and Amelunxen/McCoy/Tropp, to get these and many other results.[3]

5. The Role of Federal Funding

The success story of compressed sensing is a testament to federal funding. Federal funding of pure and applied science has endowed the United States with research universities that are the envy of the world. Federal funding of mathematical sciences has led to amazing collections of sophisticated mathematical talent within those great universities, housed in departments of mathematics, applied mathematics and statistics, electrical and systems engineering and computer science, and even biology and chemistry.

These great institutions attract terrifically talented young people from around the world to come here to learn and become part of our technical infrastructure, strengthening the next generation of US science and industry.

Federal funding enabled the breakthroughs of compressed sensing along three paths:

- Federal funding enabled *basic research* in high-dimensional geometry, which is at the heart of compressed sensing. Harold Ruben and Branko Grünbaum were housed in statistics and mathematics departments at US universities when they did their foundational work.
- Federal funding enabled *cross-disciplinary* work that identified the key questions for mathematicians to resolve. In the late 1990s, NSF funded a joint project between optimization specialists Stephen Boyd and Michael Saunders and myself at Stanford, which studied the Manhattan metric for important data processing problems. Prior to this project, most scientists used the Crow Flight metric instead of the Manhattan metric.[4] That project and its sequels funded work by Xiaoming Huo (now at The Georgia Technical Institute), Michael Elad (now at Technion), and myself that proved mathematically that the Manhattan metric could pick out the unique correct answer—when the answer we are seeking had a very strong mathematical sparsity property.
- Federal funding enabled *focused* research to go far deeper into this surprising area and dramatically weaken the required sparsity, for example by using random measurements. Researchers Emmanuel Candès (then at California Institute of Technology) and Terry Tao (University of California—Los Angeles), and myself and many others all were supported in some way by NSF to intensively study random measurements.

This is *above all* a case where the *federal research funding system has worked.* NSF support of mathematics and statistics and NIH support of electrical engineering and radiology have really delivered. The research universities, such as Stanford University, California Technical Institute, University of California—Los Angeles, and University of California—Berkeley, to mention only a few, have really delivered. As we have seen, industrial research groups at Siemens and GE have responded enthusiastically to the initial academic research breakthroughs.

Enabling the rapid transition were the great students and facilities available at Stanford University, produced by decades of patient federal support and also visionary campus planning and generous private donations. When Michael Lustig came to campus in the early 2000s as a graduate student of electrical engineering professor John Pauly, his office in Packard was less than a hundred yards away from the statistics department in Sequoia Hall, maybe 100 yards away from a GE-donated high-field MR research scanner, and maybe 300 yards away from a medical MRI research facility at Stanford Medical school. In a few hours, Lustig could literally engage in impromptu conversations about high-dimensional geometry with mathematical scientists, about specific MR pulse sequences with electrical engineers, and about specific possible clinical trials

[3] *Actually many other ways have arisen to understand these results, using sophisticated ideas from metric geometry to information theory. I would mention for example work of Roman Vershynin, Ben Recht and collaborators, and of Mihailo Stojnic. But in a brief presentation for Congressional staffers, I couldn't mention all the great work being done.*

[4] *Of course Crow Flight metric means standard Euclidean or l_2 distance.*

The cost-benefit ratio of mathematical research has been off-scale.

with doctors at Stanford Hospital. This tremendous concentration of resources allowed him in his thesis to produce results that inspired many MRI researchers to pursue compressed sensing-inspired research programs.[5] Today's Federal Funding model supported the development of such concentrated facilities and talent over decades of patient investment; the model has delivered.

The cost-benefit ratio of mathematical research has been off-scale. The federal government spends about $250 million per year on mathematics research. Yet in the US there are 40 million MRI scans per year, incurring tens of billions in Medicaid, Medicare, and other federal costs. The financial benefits of the roughly 10-to-1 productivity improvements now being seen in MRI could soon far exceed the annual NSF budget for mathematics research.

[5] *Lustig credits many other MRI researchers with decisive contributions in bringing CS into MR Imaging. He says: "I would emphasize the contributions of Tobias Block* `pubmed/17534903` *and Ricardo Otazo (with Daniel Sodickson)* `pubmed/20535813`*. These guys have taken the clinical translation of Compressed Sensing orders of magnitude forward." He also mentions key contributions of Zhi-Pei Lian of UIUC, Joshua Trzasko of Mayo Clinic, and of Alexey Samsonov and Julia Velikina of UW Madison. It seems that federal funding was important in each case.*

ACKNOWLEDGMENT. There are by now literally thousands of papers in some way concerned with compressed sensing. It's impossible to mention all the contributions that deserve note. I've mentioned here a select few that I could tie into the theme of a Congressional Lunch on June 28, 2017.

Thanks to Michael Lustig, PhD (University of California—Berkeley), Edgar Mueller (Siemens Healthineers), Jason Polzin, PhD (GE Global Research), and Shreyas Vasanawala MD (Stanford University), for patient instruction and clarifying explanation about MRI applications. Thanks to Ron Avitzur and Andrew Donoho for help with Pacific Tech Graphing Calculator Scripts used for the above figures and for movies that were used in the briefing. Thanks to Emmanuel Candès (Stanford University) and David Eisenbud (MSRI) for helpful guidance.

Image Credits

Figures for the article prepared using the Pacific Tech Graphic Calculator 4 for MAC OS. Thanks to Ron Avitzur and Andrew Donoho for help with Graphing Calculator 4.

Photo of David Donoho courtesy of David Donoho.

ABOUT THE AUTHOR

David Donoho is a trustee of the Mathematical Sciences Research Institute and a longtime member of the American Mathematical Society.

David Donoho

LETTERS TO THE EDITOR

On the Mathematization of Warfare and the Militarization of Mathematics

All mathematics is divided into three parts: cryptography (paid for by CIA, KGB and the like), hydrodynamics (supported by manufacturers of atomic submarines) and celestial mechanics (financed by military and by other institutions dealing with missiles, such as NASA).

—Vladimir Arnold, 1999

This statement by the well-known Russian mathematician was an overstatement, but also carries some truth. Intelligence, military, and government agencies have financed whole branches of mathematical research. During the Cold War, for example, the Defense Advanced Research Projects Agency (DARPA) supported large research groups like MAC (Mathematics and Computation) and the Artificial Intelligence Lab at MIT.

At the same time, military funding also influenced directions in mathematics research. Questions on the tactical and technical aspects of warfare were very present amongst researchers and applied mathematics like control theory, optimization, and the basics of cryptology gained more and more popularity.

That was decades ago. How are mathematics and the military connected today?

Wars today are fought in cyberspace via the media, with spy satellites, and with remotely operated drones—tactics that depend heavily on theoretical foundations in various subjects, particularly mathematics. Number theory is central to cryptography, optimal control underpins guidance of cruise missiles and drones, and stochastics and dynamical systems permit handling uncertainty in urban warfare. Many of these applications continue to be funded by military and intelligence agencies.

There is also marked influence in higher education. For one, military applications are mentioned in courses given at colleges and universities. But military and intelligence agencies and arms manufacturers also bond with students early on through well-paid internships, co-op programs, and summer camps. This early contact helps position military stakeholders as attractive employers to young mathematicians and allows them easy access to student knowledge.

Modern, hybrid warfare has some drastic impacts on today's society. These include attacks on citizens' privacy via increased surveillance and reconnaissance techniques as well as influencing the public dialogue through propaganda and strategic communication. There is also physical harm, for example when civilians are injured and killed in drone attacks. Battlefields have thus expanded into civil society.

Mathematics is a resource that fuels modern warfare. This raises questions that the mathematics community should address. How important is military support to contemporary mathematics? How do military applications influence mathematics research? And to what degree are we responsible for the uses to which our subject is put?

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Mathematics People

Weiss, Barish, and Thorne Awarded Nobel Prize in Physics

Rainer Weiss

Kip S. Thorne

The Royal Swedish Academy of Sciences has awarded the 2017 Nobel Prize in Physics to Rainer Weiss, Barry C. Barish, and Kip S. Thorne, all of the LIGO/Virgo Collaboration, for their "decisive contributions to the LIGO detector and the observation of gravitational waves." Weiss receives one-half of the prize; Barish and Thorne share one-half.

According to the prize citation, "LIGO, the Laser Interferometer Gravitational-Wave Observatory, is a collaborative project with over one thousand researchers from more than twenty countries. Together, they have realized a vision that is almost fifty years old. The 2017 Nobel Laureates have, with their enthusiasm and determination, each been invaluable to the success of LIGO. Pioneers Rainer Weiss and Kip S. Thorne, together with Barry C. Barish, the scientist and leader who brought the project to completion, ensured that four decades of effort led to gravitational waves finally being observed.

"Gravitational waves spread at the speed of light, filling the universe, as Albert Einstein described in his general theory of relativity. They are always created when a mass accelerates, like when an ice-skater pirouettes or a pair of black holes rotate around each other. Einstein was convinced it would never be possible to measure them. The LIGO project's achievement was using a pair of gigantic laser interferometers to measure a change thousands of times smaller than an atomic nucleus, as the gravitational wave passed the Earth."

Lydia Bieri of the University of Michigan provided the *Notices* with the following statement about the prizewinners, their work, and the LIGO project:

"In 1972, Rainer Weiss wrote down in an MIT report his ideas for building a laser interferometer that could detect gravitational waves. He had thought this through carefully and described in detail the physics and design of such an instrument. This is typically called the 'birth of LIGO.' Rai Weiss's vision, his incredible insights into the science and challenges of building such an instrument were absolutely crucial to make out of his original idea the successful experiment that LIGO has become.

"Kip Thorne has done a wealth of theoretical work in general relativity and astrophysics, in particular connected with gravitational waves. In 1975, a meeting between Rainer Weiss and Kip Thorne from Caltech marked the beginning of the complicated endeavors to build a gravitational wave detector. Rai Weiss's incredible insights into the science and challenges of building such an instrument combined with Kip Thorne's theoretical expertise with gravitational waves, as well as his broad connectedness with several areas of physics and funding agencies, set the path toward a larger collaboration. Building such an instrument requires an enormous expertise in many areas of physics, astrophysics, mathematics, and engineering. Thus many more people needed to be hired.

"Kip Thorne at Caltech created a gravitational-wave research group and hired Ron Drever from Glasgow University. Very sadly, Drever passed away in early 2017. Ron Drever was most crucial to improving the laser technique required by such a detector.

"The group consisting of Rai Weiss, Ron Drever, and Kip Thorne was called the 'LIGO triumvirate.'

"In 1989 LIGO was funded by NSF, and in 1994 construction began for the two detectors of LIGO.

"In 1994, Barry Barish joined Rai, Kip, and Ron. Barry became LIGO principal investigator that year. Barry Barish was the person who transformed the project from a small group into a large team that was required in order to construct such an experiment and lead it to success. Barry Barish's knowledge on building and managing such a huge project paired with his vision and ability to realize science led to the construction of the instruments and to the creation of the LIGO Scientific Collaboration. He was also crucial in making the successful proposal for building Advanced LIGO, which has observed gravitational waves several times already.

"Rai Weiss, Kip Thorne, Ron Drever, and Barry Barish were all absolutely crucial to make LIGO a success. Each one of them contributed in extraordinary ways."

Rainer Weiss was born in Berlin, Germany, and received his PhD in 1962 from the Massachusetts Institute of Technology, where he is currently professor of physics. Barry Barish was born in Omaha, Nebraska, and received his PhD in 1962 from the University of California. He is Linde Professor of Physics at the California Institute of Technology. Kip Thorne was born in Logan, Utah, and received his PhD in 1965 from Princeton University. He is Feynman Professor of Theoretical Physics at the California Institute of Technology.

— From a Nobel Prize announcement

EDITOR'S NOTE. The August 2017 issue of the *Notices* features the articles "How the Green Light Was Given for Gravitational Wave Search" by C. Denson Hill and Paweł Nurowski and "Gravitational Waves and Their Mathematics" by Lydia Bieri, David Garfinkle, and Nicolas Yunes.

Viazovska Awarded SASTRA Ramanujan Prize

Maryna Viazovska

Maryna Viazovska of the Swiss Federal Institute of Technology, Lausanne, has been awarded the 2017 SASTRA Ramanujan Prize. She is "an extraordinarily gifted mathematician who has made deep contributions to several fundamental problems in number theory."

The prize citation reads: "Maryna Viazovska is awarded the 2017 SASTRA Ramanujan Prize for her stunning and elegant resolution of the celebrated sphere-packing problem in dimension 8, the proof of which appeared in her paper in the *Annals of Mathematics* (2017), and for her joint 2017 paper in the *Annals of Mathematics* with Henry Cohn, Abhinav Kumar, Stephen D. Miller, and Danylo Radchenko, which resolves the sphere-packing problem in dimension 24 by building on her ideas in dimension 8. The prize also recognizes her outstanding PhD thesis of 2013 at the University of Bonn in which she resolved significant cases of the Gross-Zagier Conjecture and her work prior to her PhD with A. Bodarenko and D. Radchenko resolving a long-standing conjecture of Korevaar and Meyers on spherical designs, that appeared in the *Annals of Mathematics* in 2013. The prize notes that the modular forms techniques developed by Viazovska will have a significant future impact in discrete geometry, analytic number theory, and harmonic analysis. The award of the 2017 SASTRA Ramanujan Prize to Maryna Viazovska is in keeping with the tradition of recognizing the spectacular contributions by the most brilliant young mathematicians."

Viazovska was born in Kiev, Ukraine, in 1984. She received her PhD from the University of Bonn in 2013 under the direction of Don Zagier. She has received several awards and recognitions, such as the Salem Prize (2016), the European Prize in Combinatorics (2017), and the Clay Research Award (2017).

The SASTRA Ramanujan Prize was established in 2005 and is awarded annually for outstanding contributions by young mathematicians to areas influenced by the genius Srinivasa Ramanujan. The age limit for the prize has been set at 32 because Ramanujan achieved so much in his brief life of 32 years. The prize will be awarded in December, 2017, at the International Conference on Number Theory at SASTRA University in Kumbakonam (Ramanujan's hometown) where the prize has been given annually.

The prize committee for the 2017 SASTRA Ramanujan Prize consisted of:

- Krishnaswami Alladi, chair
- Andrew Granville
- Winfried Kohnen
- Philippe Michel
- Peter Sarnak
- Michael Schlosser
- Gisbert Wustholz

Previous winners of the SASTRA Ramanujan Prize are:

- Manjul Bhargava and Kannan Soundararajan (two full prizes), 2005
- Terence Tao, 2006
- Ben Green, 2007
- Akshay Venkatesh, 2008
- Kathrin Bringmann, 2009
- Wei Zhang, 2010
- Roman Holowinsky, 2011
- Zhiwei Yun, 2012
- Peter Scholze, 2013
- James Maynard, 2014
- Jacob Tsimerman, 2015
- Kaisa Matomaki and Maksym Radziwill (shared), 2016

—Krishnaswami Alladi, University of Florida

EDITOR'S NOTE. A feature article on Viazovska's work appears in the February 2017 issue of *Notices*.

Venkatesh Awarded Ostrowski Prize

Akshay Venkatesh

Akshay Venkatesh of Stanford University has been awarded the Ostrowski Prize for 2017 "for his groundbreaking work in number theory, the theory of automorphic forms and representation theory, homogeneous dynamics, and arithmetic geometry." The prize carries a cash award of 100,000 Swiss francs (approximately US$102,000).

The prize citation reads as follows: "Venkatesh is notable for his originality and his ability to synthesize between different fields, bringing conceptually new tools to bear against long-standing problems with striking consequences. This not only advances the state of our knowledge, but plants the seed of further progress by exploring and highlighting previously unexplored connections between different mathematical fields.

"Among his notable results are his work on subconvex estimates for L-functions, in part joint with Philippe Michel, where a unified treatment of all previous subconvex estimates for *GL_2* forms is given and new important cases of subconvexity are established by exploiting the link between subconvex estimates and effective equidistribution. Along the way, Venkatesh also proved significant new results regarding sparse equidistribution questions in homogeneous dynamics. This theme of effective equidistribution results and its connection with analytic number theory is further explored in his work with Einsiedler, Margulis, and Mohammadi on effective equidistribution for periodic orbits of semisimple groups, where, in particular, the effective approach allows Vankatesh and his collaborators to prove new equidistribution results that are not approachable even qualitatively by previous techniques.

"The fruitful interaction between a wide range of number theoretic and dynamical techniques are also displayed in his work with Einsiedler, Lindenstrauss, and Michel on a cubic analogue of Duke's well-known results on equidistribution of *CM*-points and in his work with Ellenberg about the very classical problem of the local-to-global principle for representing quadratic forms by a given quadratic form in more variables, dramatically reducing the co-dimension needed for a local-to-global result to hold.

"Recently, another unexpected connection between mathematical fields was explored by Venkatesh in collaborations with Bergeron and Calegari in the study of the diffcult problem of counting torsion classes in the cohomology of arithmetic varieties, where analytic tools from differential geometry and specifically analytic torsion are employed."

Akshay Venkatesh received his PhD in mathematics in 2002 from Princeton University under Peter Sarnak. He has held positions at the University of Western Australia, the Massachusetts Institute of Technology, and Courant Institute of Mathematical Sciences. His honors include the Salem Prize (2007), a Packard Fellowship (2007), the SASTRA Ramanujan Prize (2008), and the Infosys Prize (2016).

The Ostrowski Foundation was created by Alexander Ostrowski, for many years a professor at the University of Basel. He left his entire estate to the foundation and stipulated that the income should provide a prize for outstanding recent achievements in pure mathematics and the foundations of numerical mathematics. The prize is generally awarded every two years.

—From an Ostrowski Foundation announcement

Candès Awarded MacArthur Fellowship

Emmanuel Candès

Emmanuel Candès of Stanford University has been awarded a MacArthur Fellowship, popularly known as a "genius grant," for 2017. According to the prize citation, "Emmanuel Candès is a mathematician and statistician known for developing a unified framework for addressing a range of problems in engineering and computer science, most notably compressed sensing. Compressed sensing is a technique for efficiently reconstructing or acquiring signals that make up sounds and images. Candès's research focuses on reconstructing high-resolution images from small numbers of random measurements, as well as recovering the missing entries in massive data tables.

"Using an approach that draws on concepts from linear algebra and L^1 minimization (a concept of high-dimensional geometry), Candès and colleagues were able to reconstruct high-resolution signals from sparse measurements under specified conditions. In diagnostic healthcare, for example, reducing the number of measurements needed to create high-resolution MRI scans shortens the amount of time patients must remain still in the scanner, an outcome with particularly beneficial implications for children. The ability to process and/or reconstruct audio, visual, and wireless signals from limited data has also led to significant refinements in digital photography, radar imaging, and wireless communications. Candès has expanded this work to address problems in low-rank matrix completion, devising statistical estimation methods for inferring missing entries in data arrays. (This is analogous to trying to identify a customer's movie preferences from the partial movie ratings that the user has provided.) His framework holds promise for phase retrieval, a problem arising in many applications such as crystallography, diffraction imaging (X-ray), and astronomical instrumentation.

"Candès's work at the interface of applied and theoretical mathematics is generating new lines of research in information theory as well as laying the groundwork for improvements in many devices that make use of signal and image processing methods."

Emmanuel Candès received his PhD in 1998 from Stanford University. He was a member of the faculty of Stanford University (1998–2000) and at the California Institute of Technology (2000–2009), before returning to Stanford. His awards and honors include a Sloan Research Fellowship (2001), the Popov Prize (2001), the Wilkinson Prize in Numerical Analysis and Scientific Computing (2005), the Alan T. Waterman Award of the National Science Foundation (2006), the Polya Prize (2010), the Collatz Prize (2011), the Lagrange Prize in Continuous Optimization (2012), the Dannie Heineman Prize (2013), and the 2015 AMS–SIAM George David Birkhoff Prize in Applied Mathematics. He gave the 2011 Erdős Memorial Lecture. He is a member of the AMS and serves on the editorial board for the *Bulletin of the AMS*.

The MacArthur Fellows Program is intended to encourage people of outstanding talent to pursue their own creative, intellectual, and professional inclinations. Each fellowship awards a stipend of US$625,000 to the recipient.

—From a MacArthur Fellowship Program announcement

Schoen Awarded Lobachevsky Prize

Richard M. Schoen

Richard M. Schoen of the University of California Irvine has been awarded the 2017 Lobachevsky Medal and Prize for Outstanding Work in Geometry and Its Applications. The prize carries a cash award of US$75,000. He was honored for his work in differential geometry. According to the prize citation, "he holds fundamental theorems on positive energy in the general theory of relativity, he obtained a complete solution of the famous Yamabe problem on compact manifolds. He also made a fundamental contribution to the theory of regularity of minimal surfaces and harmonic maps."

Schoen was born in Celina, Ohio, and received his PhD in 1977 from Stanford University. His honors include a MacArthur Fellowship (1983), the Bôcher Memorial Prize (1989), the Wolf Prize (2017), the Heinz Hopf Prize (2017), and the Rolf Schock Prize (2017). He tells the *Notices*: "I grew up as the tenth child in a family of thirteen children, eight girls and five boys. We lived in rural Ohio and there were no relatives in previous generations who were educated beyond high school. I was lucky to arrive as a PhD student at Stanford in the early 1970s at a time when nonlinear methods in PDE had been rapidly developing. With my advisors Leon Simon and Shing-Tung Yau we advanced an area of mathematics which was a synthesis of nonlinear PDE with differential geometry, a field which is now called geometric analysis. I think my rural midwestern background has helped me to keep my personal and professional lives in a healthy balance."

—Elaine Kehoe

Ros-Oton Awarded Rubio de Francia Prize

Xavier Ros-Oton

Xavier Ros-Oton of the Universität Zürich has been awarded the 2016 Rubio de Francia Prize of the Royal Spanish Mathematical Society (RSME). According to the prize citation, he was recognized "for contributions to analysis and partial differential equations, including his work on regularity of stable solutions up to dimension seven in domains of double revolution (with X. Cabré), important contributions to boundary regularity for fully nonlinear integro-differential equations (with J. Serra) and to the obstacle problem for integro-differential operators (with L. Caffarelli and J. Serra), and obtaining a Pohozaev type identity for the fractional Laplacian (with J. Serra)." He received his PhD in 2014 from the Universitat Politècnica de Catalunya and served as R. H. Bing Instructor at the University of Texas at Austin from 2014 to 2017 before joining the University of Zurich. In 2017 he was awarded the Antonio Valle Prize from the Spanish Society of Applied Mathematics, given to the best researcher under thirty-four years of age. At age twenty-nine, he was the youngest recipient of the award. Ros-Oton tells the *Notices*: "I grew up in Barcelona, and lived always there until I finished my PhD in mathematics. After that, I moved to Austin, where I found a really nice city with an even warmer weather than in Barcelona. Very recently I moved to Zurich, so now I will have to get used to cold winters and to see snow in my city! I'm looking forward to trekking and exploring the mountains nearby." The prize honors the memory of renowned Spanish analyst J. L. Rubio de Francia (1949–1988). The RSME awards the prize annually to a mathematician from Spain or who has received a PhD from a university in Spain and who is at most thirty-two years of age. The prize is awarded for high-caliber contributions to any area of pure or applied mathematics.

—From a Royal Spanish Mathematical Society announcement

Pardon Awarded 2017 Packard Fellowship


John V. Pardon

John V. Pardon of Princeton University has been awarded a Packard Fellowship by the David and Lucile Packard Foundation. The Fellowship provides a grant of US$875,000 over five years to allow the recipient to pursue his or her research. Pardon's research explores problems in geometry and topology and related fields. Although topological problems are insensitive to the geometry of objects in question, geometric structures often play an unexpected role in the answer to topological questions. Pardon received his PhD in 2015 from Stanford University. He was awarded the AMS Morgan Prize in 2012 and the NSF Waterman Award in 2017. He is an AMS member.

—From a Packard Foundation announcement

2017 CAV Award


Parosh Abdulla


Alain Finkel

The 2017 CAV Award for fundamental contributions to the field of computer-aided verification has been awarded to Parosh Abdulla, Uppsala University; Alain Finkel, Ecole Normale Supérieure de Cachan, Bengt Jonsson, Uppsala University, and Philippe Schnoebelen, CNRS and ENS de Cachan, "for the development of general mathematical structures leading to general decidability results for the verification of infinite-state transition systems." The award carries a cash prize of US$10,000, shared equally among the recipients.

According to the prize citation, the recipients "showed how the abstract notion of a well-quasi-order can be used to identify a large class of infinite-state transition systems with important decidability properties." The principles espoused in their work "were shown to be applicable to an impressively large number of formalisms. Equally important, well-quasi-orders also have been at the heart of many practical contributions that target, among others, the automatic verification of multi-threaded programs and parametric systems."

Jonsson is an educated concert pianist and enjoys "the very Swedish sport of orienteering." Schnoebelen tells the *Notices*: "I am a puzzle enthusiast and started the Retrograde Analysis Corner (now at `www.janko.at/Retros/index.htm`) at some time around 1995, several years before wikis and blogs became fashionable. One of the few problems I composed is at `www.janko.at/Retros/Probleemblad/N1996-05.htm`: It is a chess position that must be reached in a legal game lasting 15 moves, where White and Black cooperate to reach the position. Solving such 'proof game' problems is like solving the *NYT* Sunday crossword: at first we only get parts of the solution, but these help finding other parts, then other parts, until some hidden Easter Egg is revealed and we finish the puzzle in some unusual state of exhilaration."


Bengt Jonsson


Philippe Schnoebelen

—From a CAV Award announcement

Kimsey Awarded Clifford Prize


David Kimsey

David Kimsey of Newcastle University has been awarded the third W. K. Clifford Prize "for his outstanding mathematical research achievements in the field of quaternionic analysis with applications in quantum mechanics." Kimsey received his PhD from Drexel University under H. Woerdeman. He later became interested in quaternionic analysis. The prize citation reads in part: "Spectral theory for normal operators on a quaternionic Hilbert space is a delicate and technical subject due to the noncommutativity of the quaternions. In particular, the proper notion of spectrum is not immediately obvious and turns out to be given by the recently discovered *S*-spectrum. Based on this notion, David Kimsey (in collaboration with Alpay and Colombo) produced a completely rigorous analogue of the spectral theorem for bounded and unbounded normal operators on a quaternionic Hilbert space. This spectral theorem is a crucial tool to formulate the axioms of quaternionic quantum mechanics and as such closed a problem formulated by Birkhoff and von Neumann in 1936." Kimsey also "initiated the study of moment problems, free analysis and interpolation in the context of quaternions."

—Hendrik De Bie, University of Ghent

AWM Inaugural Fellows Chosen

The Executive Committee of the Association for Women in Mathematics (AWM) has established the AWM Fellows Program to recognize individuals who have demonstrated a sustained commitment to the support and advancement of women in the mathematical sciences, consistent with the AWM mission: "to encourage women and girls to study and to have active careers in the mathematical sciences, and to promote equal opportunity and the equal treatment of women and girls in the mathematical sciences."

The Inaugural Class of AWM Fellows has been chosen from living members of the following groups: past presidents of the AWM, AWM Lifetime Service Award winners, and AWM Humphreys Award winners. Additionally, each Fellow of the Inaugural Class is at least twenty-five years beyond her or his terminal degree and has been a member of AWM for at least ten years.

The inaugural fellows are:

- Georgia Benkart, University of Wisconsin, Madison
- Lenore Blum, Carnegie Mellon University
- Bettye Anne Case, Florida State University
- Ruth Charney, Brandeis University
- Carolyn Gordon, Dartmouth College
- Mary W. Gray, American University
- Helen G. Grundman, AMS and Bryn Mawr College
- Ruth Haas, University of Hawaii at Manoa
- Deanna Haunsperger, Carleton College
- Rhonda J. Hughes, Bryn Mawr College
- Linda Keen, City University of New York, Lehman College and the Graduate Center
- Cathy Kessel, senior editor, *Illustrative Mathematics*
- Barbara Keyfitz, Ohio State University, Columbus
- Kristin Lauter, Microsoft Research
- Suzanne Lenhart, University of Tennessee, Knoxville
- Jill P. Mesirov, University of California San Diego
- James Morrow, University of Washington
- Jill Pipher, Brown University
- Judith Roitman, University of Kansas
- Linda P. Rothschild, University of California San Diego
- Bhama Srinivasan, University of Illinois at Chicago
- Jean E. Taylor, Rutgers University and New York University, Courant Institute
- Chuu-Lian Terng, University of California Irvine
- Sylvia M. Wiegand, University of Nebraska—Lincoln
- Carol Wood, Wesleyan University

—From an AWM announcement

Sims Awarded Australian Mathematical Society Medal

Aidan Sims of the University of Wollongong has been awarded the Australian Mathematical Society Medal for 2016 for his work in functional analysis, specifically in operator algebras and their applications in related fields. The Medal is awarded to a member of the Society under the age of forty years for distinguished research in the mathematical sciences. A significant portion of the research work should have been carried out in Australia.

—Australian Mathematical Society announcement

2017 Golden Goose Award Given

Lotfi A. Zadeh of the University of California Berkeley has been named a recipient of the 2017 Golden Goose Award for his work on fuzzy logic. In the early 1960s he recognized the need to translate fuzzy human concepts into concrete instructions for computers, and in 1965 he published the article "Fuzzy Sets." Today "fuzzy math" or "fuzzy logic" is the basis for many important innovations such as devices that understand human speech. It is used in industrial processes, medical diagnosis, home appliances, economics research, fraud detection, and consumer electronics. Zadeh passed away in 2017. The Golden Goose Award recognizes federally funded basic research that has had unexpected and significant societal impact even if the original research sounded obscure.

—From a Golden Goose Award announcement

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Clifford J. Earle: A Life in Mathematics and Music (1935–2017)

Linda Keen and Irwin Kra

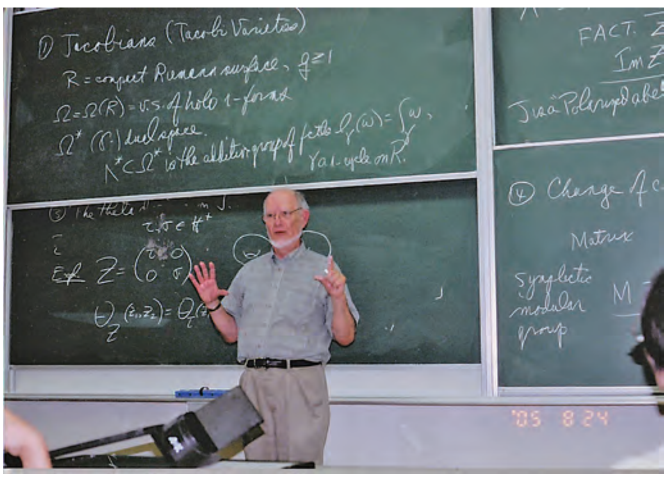

Figure 1. Clifford Earle.

Clifford J. Earle worked in complex analysis and Teichmüller space during his nearly forty years at Cornell. He earned his PhD under Lars Ahlfors at Harvard. In his first position at the Institute for Advanced Study, he persuaded Ahlfors that a set of his lecture notes should be published and undertook to turn them into publishable form. The resulting volume, *Lectures of Quasiconformal Mappings* [1], remains the standard introductory monograph for the field.

The 1950s and 1960s were a great time to be a graduate student in mathematics in the fields of Riemann surfaces and Teichmüller theory because they were undergoing a rebirth. Ahlfors and Lipman Bers had just proved what is now called the Measurable Riemann Mapping Theorem and Ahlfors was working on his Finiteness Theorem for Kleinian Groups, the most significant result on Kleinian groups in the twentieth century. It made it possible to develop the different structures of finitely generated Kleinian groups and to study various boundaries of moduli spaces. There was excitement in the air and rapid progress because so many people, including those from other fields, were attracted to it. Among these were Dennis Sullivan and William Thurston, who created a second revolution in the field. Sullivan gave a new proof of the Ahlfors Finiteness Theorem, realized that its proof also proves the No-wandering Domain Theorem in complex dynamics, and created a dictionary between Kleinian groups and complex dynamics. Thurston used Kleinian groups to develop his theory of hyperbolic three manifolds.

A Riemann surface is a topological surface with a conformal structure. Teichmüller theory provides a way to distinguish distinct conformal structures that occur with a given topology. Cliff's thesis and a lot of his later work dealt with describing natural families of Riemann surfaces and providing moduli for the members that fit together as a pleasing whole "moduli space." A particular approach to this problem, developed by Earle and his co-authors, was via vector bundles over the moduli spaces.

Among Earle's major contributions, we mention only three of the most influential. These were all joint work: first with James Eells in developing a differential geometric approach to Teichmüller theory [2]; second, his decades' long collaboration with Albert Marden on intrinsic coordinates for moduli spaces; and third, his work with Douady on conformally natural extensions of homeomorphisms of the circle.

Over his long career Cliff collaborated with almost twenty others. It was a pleasure to write a paper with Earle. He was always generous in attributions to others. According to Marden, Cliff had the patience and fortitude to carefully study a situation before putting it in print. And then what he wrote, by himself or with others, was carefully presented. His exposition was clear and got to the key points in a very expeditious manner. Most co-au-

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thors counted on him to provide the final version of a manuscript.

He was also very much a scholar. David Drasin, who knew Cliff at Cornell as a graduate student and remained in touch over many years, remembers Cliff having huge sets of handwritten notes in his office on whatever interested in him in mathematics. Bernard Maskit (Figure 2) recalls that "though our work was disparate, it was always a great pleasure to talk to him, both to listen to his carefully thought out ideas and work, and to have him carefully listen to my explanations of my own work." A long-time friend, Maskit found Cliff's decency and steadiness a great help during a rocky period in his life.

Figure 2. Earle with Bill Harvey and Bernie Maskit.

Cliff was an influential teacher. Benson Farb, who was an undergraduate at Cornell, relates this story about Cliff's graduate student days:

"I still tell the story, about twice each year, that Professor Earle told me about his time at Harvard. He arrived ready to do real analysis, but then flunked the complex analysis prelim exam. Ahlfors made him do a huge amount of work, and he studied hard to pass it. He was influenced so much that he became a complex analyst—one of the best in his generation."

He also took his role as citizen of the mathematical community very seriously.

Cliff supervised ten PhD theses and influenced many graduate students and post-docs during his years at Cornell. One of the earliest, Lawrence Harris, wrote about Cliff on the occasion of his 80th birthday:

"Cliff was my thesis advisor during 1966–1969. He gave me many advantages and much to live up to. He shaped how I view mathematical research and what it means to be a mathematician."

Cliff was not only a fine mathematician, scholar, and teacher; he also took his role as citizen of the mathematical community very seriously. At Cornell he served as chair and on many committees over his long career. He served on the editorial boards of a number of journals and as managing editor of the *Proceedings of the AMS.* He was part of the committee that restructured its editorial board. He brought his strong sense of integrity to these roles.

Earle's passion for mathematics was matched by his love of music. He was a serious amateur pianist and performed at many venues in Ithaca and particularly enjoyed accompanying singers. He was especially proud of his performance, with tenor Doug Alfors, of the Schubert song cycles.

In sum, Cliff was a man of great integrity and many talents, and a warm personal friend.

Figure 3. Earle with Albert Marden and Irwin Kra, who was proud to have Earle succeed him as managing editor of *PAMS.*

References

[1] L. V. Ahlfors, *Lectures on Quasiconformal Mappings,* Second Edition, University Lecture Series, vol. 38, *Amer. Math. Soc.*, 2006.

[2] C. J. Earle and Jr. J. Eells, On the differential geometry of Teichmüller spaces, *J. Analyse Math.* **19** (1967).

Photo Credits

Figures 1–3 courtesy of Elizabeth D. Earle.

My Father André Weil

Sylvie Weil

Figure 1: André Weil, quite the dandy! 1930.

Almost twenty years have passed since my father's death on August 6th, 1998, yet he still sometimes calls me: "Sylvie, get me out of here, I'm bored." (The French word he uses is not so polite.)

I am sure that, following Jewish tradition, André was assigned a study companion for all eternity. I had once asked him who this companion would be. "Euler," he answered, and smiled. So when he calls me to tell me he is bored, I ask: "What about Euler? Is he bored, too?"

Nothing horrified my father more than being bored or wasting time. Every moment needed to be usefully or pleasantly employed. I still have my father's letters to me when I was a teenager. He recommended extraordinary programs: evenings given to reading Euripides and Sophocles, Thursdays at the Louvre or the Comédie Française, Sunday afternoons at the Salle Pleyel to hear Beethoven…. The idealism of these letters makes me smile, but reactivates the terrible guilt I felt because, at fifteen, I just wanted to have a good time.

Meals with André could be a bit stressful: We were to have "interesting" topics of conversation. Reciting verses by Racine, or better still, Virgil, would be well received. However it was hard to avoid criticism: "My poor girl, don't

Figure 2: Theatrical André holding a Séance, with Serge Lang in white shirt listening on the floor. Chicago 1950s.

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they teach you to accent Latin verses properly?" might well be André's response. But his program wasn't necessarily austere. He loved films by Satyajit Ray and Kurosawa. He loved swimming, ice skating…. He could also be quite theatrical (see Figure 2). When my sister and I were children, he read us the comedies of Molière and was great at playing the young ingénue, putting on a falsetto voice which sent us into raptures.

Most of the time, I felt it was a privilege to grow up with a father who was not only one of the foremost mathematicians but enjoyed a worldwide reputation for being arrogant, ironical, intimidating. So intimidating that postdocs would send me into his office with my homework to test out the great man's mood. If they heard him yelling at me, they would vanish!

It is precisely André's reputation of arrogance that makes one childhood memory so delicious: the terrifying, arrogant mathematician André Weil, a raincoat thrown over his pajamas, out in the rain, running around the dimly lit courtyard of a shabby motel somewhere in the American West, knocking at doors, desperately begging for a quarter. In our awful room with the two squeaky beds, my mother, my sister, and I had been watching a movie on the coin-operated television when we ran out of coins. André failed in his quest and we never got to know what happened to the beautiful young heroine who was going deaf.

In 2008, as the centennial of Simone Weil drew near, it became obvious that a large number of books about her would be published. I had long played the dubious role of "saint's niece," a kind of "relic," one might say, since perfect strangers felt free to approach me, touch me, even kiss me, as they marveled: "Oh, how you look like her!" Now it was time to write the book that no one else could write.

Simone is, of course, much better known than André. More people are capable, or think they are capable of reading philosophical, political, or mystical writings, than mathematics! And yes, in the eyes of many, she was a saint!

But it seemed unfair not to write about André. It was a question of balance, especially since I had always seen André and Simone as a bizarre pair of twins (Figure 3). *In addition to being a saint, my aunt was a double of my father, whom she resembled like a twin. A terrifying double for me, since I looked so much like her. I resembled the double of my father.*[1]

Figure 3: A couple of bizarre twins, André and Simone, 1922.

Of course this resemblance influenced our relation. André felt that Simone had been overly sheltered from everyday life by their doting mother, so he encouraged my own independence! In one instance, when I was twelve years old, I had to travel across France to join relatives for a holiday. This involved changing trains three times. André wrote the three station masters asking them to meet me and help me. In each station I made sure the station master didn't find me and changed trains on my own. Upon my return home, I told my father. He was very pleased. "I did my duty as a father," he said, "you did your duty as a daughter."

I wanted to recapture, reconstruct a "Weil space."

What I wanted to do in my book was not to write a biography, but to recapture, reconstruct a "Weil space." I will again, if I may, quote from a chapter from my book. The title of this chapter is "The beauty of Euclid."

I read in one of Simone's notebooks:

"The axiomatic system of modern mathematicians. What are they seeking? They do mathematics without understanding its use.

(Ask André: Does he feel pleasure when he succeeds, or aesthetic joy?)"

I read this… and suddenly, without knowing why, I feel good. The parentheses harbor a small family reunion. And the family is mine….

I imagined lunch in the family kitchen, my grandmother's special sauerkraut, a nice bottle of Riesling, and the conversation between Simone and André. Did André tell his sister, as he sometimes told me, that mathematics was not a science, but an art? Did he tell her that the pleasure of experiencing thoughts following one another miraculously, and flowing from one another, is superior to sexual pleasure because it can last for several hours, even several days, as he would write later on?

A picture (Figure 4) taken during the 1938 Bourbaki meeting shows my father, in high spirits, ostentatiously ringing a bell. Simone is there, very serious, bent over her notebook! This photo was taken long before I was born, but that was the cast of characters that surrounded my childhood. The Bourbaki group[2] were passionate, idealistic and, to a certain extent, selfless, signing their articles Nicolas Bourbaki, Université de Nancago (Nancy-Chicago).

[1] *Sylvie Weil,* At Home with André and Simone Weil, *Trans. Benjamin Ivry*

[2] *See the review of* Bourbaki, A Secret Society of Mathematicians *in the October 2007* Notices `www.ams.org/notices/200709/tx070901150p.pdf`.

Figure 4: André rings a bell at a meeting of the composite author Bourbaki, 1938. To his right, Simone is studiously bent over her notebook. To his left are Jean Dieudonné, Charles Ehresmann, and Jean Delsarte. In the foreground with his back to us is probably Henri Cartan.

But the selfless and idealistic passion of the Bourbaki group was wont to loud expression! There was one infamous congress in a small hotel in the Alps, when these gentlemen screamed at each other so violently that the hotel keeper called the gendarmes, fearing someone would be murdered.

I must mention that André's very first passion seems to have been not for mathematics, but for croquet. This yielded or possibly led to a passion for geometry. Selma, my grandmother, humorously but proudly announces this shift in a letter to a friend. André is all of seven years old.

I'm afraid André's ardent nature didn't let much get in his way, especially not conventional etiquette. Once, during a concert in Princeton, there was a commotion; a person sitting in front of André was taken away on a stretcher. The concert resumed, but people whispered. My father angrily demanded they be quiet. A lady hissed at him: "That man was dead, you know !" —"So what!" André replied, "There are worse things than dying while listening to Mozart!" And that was precisely his own wish: to die listening to Mozart. Sadly, I was not able to arrange it.

The year following the cheerful 1938 Bourbaki Congress, WWII began. The story of my father's arrest, in November 1939 in Helsinki, is well known. Rolf Nevanlinna's version of the story tells how he, Nevanlinna, saved André from being shot as a Russian spy. After being transferred to various prisons in Sweden, Denmark, and England, he was imprisoned in France for not having answered the call to the army.

As I was writing my book, I found in a box of documents a small sheet of paper. I immediately called it "A family portrait." Four short sentences, four handwritings, all so familiar to me as to be, in fact, real people. A fifth person appears on the letter in the form of a large, crude dark blue zigzag, the prison supervisor, no doubt. In February 1940, André is in the Rouen prison. The family has come to see him and the guard on duty refuses them entry. No visit. I imagine my grandmother Selma having one of her very persuasive attacks of nerves. The guard is persuaded, he will accept a letter. The four go to the café, there is always a café across the street from a jail. They each write a sentence. First come three blue sentences: Selma, Simone, and Eveline, my mother. Probably my mother's pen. My grandfather refuses to write in blue ink, maybe he borrows a pen from the café. His sentence is written in black ink.

I could construct a whole Weil family scene.

A group photo would have lied, because everybody would have tried to look pleasant, perhaps to smile, feelings camouflaged. No camouflage in these four sentences! Each, while necessarily brief, is absolutely true to character, revealing the emotional relationship each person has with André. Selma and Eveline compete in expressions of affection, Simone hopes her brother is writing poems and dreaming up beautiful theorems. Bernard, less effusive than the women, hopes they will have the pleasure of seeing André again soon. From this slip of paper, I could construct a whole Weil family scene. As if I had been there.

How did it end? After some months, André was judged, went to the army (see Figure 5), then was evacuated to England.

In 1994, my father was awarded the Kyoto Prize. I accompanied him to Japan. Japan was a mythical place for me, an imaginary land described by André when my sister and I were young girls. In 1955, after a stay in Japan, he had returned obsessed with Japan. He taught

Figure 5: Military man! (in French Army) 1940.

us to bow, eat with chopsticks, use tiny bath towels. When the phone rang we would rush to pick it up and answer *Mush'mashi!* "In Japan," André explained, "you never display your feelings. It is impolite. You must always smile." We practiced being polite, hiding little giggles behind our hands. We were Japanese.

Now I was in Kyoto with my father. The first two evenings, we left the luxury hotel which he despised, to go have dinner in a modest restaurant recommended by one of the hotel maids. As we walked slowly down a dark little street, André explaining and commenting on some of the things we were seeing, I felt I had returned to the imaginary Japan of my childhood.

André was happy to meet Akira Kurosawa. "I have a great advantage over you," he said to the famous director, "I can love and admire your work, whereas you cannot love or admire my work." Some people saw this as a backhanded compliment. They were wrong. André was perfectly sincere.

The last morning in Tokyo, as we were waiting for taxis to take us to the Imperial Palace, the silence was oppressive and André became bored. He turned towards Kurosawa: "Does the Emperor like your films?" There was a brief silence, then, with a small bow, came the answer: "His majesty is a great Emperor."

In Kyoto, there were endless ceremonies. These demanded "performances." But now André was old. He didn't want to perform or bow. He no longer wanted to be Japanese. I was in charge of him. At times I felt I was a Bunraku puppeteer and he was my puppet! Sometimes I wished I could get a beautiful Japanese mask, or even a terrifying red and gold demon mask for my old father who didn't want to smile or be polite.

In the end, he didn't need a mask. I look at the official photo of the three recipients of the Kyoto Prize (Figure 6). Kurosawa's slightly amused distant little smile, the tall, portly American scientist's wide happy smile. And André, the old dwarf jammed between the two giants, having the last laugh. He has pulled his hand away from the pile of hands, a good strong hand. He is free!

Figure 6: The three Kyoto Prize laureates 1994: Paul Christian Lauterbur, André Weil, Akira Kurosawa.

EDITOR'S NOTE. See the review of Sylvie Weil's book, *At Home with André and Simone Weil*, in the May 2011 *Notices,*[3] André Weil memorial articles in the April, June–July, and September 1999 *Notices,* and a piece in the March 2005 *Notices.*[4]

Photo Credits

Figures 1–5 courtesy of Sylvie Weil.
Figure 6 courtesy of Inamori Foundation Media.
Author photo courtesy of Sylvie Weil.

ABOUT THE AUTHOR

Sylvie Weil is the daughter of mathematician André Weil and the niece of philosopher Simone Weil. Her book, *At Home with André and Simone Weil,* an intimate portrait of one of the twentieth century's most intriguing intellectual families, has been translated from the original French into many languages.

Sylvie Weil

[3] www.ams.org/notices/201105/rtx110500697p.pdf

[4] www.ams.org/notices/200503/fea-weil.pdf

Allyn Jackson Concludes 31-year Career at the AMS

We celebrate Allyn Jackson's 31-year career at the AMS, including service as *Notices* senior writer and deputy editor, which concluded in 2017.

Among the most memorable articles she wrote for the *Notices* are a two-part biography of Alexander Grothendieck (October and November 2004), a piece on the demise of the NSF Young Scholars Program (March 1998), a two-part article on the "math wars" (July/July and August 1997), and an investigative piece on the tenure case of Jenny Harrison (July/August 1994). Jackson described the workings of ten mathematical sciences institutes in a series of articles and did in-depth interviews with major mathematicians. One of her most recent and widely read articles is "Can Mathematics Light the Way?" (May 2017).

As deputy editor, she shepherded into print hundreds of articles on a wide variety of topics. In 2002 she founded the "WHAT IS...?" column, soliciting and editing more than 100 pieces. She also solicited and edited nearly all of the book reviews published in the *Notices* over the past 20 years.

She performed a myriad of other writing and editing tasks for the Society, including launching the "Math Digest" feature on the AMS website and writing numerous news releases and other press material.

Jackson's wisdom and her broad knowledge of mathematics, mathematicians, and the *Notices* are irreplaceable, and we can only wonder what we will do without them. We cannot thank enough this person whose name has been most closely associated with *Notices* and its history. We wish her every happiness and success in the years ahead.

—Frank Morgan, Editor-in-Chief

Native Script in MathSciNet: Celebrating Diversity

by Allyn Jackson

Mathematics has forever been an international subject, carried out by people all over the planet. Today the professional output of the field is dominated not only by the English language but also by the Roman alphabet. There are many legitimate reasons for this dominance, but it does tend to mask the diversity of the people who do mathematics. One small way of revealing and celebrating this diversity is being carried out through the "native script" capability in MathSciNet®.

Each of the approximately 857,000 authors in MathSciNet has an individual author page that presents data about the person: full name, names of co-authors, number of publications, areas of those publications, and other information. Authors can personalize their pages by, for example, adding a photo of themselves or a link to a personal website.

In addition, those whose names are properly written using characters or an alphabet other than Roman can now have their names appear on their MathSciNet author page in native script. For example, on the author page for Japanese Fields Medalist Shigefumi Mori, one also sees his name as 森 重文. For some deceased mathematicians, *Math Reviews* staff have added names in native script, so that, for example, the Russian mathematician Andreĭ Nikolaevich Kolmogorov also appears as Андрей Николаевич Колмогоров.

In some cases, names that differ in native script end up identical when transliterated into Roman letters. The native script capability helps to avoid such ambiguity and to allow clearer identification of authors.

The availability of native script in MathSciNet was made possible by a donation from TEX creator Donald Knuth (see sidebar on Knuth, page 61), who has an abiding love for all things typographical—as well as a deep appreciation of diversity. "I view mathematics and computer science as a collaboration between thousands and thousands of people from many different cultures," he said. "Many historians present mathematics as consisting of a few great people who reached some milestones. I think of it more like the Great Wall of China—it's made out of a million individual bricks, each of which were important stones."

TEX creator Donald Knuth's donation made possible native script in MathSciNet author pages. Here he is lecturing at Case Western Reserve University in May 2010.

Knuth has long paid meticulous attention to how people's names are written. In 1967, when he prepared the index to his now-classic *The Art of Computer Programming (TAOCP),* he decided to put in the full name—including middle name, not just initials—of every author who appeared there. In the 1980s, he used an early prototype of TEX to add Chinese and Japanese names in native script, and by the 1990s, he could include names in a wide range of alphabets and characters. "At last I did not have to rely entirely on transliteration when listing the name of the father of algorithms, Abu Ja'far Mohammed ibn Mūsā al-

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Khowārizmī," wrote Knuth in a piece that appears in the "Recent News" section of his website.[1] The name is written in Arabic as الخوارزمی موسی بن محمد.

Over the years, many people commented favorably to Knuth about this feature of *TAOCP*. "It has turned out to be a beautiful way to relish the fact that computer science is the result of thousands of individual contributions from people with a huge variety of cultural backgrounds," he wrote. To this day Knuth still works hard to include people's full names in his writing, often going to great lengths to uncover the names initials stand for. He even has a help page on his website where he offers US$2.56 to those who can supply a missing name. "A lot of people have earned US$2.56 in this way," he noted. His own MathSciNet author page includes his middle name (Ervin) as well as his Chinese name, 高德納.

Supporting the MathSciNet native script capability is of a piece with Knuth's appreciation for the beauty of names and their typographical representation. "My goal is to have the message [of the native script capability] get out to the people whose names up until now never appeared in an AMS journal because they were written in an exotic alphabet," Knuth said. "But also, I hope ordinary Americans and Germans and Poles and so on will update their MathSciNet author profiles as well." Updating is easy: Just visit your page in MathSciNet and click on the blue "Edit Author Profile" link.

The author profile page on MathSciNet provides an unostentatious way for authors to clearly identify themselves "among the billions of people on this planet," said Knuth. And having names appear in native script provides a new way to celebrate the human side of our discipline. "It shows how much we owe to all the different strands that make up mathematics."

Photo Credits

Photo of Donald Knuth lecturing courtesy of Dasha Slobozhanina.
Sidebar photo of Donald Knuth at the keyboard courtesy of Peter Badge.

Native Script Full Names of a Few Randomly Chosen Mathematicians

Ngô Bảo Châu: ŋo ɓãːʊ̯ cəʊ̯

Elon Lindenstrauss: לינדנשטראוסאילון

Heisuke Hironaka: 広中 平祐

Shiing Shen Chern: 陳省身

Srinivasa Iyengar Ramanujan: ஸ்ரீநிவாச இராமானுஜன்

Marina Evseevna Ratner: Мари́на Евсе́евна Ра́тнер

Archimedes: Ἀρχιμήδης

Karen K. Uhlenbeck: Karen Keskulla Uhlenbeck

V. F. R. Jones: Vaughan Frederick Randal Jones

Michael Atiyah: Michael Francis Atiyah (عطية مايكل)

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[1] www-cs-faculty.stanford.edu/~knuth/news17.html.

Math, Music, Mystery: A Brief Profile of Donald Knuth

Donald Knuth playing the organ in 2012. An accomplished musician and composer, he had a hard time deciding to major in physics rather than music when he was an undergraduate at Case Institute of Technology (now part of Case Western Reserve University).

Now 79 years of age, Donald Knuth is a legendary figure in the computer and mathematical sciences. He earned his PhD in mathematics in 1963 at Caltech, where his adviser was Marshall Hall. Knuth has made foundational contributions to several areas of computer science and also had an enormous impact through his multi-volume opus *The Art of Computer Programming.* His invention of the TeX typesetting system revolutionized publication in the sciences and had a huge influence on communication among mathematicians. His long list of honors includes the 1974 Turing Award of the Association for Computing Machinery, the 1979 US National Medal of Science, and the 1996 Kyoto Prize of the Inamori Foundation.

Erudite and possessing broad knowledge, Knuth is nevertheless exceedingly modest and easy to talk to. He seems to say the first thing that pops into his mind, with a benevolent candor that's leavened by a quirky sense of humor. This unusual mix of qualities accounts for the success of his periodic lectures that go under the title "All Questions Answered," in which people show up and ask Knuth anything they want.[2]

Taking the interview for this article as a mini-instance of "All Questions Answered," this reporter asked about the question of P versus NP. "It's probably true that P equals NP, but we will never know why," Knuth answered. The question has two aspects, he explained. The first: Given a computational problem, does there exist a polynomial-time algorithm for its solution? And the second: Is that algorithm knowable—that is, can we actually write it down? "What I suspect is that there is some algorithm, it's out there, but it's so complicated that for practical purposes, it makes no difference because nobody will ever know what it is," he said. A suggestive example comes from the Robertson-Seymour theorem, which says that for any minor-closed family of graphs, there exists a polynomial-time algorithm to recognize whether a given graph belongs to the family. But "almost never do we know what the algorithm is."

Another totally different question: Why does he believe in God? He discussed this topic in six public lectures he gave at MIT in 1999 titled "Things a Computer Scientist Rarely Talks About."[3] "So many people have crazy opinions about religion, and that has given it a bad name," Knuth remarked. When he said in the lectures that he is a Christian, it felt "a little bit like coming out of the closet." It's different for Jews, he noted: in the audience for his lectures, there was little respect for someone being serious about religion, unless the person was Jewish.

The lectures didn't attempt to tell anyone what to think or believe. "I only said, these are important things and hard to contemplate, but you don't use standards of proof on them," he said. The question of the existence of God is "inherently unprovable." "I kind of like that," he went on. "I think my life would be incomplete if I knew everything, if there were no mystery. On the other hand, it would also be incomplete if it were all mystery. So I am willing to recognize the limits of what I know, but I'm also glad that mathematics gives me a chance to surround something and say, 'Yes, I do have some idea of truth.'"

And music? It's a major passion for Knuth and figures in his unusual plan for celebrating his 80th birthday on January 10, 2018. Fulfilling a lifelong dream, Knuth has composed a 90-minute organ work called "Fantasia Apocalyptica,"[4] based on the *Book of Revelation.* On his 80th birthday, one of the world's foremost organists, Jan Overduin, will perform the piece at the Studio Acusticum in Piteå, Sweden, which has a state-of-the-art organ. Projected onto screens in the concert hall will be several video tracks: one shows the *Book of Revelation* text in the original Greek with English translation; another displays the organist's score; and a third gives a comic-book style depiction of the action. The latter was created by Duane Bibby, who illustrated *The TeXbook.* The premiere performance will cap a three-day international symposium called Knuth80, at which several dozen leading computer scientists, mathematicians, and typographers will speak.

"This event is really crazy, because who wants to go to northern Sweden in January?" Knuth remarked. "On the other hand, it's just crazy enough that it appeals to me. It's once in a lifetime. Sweden is actually a lot warmer than Chicago at that time of year. It's also got Northern Lights. And there won't be trouble getting hotels."

[2] *An edited transcript of one such lecture, held in Munich in 2001, appeared in the March 2002 issue of the* Notices, `www.ams.org/notices/200203/fea-knuth.pdf`.

[3] *Annotated transcripts were published in 2001 under the same title.*

[4] `www-cs-faculty.stanford.edu/~knuth/fant.html`.


The AMS Mathematics Calendar is now an online-only product!
Please note that the *Notices of the American Mathematical Society* no longer includes these listings.
You can submit an entry to the Mathematics Calendar at **www.ams.org/cgi-bin/mathcal/mathcal-submit.pl**
Questions and answers regarding this page can be sent to **mathcal@ams.org**.
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How Mathematics Research Journals Select Articles

by Harriet Pollatsek

The *Notices* invited editors of four highly rated non-specialized research journals in mathematics to respond to questions about how they handle submission and consideration of papers.

Notices: What kinds of referee reports typically lead to acceptance or rejection of a paper?

Annals of Mathematics of Mathematics: With the quick reports, all reports should be enthusiastic to proceed. Once a full report is sought and received and indicates that the "serious" referee thinks the paper is suitable and is readable and is correct, then it is brought to the editors meeting for a vote.

Duke Mathematical Journal Mathematical Journal: The reports need to convince me that the paper has importance and novelty, represents a substantial advance in a subject, and is not better suited for a more specialized journal.

Pacific Journal of Mathematics Journal of Mathematics: Acceptance: a strongly positive report by a respected and active research mathematician on a paper in his or her own field. Rejection: A lukewarm or negative report(s) by the same.

Transactions of the American Mathematical Society: For acceptance, it should be a substantial report explaining why the results in the paper are new and significant.

Notices: Do editors typically confer on the decision to accept or reject a paper?

Annals of Mathematics of Mathematics: For acceptance yes; for rejection at an early stage, no.

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Duke Mathematical Journal: The managing editor always confers with one of the editors, if possible; but the managing editor alone is
responsible for the decision on acceptance or rejection.

Pacific Journal of Mathematics: Rejection decisions are made by the handling editor without oversight.

Acceptance recommendations are made by handling editors to the managing editor. The managing editor either agrees (the typical case) or refers the paper back to the handling editor with a request for further explanation of the decision or further review.

Transactions of the American Mathematical Society: Usually our editors make the decisions autonomously. On occasion we confer. I am always inclined to defer to my colleagues' expertise in their field.

Notices: When a paper is received by an editor, is it always sent to at least one referee? Under what circumstances is it sent to more than one referee?

Annals of Mathematics: No. Certain papers are rejected automatically if they make no mathematical or other sense. Other than the auto-reject papers, a quick opinion as to suitability for the *Annals of Mathematics* is requested from one or more experts. If the opinions are positive one seeks to have the paper fully refereed by one or more experts.

Duke Mathematical Journal: All papers are received by the managing editor. Initial opinions from several experts are solicited; the majority of submissions are thus rejected at this stage. If there is unanimous support, at least two full referee reports are solicited.

Pacific Journal of Mathematics: First part: No, *PJM* editors do receive manuscripts that are manifestly inappropriate for it. Editors will reject these without review, using a customizable form letter. Second part: Most editors use just one referee, but several routinely ask several referees.

We also have a formal "informal opinion" category that is very useful for preliminary evaluation.

Transactions of the American Mathematical Society: We start by asking for a quick opinion by an expert. If that is positive then it's sent to a referee. Multiple referees are rare but might be necessary for long manuscripts for the *Memoirs.*

Notices: If a paper is sent to more than one referee, how does the editor handle the multiple reports?

Annals of Mathematics: As far as quick opinions, if there are two or more reports and not all are favorable the paper is rejected (and quickly) by one of the editors.

Duke Mathematical Journal: The managing editor reads them and decides whether a decision is clear; if not, he solicits more reports.

Pacific Journal of Mathematics: This is the handling editor's choice.

Transactions of the American Mathematical Society: Multiple reports would inform the editor and author collectively. I have never seen radically different reports.

Notices: Are papers considered singly or in groups and in comparison to each other?

Annals of Mathematics: Once a paper has gone through the full refereeing process and all reports recommend acceptance, the paper is presented to all the editors at a bimonthly meeting. It is at that meeting that papers are voted on and finally accepted.

Duke Mathematical Journal: Papers are generally considered singly, except when a group of papers on similar subjects are being considered at the same time.

Pacific Journal of Mathematics: Singly.

Transactions of the American Mathematical Society: They are considered singly but with full knowledge of the quality and scope of the journal and hence of other submissions. Referees do a good job of putting papers in context.

Notices: Is there a mechanism by which an author can indicate a possible conflict of interest by one or more potential referees? If so, how would you handle it?

Annals of Mathematics: The author can make such a request in his or her submission letter.

Duke Mathematical Journal: Authors will sometimes include a cover letter explaining their concerns. More frequently, a referee will back out because of a possible conflict of interest. I sometimes ask a neutral third party about a perceived conflict.

Pacific Journal of Mathematics: No, but authors with concerns may (and do) email the handling editor. In my opinion it would be good to have entries for conflicts and referee suggestions on the online submission form.

Transactions of the American Mathematical Society: This is extremely rare. I suppose authors could inform the editor.

Notices: Do you recommend that an author communicate with the appropriate editor before submitting his or her manuscript?

Annals of Mathematics: No.

Duke Mathematical Journal: No.

Pacific Journal of Mathematics: No.

Transactions of the American Mathematical Society: Probably not. The journal should have a good FAQ page though.

Notices thanks the four editors:

Annals of Mathematics of Mathematics: Peter Sarnak is one of six members of the editorial board; there is no managing editor. There are also six associate editors.

Duke Mathematical Journal Mathematical Journal: Jonathan Wahl is managing editor. He serves with four main editors and with the cooperation of 17 associate editors.

Pacific Journal of Mathematics Don Blasius is managing editor. He serves with 10 editors.

Transactions of the American Mathematical Society of the AMS: Alejandro Adem is managing editor. He serves with a 20-person editorial board, four of whom are designated coordinating editors.

EDITOR'S NOTE. See also "How to Deal with a Mathematics Journal" by Scott Chapman, then editor of *The American Mathematical Monthly,* in the February 2016 *Notices* `www.ams.org/publications/journals/notices/201602/rnoti-p182.pdf` and the London Mathematical Society guide for authors `www.lms.ac.uk/sites/lms.ac.uk/files/Publications/AuthorGuide.pdf.`

Inside the AMS

Trjitzinsky Awards Given

The AMS has made awards to seven undergraduate students through the Waldemar J. Trjitzinsky Memorial Fund. The fund is made possible by a bequest from the estate of Waldemar J., Barbara G., and Juliette Trjitzinsky. The will of Barbara Trjitzinsky stipulates that the income from the bequest should be used to establish a fund in honor of the memory of her husband to assist needy students in mathematics

For the 2017 awards, the AMS chose seven geographically distributed schools to receive one-time awards of US$3,000 each. The mathematics departments at those schools then chose students to receive the funds to assist them in pursuit of careers in mathematics. The schools are selected in a random drawing from the pool of AMS institutional members.

Waldemar J. Trjitzinsky was born in Russia in 1901 and received his doctorate from the University of California, Berkeley, in 1926. He taught at a number of institutions before taking a position at the University of Illinois, Urbana-Champaign, where he remained for the rest of his professional life. He showed particular concern for students of mathematics and in some cases made personal efforts to ensure that financial considerations would not hinder their studies. Trjitzinsky was the author of about sixty mathematics papers, primarily on quasi-analytic functions and partial differential equations. A member of the AMS for forty-six years, he died in 1973.

Following are the names of the selected schools for 2017, the names of the students receiving the awards, and brief biographical sketches of the students.

Skidmore College: Alexandra Cassell is a declared double major in mathematics and computer science at Skidmore College. She was inducted into the New York Alpha Theta Chapter of Pi Mu Epsilon as a sophomore. Alexandra is also a member of the Periclean Honors Forum, the Skidmore College Honors program. She has one of the highest GPAs among Skidmore College mathematics majors and has made the dean's list every semester. This past summer, she participated in the collaborative research program at Skidmore College, working on a project in mathematical biology, where she and another student studied mathematical oscillator models for group behavior of fruit flies. She will work for the mathematics department this year as a quantitative reasoning tutor helping students who need help with basic mathematics. At school, she is interested in everything from mathematical biology to artificial intelligence and engineering. Her dream job would be to work for NASA programming the Mars Rovers.

Providence College: Keysha Rodriguez is a member of the class of 2019 at Providence College. She has declared a Mathematics major with a Business Studies certificate. She also holds a number of leadership positions on campus: Coordinator for the Peer Mentoring Program for first-generation students, Treasurer for the Organization of Latin American Students, and an avid member of the Board of Multicultural Students. As a first-generation student herself, it was hard for her to enter the college atmosphere, let alone thrive in it, but she continues to do so. She has an overarching love for mathematics that is rare in itself; she will spend hours upon hours at her whiteboard practicing problems and listening to music. She declared Mathematics her sophomore year and she says that she does not specifically know why; she just figures one can never go wrong with something they love.

Eastern Michigan University: Katherine Maza is a recent transfer student to Eastern Michigan University. After taking a long break from her studies to stay at home with her children, she returned to school with the intent to complete a bachelor's degree in Architecture. Upon taking a placement mathematics test and placing at a developmental mathematics level, she became determined to increase her knowledge with self-preparation to become eligible for college-level mathematics courses. After two months of diligent study, Katherine successfully placed into Precalculus. She discovered in the process how much she enjoys mathematics and decided she would like to teach mathematics and show others how valuable and enjoyable it can be. Katherine changed her major to mathematics and is working toward the B.S. Mathematics degree.

University of Dallas: Mary Kate Tomassi is a sophomore math major from Portland, Oregon. She has an interest in both mathematics and computer science, and she is currently exploring what direction her undergraduate studies might take her. Mary Kate is one of those rare students who not only excels in mathematics but who also embraces the mathematics culture at the University of Dallas by working in the department office. She embodies the diligent student who loves learning mathematics and gives fully to the department and her peers.

Lamar University: Destiny Faith Allain graduated valedictorian from Warren High School, Warren, Texas, in

2014 and is a senior working on her degree in mathematics at Lamar University. She struggled with the decision of which degree to pursue. She started at Lamar University in the fall of 2014 as a social work major, then moved to biology. Along the path, she also considered mechanical engineering. For the 2017–2018 academic year, she was awarded the prestigious Mary Katherine Bell Regents Scholarship in Mathematics. Upon graduation, she plans on pursuing a career as an applied mathematician in the oil industry.

University of California Riverside: Eric Zarate has always loved math. "I remember my fifth grade teacher saying, 'today we are going to learn Algebra; everyone pay attention because if you do not understand algebra now, it will be your worst nightmare for the rest of your life.'" Eric took that as a challenge and sought tutoring from his father, a man who understands numbers. His first language is Spanish, but he tutored Erik, speaking in broken English, which made it hard to understand him. "I spoke Spanish all my life but my dad taught me that numbers are a completely different language in itself." Eric applied to UCR because it was close to home, but there were too many variables in the equation and too little time. His family lost their house, he didn't have a ride to school, and money was too tight. He was the first of his siblings to make it to a university and the first to drop out. After two years, he was able to return, and he recently finished his first year at UCR. "Life is always going to throw obstacles my way, but I am determined to finish what I started."

St. Mary's College of California: Sabrina Garcia is a senior mathematics major at St. Mary's College. A native of San Jose, Sabrina attended Oak Grove High School, where she participated in soccer and softball, as well as excelling at her studies. Sabrina is the first of her family to attend college and took classes at several junior colleges in and around San Jose, as well as spending a frigid year in Boston at Northeastern before returning to California .

—AMS Trjitzinsky Fund announcement

Erdős Memorial Lecture

The Erdős Memorial Lecture is an annual invited address named for the prolific mathematician Paul Erdős (1913–1996). The lectures are supported by a fund created by Andrew Beal, a Dallas banker and mathematics enthusiast. The Beal Prize Fund is being held by the AMS until it is awarded for a correct solution to the Beal Conjecture (see www.math.unt.edu/~mauldin/beal.html). At Mr. Beal's request, the interest from the fund is used to support the Erdős Memorial Lecture.

Andrea Bertozzi of the University of California Los Angeles will present the 2018 Erdős Memorial Lecture during the Spring Southeastern Sectional Meeting at Vanderbilt University, Nashville, Tennessee, April 14–15, 2018. For more details, see www.ams.org/meetings/lectures/meet-erdos-lect.

—AMS announcement

JPBM Statement on NSF Big Ideas

In 2016, the National Science Foundation (NSF) introduced its Ten Big Ideas, identifying areas of national importance for future investment at the frontiers of science and engineering (S&E). These ideas include several in which mathematics and statistics play a key role, including Harnessing the Data Revolution, Understanding the Rules of Life, and the Quantum Leap. In her testimony before the Subcommittee on Research and Technology for the Committee on Science, Space, and Technology of the US House of Representatives, NSF Chief Operating Officer Joan Ferrini-Mundy signaled the role these ideas may play in funding decisions: "Funding the research that will advance these ideas, and efforts to develop the talented people who can pursue them, will push forward the frontiers of US-based science and engineering, contribute to innovative approaches to solving some of the most pressing problems the world faces, and lead to unimagined discoveries that can change lives."

The NSF is the only federal agency that funds basic research in all fields of fundamental S&E, and over 60 percent of funding for research in the mathematical sciences comes from the NSF. The Ten Big Ideas are already driving forward new funding opportunities for the mathematical and statistical science research communities, such as Transdisciplinary Research in Principles of Data Science (TRIPODS) and the NSF-Simons Research Centers for Mathematics of Complex Biological Systems (MathBioSys).

The Joint Policy Board for Mathematics—consisting of the American Mathematical Society (AMS), American Statistical Association (ASA), Mathematical Association of America (MAA), and Society for Industrial and Applied Mathematics (SIAM)—has issued a statement on ***Mathematics and Statistics Community Engagement with NSF Big Ideas*** that can be found here: bit.ly/2z9ZqK1.

—AMS Washington Office announcement

AMERICAN MATHEMATICAL SOCIETY

Advancing research. Creating connections.

We are delighted to share the new AMS logo with you. The official launch will take place at the 2018 Joint Mathematics Meetings.

Our new logo underscores that the AMS is a society of individuals with diverse backgrounds, inspirations, and goals who collectively pursue and advance mathematics research and education. This new logo will favorably represent our organization and help us stand out in the landscape of other mathematical and scientific organizations.

Please note:

- The three spiraling streams of dots of different sizes departing from a distinct origin are dynamically moving forward. Each stream of dots also represents the varied and expanding activities of the Society, as well as our goal to serve mathematicians throughout their long careers.
- Eliminating the closed circle from our old logo demonstrates we are an open and welcoming membership organization.
- The reason the image does not reference any specific mathematical symbol is to reflect our support for the diversity of disciplines within mathematics.
- The font and the blue color from our old logo have been retained to provide continuity. Orange has been added as an energetic and welcoming color, which you may also recognize as a match to the covers of the old paper copies of *Mathematical Reviews* (now available as MathSciNet®).

The new logo is part of the implementation of our 2016–2020 Strategic Plan, which comprises six initiatives addressing diversity; membership; visibility; enhancements to MathSciNet; growth and innovation in our publishing program; and coherence in our portfolio of programs, meetings, publications, and professional services. This plan will strengthen the Society in many key ways and position us well for the coming decades.

The initiative entitled Advocacy, Awareness & Visibility directs us to "create new and consistent branding across the AMS for its publications, programs, and services."

Survey research revealed that AMS members are less aware of the range of our publications, programs, and services than we realized. The rationale for creating new branding is based, in part, on a desire to increase this awareness and to signal to the mathematics community that the AMS is moving forward in new and important ways. A new logo is just the first step. Moreover, market research on our current logo revealed that the connection between a Greek temple and a mathematical society has become increasingly tenuous among non-members and younger mathematicians, who associate the Greek temple with a financial institution.

This new logo is distinct from that of all other mathematical organizations today. We believe its consistent use across all of our communications will help the AMS stand at the forefront of the mathematics community.

We hope you share our excitement about our new logo and tagline. You can always, of course, reach out to any of us to share any of your thoughts, concerns, and ideas at **branding@ams.org**.

Thank you.

Kenneth A. Ribet
AMS President

Robert Lazarsfeld
Chair, AMS Board of Trustees

Jennifer Taback
Executive Committee of the AMS Council

Catherine A. Roberts
AMS Executive Director

Find the data of interest to you!

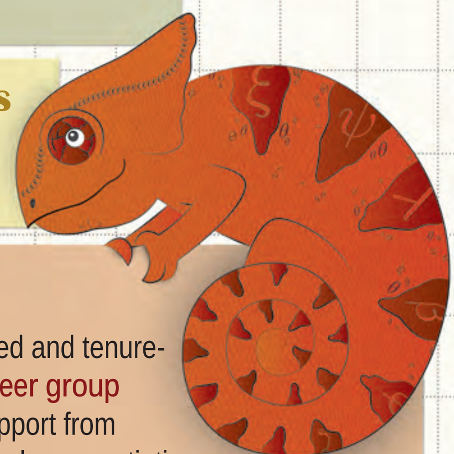
www.ams.org/annual-survey

New Doctoral Recipients

Find out about new PhD graduates, their employment plans, demographics, and starting salaries. The report provides valuable information for both prospective graduate students and the wider mathematics community.

Doctoral Degrees and Thesis Titles

See a listing of PhD graduates, their thesis titles, and where they earned their degrees.



Faculty Salaries

See the recent academic salaries for full-time tenured and tenure-track Mathematical Sciences faculty. Specialized peer group analyses are used by department chairs to gain support from administrators for program expansions and faculty salary negotiations.

Recruitment & Hiring

Get a comprehensive picture of the academic job market in the Mathematical Sciences. Check out the number of full-time faculty positions (tenure-track and non-tenure-track, postdocs, instructors, lecturers, etc.) that were under recruitment and filled.

Departmental Profile

Learn about the number of faculty in various categories, undergraduate and graduate course enrollments, the number of graduate students, and the number of masters and bachelors degrees awarded in the Mathematical Sciences.

Annual Survey
of the Mathematical Sciences
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Mathematics Opportunities

Listings for upcoming math opportunities to appear in Notices *may be submitted to* notices@ams.org *.*

AMS-Simons Travel Grants Program

Starting February 1, 2018, the AMS will begin accepting applications for the AMS-Simons Travel Grants program, with support from the Simons Foundation. Grants provide early-career mathematicians with US$2,000 per year for two years to reimburse travel expenses. Apply at www.mathprograms.org. The deadline is **March 31, 2018**.

—AMS announcement

Simons Foundation Collaboration Grants for Mathematicians

The Simons Foundation invites applications for Collaboration Grants for Mathematicians (US$8,400 per year for five years). The application deadline is **January 31, 2018**. See tinyurl.com/zvoqm5t.

—From a Simons Foundation announcement

*NSF Collaborative Research in Computational Neuroscience

The National Science Foundation, the National Institutes of Health, the German Federal Ministry of Education and Research, the French National Research Agency, the United States-Israel Binational Science Foundation, and Japan's National Institute of Information and Communications Technology support collaborative activities advancing the understanding of nervous system structure and function, mechanisms underlying nervous system disorders, and computational strategies used by the nervous system. The deadline for full proposals is **January 5, 2018**. See nsf.gov/funding/pgm_summ.jsp?pims_id=5147.

—From an NSF announcement

*NSF Major Research Instrumentation Program

The National Science Foundation Major Research Instrumentation program helps institutions of higher education or nonprofit organizations increase access to shared scientific and engineering instruments for research and research training. The deadline is **January 10, 2018**; see www.nsf.gov/funding/pgm_summ.jsp?pims_id=5260&org=MPS&sel_org=MPS&from=fund.

—From an NSF announcement

*NSF Algorithms for Threat Detection Program

The Algorithms for Threat Detection (ATD) program will support research projects to develop the next generation of mathematical and statistical algorithms for analysis of large spatiotemporal datasets with application to quantitative models of human dynamics. The program is a partnership between the Division of Mathematical Sciences (DMS) at the National Science Foundation and the National Geospatial Intelligence Agency. The deadline for full proposals is **February 20, 2018**. See www.nsf.gov/funding/pgm_summ.jsp?pimsid=503427&org=DMS&sel_org=DMS&from=fund.

—From an NSF announcement

**The most up-to-date listing of NSF funding opportunities from the Division of Mathematical Sciences can be found online at:* www.nsf.gov/dms *and for the Directorate of Education and Human Resources at* www.nsf.gov/dir/index.jsp?org=ehr. *To receive periodic updates, subscribe to the DMSNEWS listserv by following the directions at* www.nsf.gov/mps/dms/about.jsp.

PIMS Education Prize

The Pacific Institute for the Mathematical Sciences (PIMS) annually awards a prize to a member of the PIMS community who has made a significant contribution to education in the mathematical sciences. The deadline for nominations is **March 15, 2018**. See `www.pims.math.ca/pims-glance/prizes-awards`.

—From a PIMS announcement

CAIMS/PIMS Early Career Award

The Canadian Applied and Industrial Mathematics Society (CAIMS) and the Pacific Institute for Mathematical Sciences (PIMS) sponsor the Early Career Award in Applied Mathematics to recognize exceptional research in applied mathematics, interpreted broadly, by a researcher fewer than ten years past earning a PhD. The nominee's research should have been conducted primarily in Canada or in affiliation with a Canadian university. The deadline for nominations is **January 31, 2018**. See `www.pims.math.ca/pims-glance/prizes-awards`.

—From a PIMS announcement

*NSF Mathematical Sciences Graduate Internship Program

The Oak Ridge Institute for Science and Education (ORISE) is accepting applications for the National Science Foundation Mathematical Sciences Graduate Internship Program, a ten-week summer program for graduate students pursuing a doctoral degree in mathematics, statistics, or applied mathematics. Call for applications closes **February 1, 2018**. See `orise.orau.gov/nsf-msgi`.

—From an ORISE announcement

Modeling with Differential Equations Competition

SIMIODE (Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations) announces the first annual Student Competition Using Differential Equation Modeling (SCUDEM), to be held on April 21, 2018. Registration for student teams and the faculty development workshop opens on **February 1, 2018**. SCUDEM is hosted at local sites around the United States and beyond. For complete details see `www.simiode.org`.

—SIMIODE announcement

Associate Editor, Mathematical Reviews
Algebra | 2018

Applications are invited for a full-time position as an Associate Editor of Mathematical Reviews/MathSciNet®, to commence as early as possible in late spring/early summer 2018. We also welcome applications from suitably qualified candidates interested in spending a sabbatical year from their institution in the position and continuing as an editor working remotely under contract afterwards. The Mathematical Reviews (MR) division of the American Mathematical Society (AMS) is located in Ann Arbor, Michigan, in a beautiful, historic building close to the campus of the University of Michigan. The editors are employees of the AMS; they also enjoy certain privileges at the university. At present, the AMS employs approximately eighty people at Mathematical Reviews, including sixteen mathematical editors. MR's mission is to develop and maintain the MR Database, from which MathSciNet is produced.

An Associate Editor is responsible for broad areas of the mathematical sciences. Editors select articles and books for coverage, classify these items, determine the type of coverage, assign selected items for review to reviewers, and edit the reviews on their return.

The successful applicant will have mathematical breadth with an interest in current developments, and will be willing to learn new topics in pure and applied mathematics. In particular, we are looking for an applicant with expertise in algebra and an interest in a range of algebraic topics, such as representation theory, nonassociative algebras, and group theory. The ability to write well in English is essential. The applicant should normally have several years of relevant academic (or equivalent) experience beyond the PhD Evidence of written scholarship in mathematics is expected. The twelve-month salary will be commensurate with the experience that the applicant brings to the position.

The review of the applications will begin on February 12, 2018 and will continue until the position is filled.

Applications can be submitted through MathJobs at **www.mathjobs.org/jobs/jobs/10667**

Inquiries may be sent to the Executive Editor:

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AMS AMERICAN MATHEMATICAL SOCIETY

The AMS Committee on Publications had its annual meeting in Chicago, October 13–14, 2017. Members may be interested in the following news item reported there and posted at `ams.org`.

—Carla D. Savage, AMS Secretary

American Mathematical Society and Mathematical Association of America Announce AMS Acquisition of MAA Book Program

Friday September 29th 2017

PROVIDENCE, RI - The American Mathematical Society and the Mathematical Association of America announced September 29 an agreement for the AMS to acquire the MAA's book publishing program. The high-quality mathematics titles and textbooks developed and edited by MAA Press will now be published as an imprint of the AMS Book Program. This agreement includes all existing MAA Press books, as well as a renewable license to continue to publish new books under the MAA Press imprint.

"The MAA Press titles are a wonderful complement to the AMS's existing list, expanding it into the realm of undergraduate and popular mathematics. The AMS will use its wide international distribution network to bring these titles, and future MAA Press imprint titles, to the attention of mathematicians around the world," said Robert Harington, Associate Executive Director for Publishing at the AMS.

"We look forward to the continuation of the MAA Press tradition of high-quality mathematics exposition and innovative textbooks. The agreement with AMS will allow us to expand the audience for the excellent books for which MAA Press is known," said Michael Pearson, Executive Director of the Mathematical Association of America.

The AMS will now publish new titles under the MAA Press imprint, and MAA Senior Acquisitions Editor Stephen Kennedy will join the AMS acquisitions team to continue to acquire new manuscripts for the imprint.

"The addition of these titles to our publishing program will broaden our reach to new audiences, help us better fulfill our mission to support mathematical scholarship, and expand our many ongoing collaborations with MAA," said Catherine Roberts, Executive Director of the AMS.

MAA Press titles will be available alongside AMS books at the AMS Bookstore (bookstore.ams.org) and at AMS exhibits at mathematics meetings by the end of this year. MAA members will continue to enjoy discounts on MAA Press titles, and will now receive discounts on AMS books, as well. AMS members will get similar discounts on both AMS and MAA titles.

Contacts: Mike Breen and Annette Emerson
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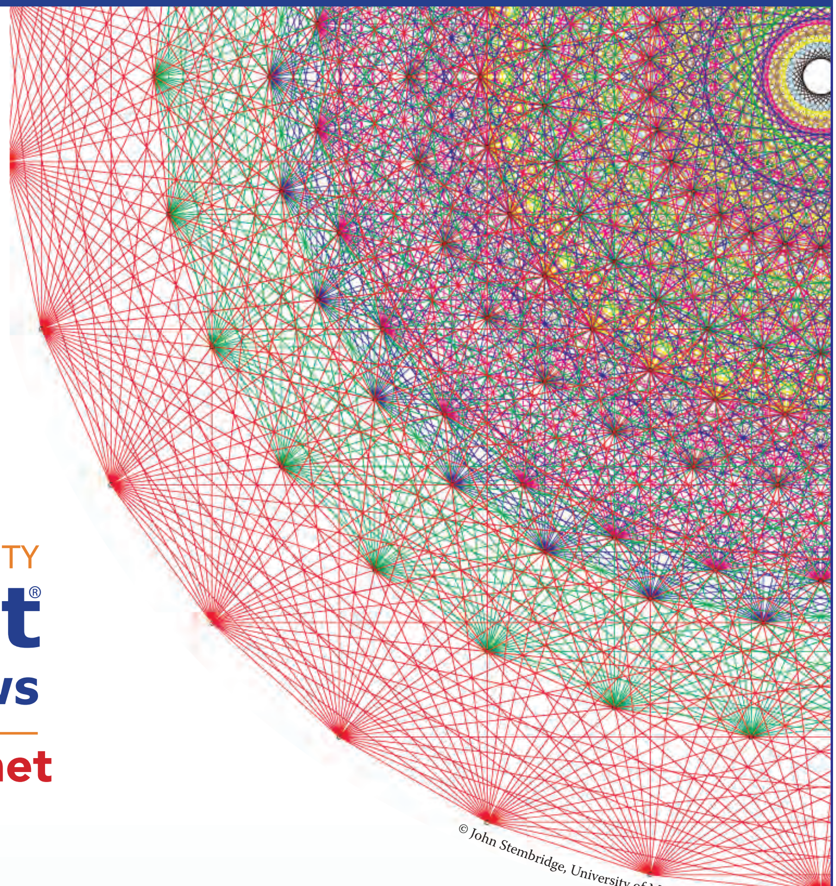

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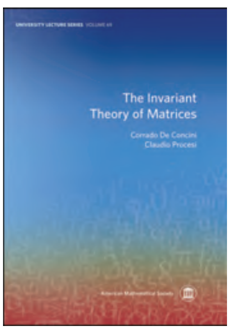
Algebra and Algebraic Geometry



The Invariant Theory of Matrices

Corrado De Concini, *Sapienza Università di Roma, Rome, Italy*, and **Claudio Procesi**, *Sapienza Università di Roma, Rome, Italy*

This book gives a unified, complete, and self-contained exposition of the main algebraic theorems of invariant theory for matrices in a characteristic free approach. More precisely, it contains the description of polynomial functions in several variables on the set of $m \times m$ matrices with coefficients in an infinite field or even the ring of integers, invariant under simultaneous conjugation.

Following Hermann Weyl's classical approach, the ring of invariants is described by formulating and proving

- the first fundamental theorem that describes a set of generators in the ring of invariants, and
- the second fundamental theorem that describes relations between these generators.

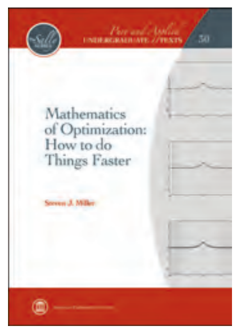
The authors study both the case of matrices over a field of characteristic 0 and the case of matrices over a field of positive characteristic. While the case of characteristic 0 can be treated following a classical approach, the case of positive characteristic (developed by Donkin and Zubkov) is much harder. A presentation of this case requires the development of a collection of tools. These tools and their application to the study of invariants are exlained in an elementary, self-contained way in the book.

Contents: Introduction and preliminaries; The classical theory; Quasi-hereditary algebras; The Schur algebra; Matrix functions and invariants; Relations; The Schur algebra of a free algebra; Bibliography; General index; Symbol index.

University Lecture Series, Volume 69

January 2018, 153 pages, Softcover, ISBN: 978-1-4704-4187-6, LC 2007041943, 2010 *Mathematics Subject Classification:* 15A72, 14L99, 20G20, 20G05, **AMS members US$35.20**, List US$44, Order code ULECT/69

Applications



Mathematics of Optimization: How to do Things Faster

Steven J. Miller, *Williams College, Williamstown, MA*

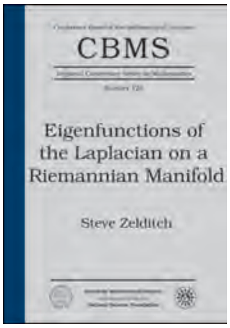
Optimization Theory is an active area of research with numerous applications; many of the books are designed for engineering classes, and thus have an emphasis on problems from such fields. Covering much of the same material, there is less emphasis on coding and detailed applications as the intended audience is more mathematical. There are still several important problems discussed (especially scheduling problems), but there is more emphasis on theory and less on the nuts and bolts of coding. A constant theme of the text is the "why" and the "how" in the subject. Why are we able to do a calculation efficiently? How should we look at a problem? Extensive effort is made to motivate the mathematics and isolate how one can apply ideas/perspectives to a variety of problems. As many of the key algorithms in the subject require too much time or detail to analyze in a first course (such as the run-time of the Simplex Algorithm), there are numerous comparisons to simpler algorithms which students have either seen or can quickly learn (such as the Euclidean algorithm) to motivate the type of results on run-time savings.

Contents: *Classical algorithms:* Efficient multiplication, I; Efficient multiplication, II; *Introduction to linear programming:* Introduction to linear programming; The canonical linear programming problem; Symmetries and dualities; Basic feasible and basic optimal solutions; The simplex method; *Advanced linear programming:* Integer programming; Integer optimization; Multi-objective and quadratic programming; The traveling salesman problem; Introduction to stochastic linear programming; *Fixed point theorems:* Introduction to fixed point theorems; Contraction maps; Sperner's lemma; Brouwer's fixed point theorem; *Advanced topics:* Gale-Shapley algorithm; Interpolating functions; The four color problem; The Kepler conjecture; Index; Bibliography.

Pure and Applied Undergraduate Texts, Volume 30

December 2017, 327 pages, Hardcover, ISBN: 978-1-4704-4114-2, LC 2017029521, 2010 *Mathematics Subject Classification:* 46N10, 65K10, 90C05, 97M40, 58C30, 11Y16, 68Q25, **AMS members US$55.20**, List US$69, Order code AMSTEXT/30

Differential Equations



Eigenfunctions of the Laplacian on a Riemannian Manifold

Steve Zelditch, *Northwestern University, Evanston, IL*

Eigenfunctions of the Laplacian of a Riemannian manifold can be described in terms of vibrating membranes as well as quantum energy eigenstates. This book is an introduction to both the local and global analysis of eigenfunctions. The local analysis of eigenfunctions pertains to the behavior of the eigenfunctions on wavelength scale balls. After re-scaling to a unit ball, the eigenfunctions resemble almost-harmonic functions. Global analysis refers to the use of wave equation methods to relate properties of eigenfunctions to properties of the geodesic flow.

The emphasis is on the global methods and the use of Fourier integral operator methods to analyze norms and nodal sets of eigenfunctions. A somewhat unusual topic is the analytic continuation of eigenfunctions to Grauert tubes in the real analytic case, and the study of nodal sets in the complex domain.

The book, which grew out of lectures given by the author at a CBMS conference in 2011, provides complete proofs of some model results, but more often it gives informal and intuitive explanations of proofs of fairly recent results. It conveys inter-related themes and results and offers an up-to-date comprehensive treatment of this important active area of research.

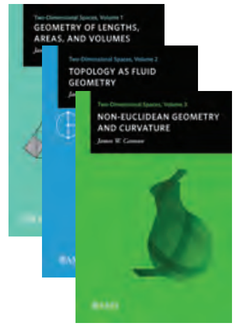
This item will also be of interest to those working in analysis.

Contents: Introduction; Geometric preliminaries; Main results; Model spaces of constant curvature; Local structure of eigenfunctions; Hadamard parametrics on Riemannian manifolds; Lagrangian distributions and Fourier integral operators; Small time wave group and Weyl asymptotics; Matrix elements; L^p nroms; Quantum integrable systems; Restriction theorems; Nodal sets: Real domain; Eigenfunctions in the complex domain; Index.

CBMS Regional Conference Series in Mathematics, Number 125

January 2018, 394 pages, Softcover, ISBN: 978-1-4704-1037-7, 2010 *Mathematics Subject Classification:* 34L20, 35P20, 35J05, 35L05, 53D25, 58J40, 58J50, **AMS members US$63.20**, List US$79, Order code CBMS/125

Geometry and Topology



Two-Dimensional Spaces, Volumes 1, 2, and 3

James W. Cannon, *Brigham Young University, Provo, UT*

This three-volume collection is devoted to the geometry, topology, and curvature of 2-dimensional spaces. The collection provides a guided tour through a wide range of topics by one of the twentieth century's masters of geometric topology. The books are accessible to college and graduate students and provide perspective and insight to mathematicians at all levels who are interested in geometry and topology.

Each volume in this set is sold separately. For a description of each volume, see the New Publication entries that follow.

Set: December 2017, 389 pages, Softcover, ISBN: 978-1-4704-4323-8, 2010 *Mathematics Subject Classification:* 51-01, 51M10, 53A35, 53A05, **AMS members US$79.20**, List US$99, Order code MBK-CANNON-SET

Geometry of Lengths, Areas, and Volumes

Two-Dimensional Spaces, Volume 1

James W. Cannon, *Brigham Young University, Provo, UT*

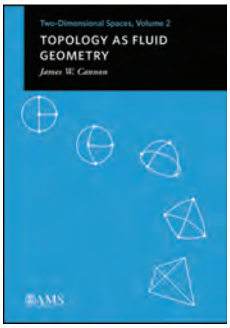
This is the first of a three volume collection devoted to the geometry, topology, and curvature of 2-dimensional spaces. The collection provides a guided tour through a wide range of topics by one of the twentieth century's masters of geometric topology. The books are accessible to college and graduate students and provide perspective and insight to mathematicians at all levels who are interested in geometry and topology.

The first volume begins with length measurement as dominated by the Pythagorean Theorem (three proofs) with application to number theory; areas measured by slicing and scaling, where Archimedes uses the physical weights and balances to calculate spherical volume and is led to the invention of calculus; areas by cut and paste, leading to the Bolyai-Gerwien theorem on squaring polygons; areas by counting, leading to the theory of continued fractions, the efficient rational approximation of real numbers, and Minkowski's theorem on convex bodies; straight-edge and compass constructions, giving complete proofs, including the transcendence of e and π, of the impossibility of squaring the circle, duplicating the cube, and trisecting the angle; and finally to a construction of the Hausdorff-Banach-Tarski paradox that shows some spherical sets are too complicated and cloudy to admit a well-defined notion of area.

Contents: Lengths—The Pythagorean theorem; Consequences of the Pythagorean theorem; Areas; Areas by slicing and scaling;

Areas by cut and paste; Areas by counting; Unsolvable problems in Euclidean geometry; Does every set have a size?; Bibliography.

December 2017, 119 pages, Softcover, ISBN: 978-1-4704-3714-5, LC 2017024690, 2010 *Mathematics Subject Classification:* 51-01, **AMS members US$39.20**, List US$49, Order code MBK/108



Topology as Fluid Geometry

Two-Dimensional Spaces, Volume 2

James W. Cannon, *Brigham Young University, Provo, UT*

This is the second of a three volume collection devoted to the geometry, topology, and curvature of 2-dimensional spaces. The collection provides a guided tour through a wide range of topics by one of the twentieth century's masters of geometric topology. The books are accessible to college and graduate students and provide perspective and insight to mathematicians at all levels who are interested in geometry and topology.

The second volume deals with the topology of 2-dimensional spaces. The attempts encountered in Volume 1 to understand length and area in the plane lead to examples most easily described by the methods of topology (fluid geometry): finite curves of infinite length, 1-dimensional curves of positive area, space-filling curves (Peano curves), 0-dimensional subsets of the plane through which no straight path can pass (Cantor sets), etc. Volume 2 describes such sets. All of the standard topological results about 2-dimensional spaces are then proved, such as the Fundamental Theorem of Algebra (two proofs), the No Retraction Theorem, the Brouwer Fixed Point Theorem, the Jordan Curve Theorem, the Open Mapping Theorem, the Riemann-Hurwitz Theorem, and the Classification Theorem for Compact 2-manifolds. Volume 2 also includes a number of theorems usually assumed without proof since their proofs are not readily available, for example, the Zippin Characterization Theorem for 2-dimensional spaces that are locally Euclidean, the Schoenflies Theorem characterizing the disk, the Triangulation Theorem for 2-manifolds, and the R. L. Moore's Decomposition Theorem so useful in understanding fractal sets.

Contents: The fundamental theorem of algebra; The Brouwer fixed point theorem; Tools; Lebesgue covering dimension; Fat curves and Peano curves; The arc, the simple closed curve, and the Cantor set; Algebraic topology; Characterization of the 2-sphere; 2-manifolds; Arcs in $\mathbb{S}^2$ are tame; R. L. Moore's decomposition theorem; The open mapping theorem; Triangulation of 2-manifolds; Structure and classification of 2-manifolds; The torus; Orientation and Euler characteristic; The Riemann-Hurwitz theorem; Bibliography.

December 2017, 165 pages, Softcover, ISBN: 978-1-4704-3715-2, LC 2017024690, 2010 *Mathematics Subject Classification:* 57-01, 57M20, **AMS members US$39.20**, List US$49, Order code MBK/109

Non-Euclidean Geometry and Curvature

Two-Dimensional Spaces, Volume 3

James W. Cannon, *Brigham Young University, Provo, UT*

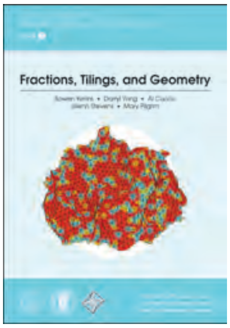
This is the final volume of a three volume collection devoted to the geometry, topology, and curvature of 2-dimensional spaces. The collection provides a guided tour through a wide range of topics by one of the twentieth century's masters of geometric topology. The books are accessible to college and graduate students and provide perspective and insight to mathematicians at all levels who are interested in geometry and topology.

Einstein showed how to interpret gravity as the dynamic response to the curvature of space-time. Bill Thurston showed us that non-Euclidean geometries and curvature are essential to the understanding of low-dimensional spaces. This third and final volume aims to give the reader a firm intuitive understanding of these concepts in dimension 2. The volume first demonstrates a number of the most important properties of non-Euclidean geometry by means of simple infinite graphs that approximate that geometry. This is followed by a long chapter taken from lectures the author gave at MSRI, which explains a more classical view of hyperbolic non-Euclidean geometry in all dimensions. Finally, the author explains a natural intrinsic obstruction to flattening a triangulated polyhedral surface into the plane without distorting the constituent triangles. That obstruction extends intrinsically to smooth surfaces by approximation and is called curvature. Gauss's original definition of curvature is extrinsic rather than intrinsic. The final two chapters show that the book's intrinsic definition is equivalent to Gauss's extrinsic definition (Gauss's "Theorema Egregium" ("Great Theorem")).

Contents: A graphical introduction to hyperbolic geometry; Hyperbolic geometry; Gravity as curvature; Curvature by polyhedral approximation; Curvature as a length derivative; Theorema egregium; Curvature appendix; Bibliography.

December 2017, 105 pages, Softcover, ISBN: 978-1-4704-3716-9, LC 2017024690, 2010 *Mathematics Subject Classification:* 51M10, 53A35, 53A05, **AMS members US$39.20**, List US$49, Order code MBK/110

Math Education



Fractions, Tilings, and Geometry

Bowen Kerins, *Education Development Center, Inc., Waltham, MA*, **Darryl Yong**, *Harvey Mudd College, Pomona, CA*, **Al Cuoco**, *Education Development Center, Inc., Waltham, MA*, **Glenn Stevens**, *Boston University, MA*, and **Mary Pilgrim**, *Colorado State University, Fort Collins, CO*

Designed for precollege teachers by a collaborative of teachers, educators, and mathematicians, *Fractions, Tilings, and Geometry* is based on a course offered in the Summer School Teacher Program at the Park City Mathematics Institute.

The overall goal of the course is an introduction to non-periodic tilings in two dimensions and space-filling polyhedra. While the course does not address quasicrystals, it provides the underlying mathematics that is used in their study. Because of this goal, the course explores Penrose tilings, the irrationality of the golden ratio, the connections between tessellations and packing problems, and Voronoi diagrams in 2 and 3 dimensions. These topics all connect to precollege mathematics, either as core ideas (irrational numbers) or enrichment for standard topics in geometry (polygons, angles, and constructions).

But this book isn't a "course" in the traditional sense. It consists of a carefully sequenced collection of problem sets designed to develop several interconnected mathematical themes. These materials provide participants with the opportunity for authentic mathematical discovery—participants build mathematical structures by investigating patterns, use reasoning to test and formalize their ideas, offer and negotiate mathematical definitions, and apply their theories and mathematical machinery to solve problems.

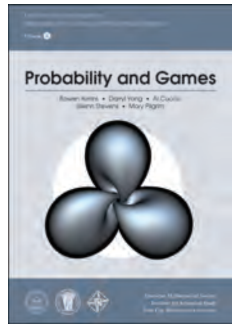
Fractions, Tilings, and Geometry is a volume of the book series "IAS/PCMI—The Teacher Program Series" published by the American Mathematical Society. Each volume in this series covers the content of one Summer School Teacher Program year and is independent of the rest.

Titles in this series are co-published with the Institute for Advanced Study/Park City Mathematics Institute.

Contents: Problems sets; Facilitator notes; Solutions.

IAS/PCMI—The Teacher Program Series, Volume 7

January 2018, 157 pages, Softcover, ISBN: 978-1-4704-4064-0, LC 2015022685, 2010 *Mathematics Subject Classification:* 00-01; 00A07, **AMS members US$23.20**, List US$29, Order code SSTP/7



Probability and Games

Bowen Kerins, *Education Development Center, Inc., Waltham, MA*, **Darryl Yong**, *Harvey Mudd College, Pomona, CA*, **Al Cuoco**, *Education Development Center, Inc., Waltham, MA*, **Glenn Stevens**, *Boston University, MA*, and **Mary Pilgrim**, *Colorado State University, Fort Collins, CO*

Designed for precollege teachers by a collaborative of teachers, educators, and mathematicians, *Probability and Games* is based on a course offered in the Summer School Teacher Program at the Park City Mathematics Institute.

This course leads participants through an introduction to probability and statistics, with particular focus on conditional probability, hypothesis testing, and the mathematics of election analysis. These ideas are tied together through low-threshold entry points including work with real and fake coin-flipping data, short games that lead to key concepts, and inroads to connecting the topics to number theory and algebra.

But this book isn't a "course" in the traditional sense. It consists of a carefully sequenced collection of problem sets designed to develop several interconnected mathematical themes. These materials provide participants with the opportunity for authentic mathematical discovery—participants build mathematical structures by investigating patterns, use reasoning to test and formalize their ideas, offer and negotiate mathematical definitions, and apply their theories and mathematical machinery to solve problems.

Probability and Games is a volume of the book series "IAS/PCMI—The Teacher Program Series" published by the American Mathematical Society. Each volume in this series covers the content of one Summer School Teacher Program year and is independent of the rest.

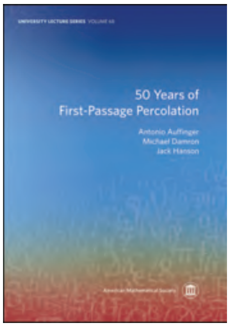
Titles in this series are co-published with the Institute for Advanced Study/Park City Mathematics Institute.

Contents: Problem sets for participants; Facilitator notes; Solutions.

IAS/PCMI—The Teacher Program Series, Volume 6

January 2018, 137 pages, Softcover, ISBN: 978-1-4704-4063-3, LC 2015022685, 2010 *Mathematics Subject Classification:* 00-01; 00A07, **AMS members US$23.20**, List US$29, Order code SSTP/6

Probability and Statistics



50 Years of First-Passage Percolation

Antonio Auffinger, *Northwestern University, Evanston, IL*, **Michael Damron**, *Georgia Institute of Technology, Atlanta, GA*, and **Jack Hanson**, *The City College of New York, NY*

First-passage percolation (FPP) is a fundamental model in probability theory that has a wide range of applications to other scientific areas (growth and infection in biology, optimization in computer science, disordered media in physics), as well as other areas of mathematics, including analysis and geometry. FPP was introduced in the 1960s as a random metric space. Although it is simple to define, and despite years of work by leading researchers, many of its central problems remain unsolved.

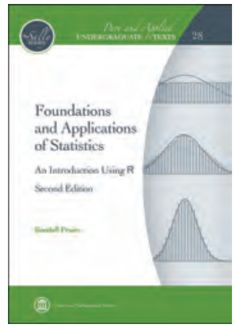
In this book, the authors describe the main results of FPP, with two purposes in mind. First, they give self-contained proofs of seminal results obtained until the 1990s on limit shapes and geodesics. Second, they discuss recent perspectives and directions including (1) tools from metric geometry, (2) applications of concentration of measure, and (3) related growth and competition models. The authors also provide a collection of old and new open questions. This book is intended as a textbook for a graduate course or as a learning tool for researchers.

This item will also be of interest to those working in mathematical physics.

Contents: Introduction; The time constant and the limit shape; Fluctuations and concentration bounds; Geodesics; Busemann functions; Growth and competition models; Variants of FPP and related models; Summary of open questions; Bibliography.

University Lecture Series, Volume 68

December 2017, 161 pages, Softcover, ISBN: 978-1-4704-4183-8, LC 2017029669, 2010 *Mathematics Subject Classification:* 60K35, 82B43, **AMS members US$35.20**, List US$44, Order code ULECT/68



Foundations and Applications of Statistics

An Introduction Using R, Second Edition

Randall Pruim, *Calvin College, Grand Rapids, MI*

Foundations and Applications of Statistics simultaneously emphasizes both the foundational and the computational aspects of modern statistics. Engaging and accessible, this book is useful to undergraduate students with a wide range of backgrounds and career goals.

The exposition immediately begins with statistics, presenting concepts and results from probability along the way. Hypothesis testing is introduced very early, and the motivation for several probability distributions comes from p-value computations. Pruim develops the students' practical statistical reasoning through explicit examples and through numerical and graphical summaries of data that allow intuitive inferences before introducing the formal machinery. The topics have been selected to reflect the current practice in statistics, where computation is an indispensible tool. In this vein, the statistical computing environment R is used throughout the text and is integral to the exposition. Attention is paid to developing students' mathematical and computational skills as well as their statistical reasoning. Linear models, such as regression and ANOVA, are treated with explicit reference to the underlying linear algebra, which is motivated geometrically.

Foundations and Applications of Statistics discusses both the mathematical theory underlying statistics and practical applications that make it a powerful tool across disciplines. The book contains ample material for a two-semester course in undergraduate probability and statistics. A one-semester course based on the book will cover hypothesis testing and confidence intervals for the most common situations.

In the second edition, the R code has been updated throughout to take advantage of new R packages and to illustrate better coding style. New sections have been added covering bootstrap methods, multinomial and multivariate normal distributions, the delta method, numerical methods for Bayesian inference, and nonlinear least squares. Also, the use of matrix algebra has been expanded, but remains optional, providing instructors with more options regarding the amount of linear algebra required.

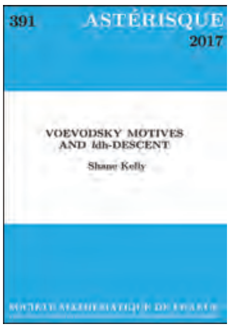
Contents: Data; Probability and random variables; Continuous distributions; Parameter estimation and testing; Likelihood; Introduction to linear models; More linear models; A brief introduction to R; Some mathematical preliminaries; Geometry and linear algebra review; Hints, answers, and solutions to selected exercises; Bibliography; Index to R functions, packages, and data sets; Index.

Pure and Applied Undergraduate Texts, Volume 28

April 2018, approximately 823 pages, Hardcover, ISBN: 978-1-4704-2848-8, LC 2017013678, 2010 *Mathematics Subject Classification:* 62-01, **AMS members US$111.20**, List US$139, Order code AMSTEXT/28

New AMS-Distributed Publications

Algebra and Algebraic Geometry



Voevodsky Motives and *l*dh-Descent

Shane Kelly, *FB Mathematik und Informatik, Berlin, Germany*

This work applies Gabber's theorem on alterations to Voevodsky's work on mixed motives. The author extends many fundamental theorems to $\mathrm{DM}(k, \mathbb{Z}[1/p])$ where p is the exponential characteristic of the perfect field k. Two applications are an isomorphism of Suslin that compares higher Chow groups and étale cohomology, and calculation of the motivic Steenrod algebra.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Astérisque, Number 391

October 2017, 125 pages, Softcover, ISBN: 978-2-85629-861-9, 2010 *Mathematics Subject Classification:* 14-XX, 14F42, 32S45, 14C15, 14E15, 14G17, 19E15, 14C25, 14F20, 18E30, 13D15, **AMS members US$41.60**, List US$52, Order code AST/391

Variétés Algébriques Réelles

Fréderic Mangolte, *Université d'Angers, France*

Real algebraic varieties are ubiquitous. They are the first objects students encounter while learning about coordinates. But the systematic study of these objects, however elementary they may be, is formidable.

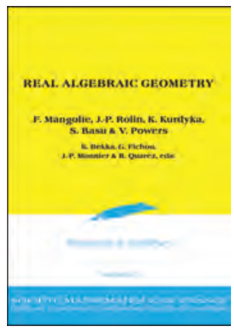
This book is will provide the basics of this rich theory and for more advanced readers, will provide many fundamental results often missing from the available literature, the "folklore". In particular, the introduction of topological methods of the theory to non-specialists is one of the original features of the book.

The first three chapters introduce the basis and classical methods of real and complex algebraic geometry. The last three chapters each focus on one more specific aspect of real algebraic varieties. The book offers a panorama of classical knowledge and also addresses the major developments of the last twenty years in terms of the topology and geometry of varieties of dimension two and three, without forgetting the curves, the central subject of Hilbert's famous sixteenth problem. Various level exercises are given, and the solutions to many of them are provided at the end of each chapter.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Cours Spécialisés—Collection SMF, Number 24

October 2017, 484 pages, Hardcover, ISBN: 978-2-85629-864-0, 2010 *Mathematics Subject Classification:* 14P25, 14P05, 14Pxx, 14E05, 14E07, 14R05, 14Jxx, 14Hxx, 26C15, 32Jxx, 57M50, 57Mxx, 57N10, **AMS members US$84**, List US$105, Order code COSP/24



Real Algebraic Geometry

Jean-Philippe Monnier, *LUNAM Université, LAREMA, Université d'Angers, France*, **Karim Bekka**, *IRMAR, Université de Rennes 1, France*, **Goulwen Fichou**, *IRMAR, Université de Rennes 1, France*, and **Ronan Quarez**, *IRMAR, Université de Rennes 1, France*, Editors; **Fréderic Mangolte**, *LUNAM Université, LAREMA, Université d'Angers, France*, **Jean-Philippe Rolin**, *IMB, Université de Bourgogne, Dijon, France*, **Krzysztof Kurdyka**, *Université de Savoie Mont-Blanc, Le Bourget-du-Lac, France*, **Saugata Basu**, *Purdue University, West Layafette, Indiana*, and **Victoria Powers**, *Emory University, Atlanta, Georgia*

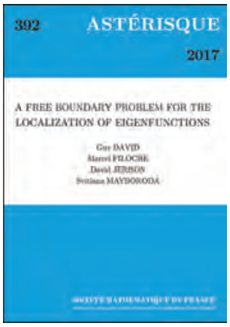
This volume presents an overview of the research in real algebraic geometry. The volume contains an introduction and five survey articles. The topics are real rational surfaces, o-minimal geometry, analytic arcs and real analytic singularities, algorithms in real algebraic geometry, positive polynomials, and sums of squares.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Panoramas et Synthèses, Number 51

October 2017, 180 pages, Softcover, ISBN: 978-2-85629-857-2, 2010 *Mathematics Subject Classification:* 03C64, 11G99, 11U09, 14A10, 14P10, 14P15, 14Pxx, 26E10, 32B10, 68W30, **AMS members US$53.60**, List US$67, Order code PASY/51

Analysis



A Free Boundary Problem for the Localization of Eigenfunctions

Guy David, *Université de Paris-Sud, France*, **Marcel Filoche**, *École Polytechnique, Palaiseau, France*, **David Jerison**, *Massachusetts Institute of Technology, Cambridge, MA*, and **Svitlana Mayboroda**, *University of Minnesota, School of Mathematics, Minneapolis, MN*

The authors study a variant of the Alt, Caffarelli, and Friedman free boundary problem, with many phases and a slightly different volume term, which the authors originally designed to guess the localization of eigenfunctions of a Schrödinger operator in a domain. The authors prove Lipschitz bounds for the functions and some nondegeneracy and regularity properties for the domains.

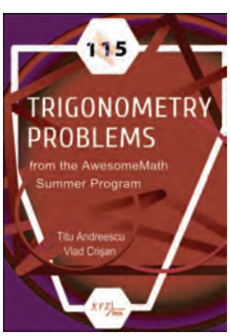
This item will also be of interest to those working in differential equations.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Astérisque, Number 392

October 2017, 203 pages, Softcover, ISBN: 978-2-85629-863-3, 2010 *Mathematics Subject Classification:* 49Q20, 35B65, **AMS members US$53.60**, List US$67, Order code AST/392

General Interest



115 Trigonometry Problems from the AwesomeMath Summer Program

Titu Andreescu, *University of Texas at Dallas*, and **Vlad Crişan**, *University of Göttingen, Germany*

This book offers a comprehensive overview of the trigonometric functions and contains a collection of 115 carefully selected introductory and advanced problems in trigonometry from world-wide renowned Olympiads and mathematical magazines, as well as original problems designed by the authors. Together with the beautiful examples and the creative solutions, the present text is a valuable resource and teaching tool for anybody who wants to explore the beauty of trigonometry.

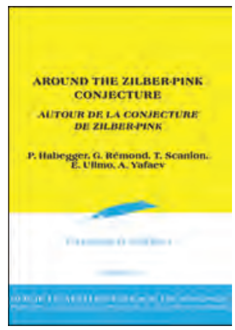
This item will also be of interest to those working in math education.

A publication of XYZ Press. Distributed in North America by the American Mathematical Society.

XYZ Series, Volume 28

November 2017, 200 pages, Hardcover, ISBN: 978-0-9993428-0-0, 2010 *Mathematics Subject Classification:* 00A05, 00A07, 97U40, 97D50, **AMS members US$47.96**, List US$59.95, Order code XYZ/28

Logic and Foundations



Around the Zilber-Pink Conjecture

Philipp Habegger, *University of Basel, Switzerland*, **Gäel Rémond**, *Institut Fourier, Grenoble, France*, **Thomas Scanlon**, *University of California, Berkeley*, **Emmanuel Ullmo**, *Institut des Hautes Études Scientifiques, Bures sur Yvette, France*, and **Andrei Yafaev**, *University College London, United Kingdom*

Following Faltings and Vojta's work proving the Mordell-Lang conjecture for abelian varieties and Raynaud's work proving the Manin-Mumford conjecture, many new diophantine questions appeared, often described as problems of unlikely intersections. The arithmetic of moduli spaces of abelian varieties and, more generally, Shimura varieties has been parallel-developed around the central André-Oort conjecture. These two themes can be placed in a common frame—the Zilber-Pink conjecture.

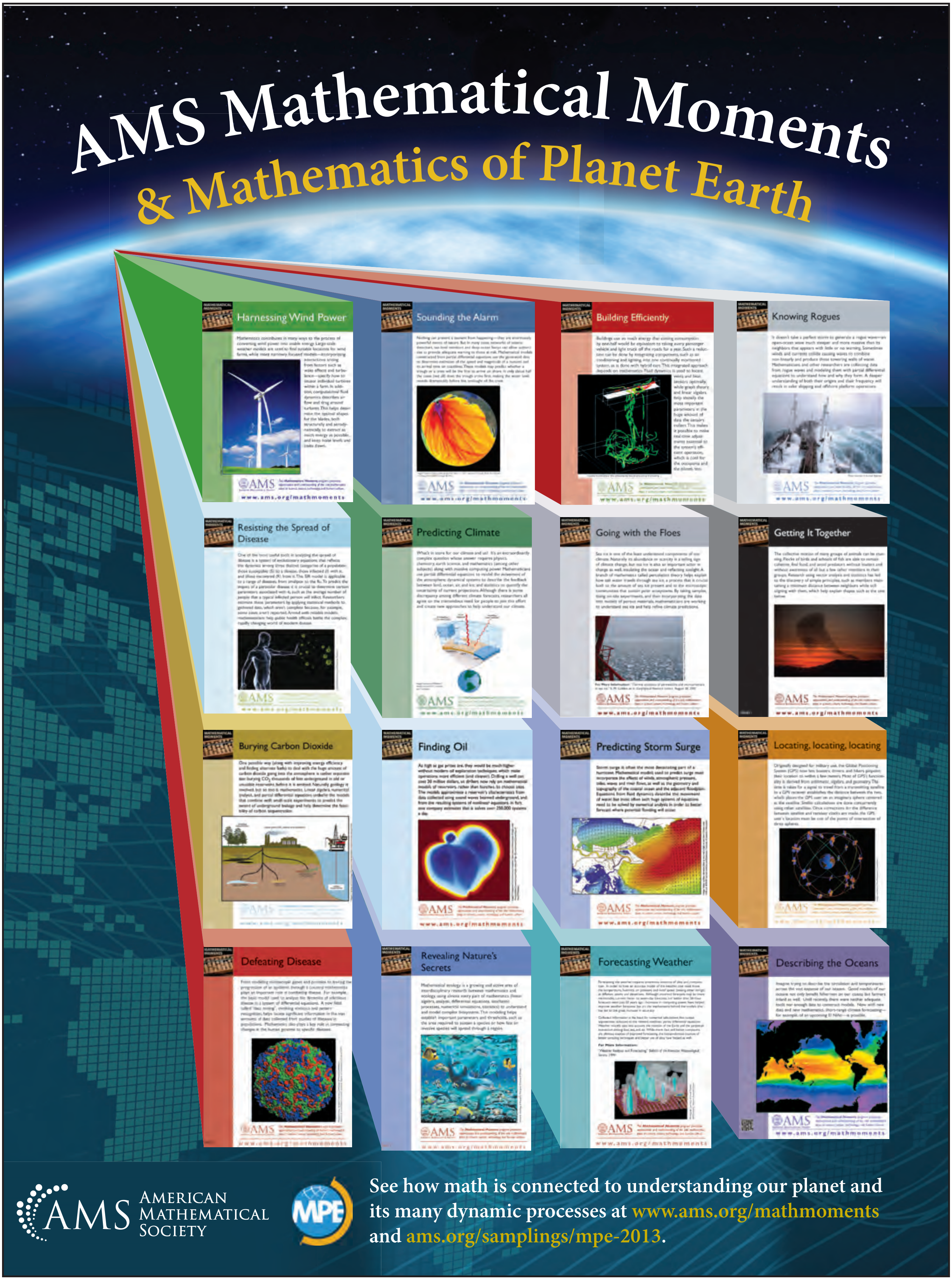
This volume is an introduction to these problems and to the various techniques used: geometry, height theory, reductive groups and Hodge theory, Shimura varieties, and model theory via the notion of o-minimal structure. The volume contains texts corresponding to courses presented at CIRM in May 2011 by Philipp Habegger, Gaël Rémond, Thomas Scanlon, Emmanuel Ullmo, and Andrei Yafaev and an ample introduction by E. Ullmo centered on the notion of bi-algebraicity aimed at a presentation of the general setting.

This item will also be of interest to those working in number theory.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Panoramas et Synthèses, Number 52

October 2017, 284 pages, Softcover, ISBN: 978-2-85629-856-5, 2010 *Mathematics Subject Classification:* 03C64, 11G10, 11G15, 11G50, 11J81, 14G35, 14K15, 14G40, 14J20, 14T05, 22E40, **AMS members US$65.60**, List US$82, Order code PASY/52



AMS Mathematical Moments
& Mathematics of Planet Earth
Harnessing Wind Power
Sounding the Alarm
Building Efficiently
Knowing Rogues
Resisting the Spread of Disease
Predicting Climate
Going with the Floes
Getting It Together
Burying Carbon Dioxide
Finding Oil
Predicting Storm Surge
Locating, locating, locating
Defeating Disease
Revealing Nature's Secrets
Forecasting Weather
Describing the Oceans
AMS American Mathematical Society
MPE
See how math is connected to understanding our planet and its many dynamic processes at www.ams.org/mathmoments and ams.org/samplings/mpe-2013.

Classified Advertising

Positions available, items for sale, services available, and more

KANSAS

University of Kansas
Department of Mathematics

The Department of Mathematics at the University of Kansas invites applications for a non-tenure-track, Visiting Assistant Professor position in Probability expected to begin as early as August 18, 2018. This is a limited term appointment for three consecutive academic years. For a complete announcement and to apply online, go to `https://employment.ku.edu/academic/10399BR`. In addition, at least four recommendation letters (teaching ability must be addressed in at least one letter) should be submitted electronically to `https://www.mathjobs.org/jobs/jobs/11340`. Initial review of applications will begin December 31, 2017.

KU is an EO/AAE. All qualified applicants will receive consideration for employment without regard to race, color, religion, sex (including pregnancy), age, national origin, disability, genetic information or protected Veteran status.

00005

University of Kansas
Department of Mathematics

The Department of Mathematics, University of Kansas invites applications for an Assistant Teaching Specialist in calculus expected to begin August 18, 2018. For a complete announcement and to apply online go to `https://employment.ku.edu/academic/10339BR`.
In addition three recommendation letters addressing the required qualifications should be submitted to MathJobs.Org at `https://www.mathjobs.org/jobs/jobs/11278`.
Review of applications will begin November 27, 2017.

KU is an EO/AAE. All qualified applicants will receive consideration for employment without regard to race, color, religion, sex (including pregnancy), age, national origin, disability, genetic information or protected Veteran status.

00002

MISSOURI

University of Central Missouri
School of Computer Science and Mathematics

The School of Computer Science and Mathematics at the University of Central Missouri invites applications for a tenure track assistant professor position in Actuarial Science to begin in August 2018. Applicants must have a PhD degree in actuarial science, mathematics, or statistics by August 2018. For more information and to apply online, go to `https://jobs.ucmo.edu`. Apply to position #998534. Initial screening of applications begins December 15, 2017, and continues until position is filled.

00003

CHINA

Tianjin University, China
Tenured/Tenure-Track/Postdoctoral Positions at the Center for Applied Mathematics

Dozens of positions at all levels are available at the recently founded Center for Applied Mathematics, Tianjin University, China. We welcome applicants with backgrounds in pure mathematics, applied mathematics, statistics, computer science, bioinformatics, and other related fields. We also welcome applicants who are interested in practical projects with industries. Despite its name attached with an accent of applied mathematics, we also aim to create a strong presence of pure mathematics. Chinese citizenship is not required.

Light or no teaching load, adequate facilities, spacious office environment and strong research support. We are prepared to make quick and competitive offers to self-motivated hard workers, and to potential stars, rising stars, as well as shining stars.

The Center for Applied Mathematics, also known as the Tianjin Center for Applied Mathematics (TCAM), located by a lake in the central campus in a building protected as historical architecture, is jointly sponsored by the Tianjin municipal government and the university. The initiative to establish this center was taken by Professor S. S. Chern. Professor Molin Ge is the Honorary Director, Professor Zhiming Ma is the Director of the Advisory Board. Professor William Y. C. Chen serves as the Director.

TCAM plans to fill in fifty or more permanent faculty positions in the next few years. In addition, there are a number of temporary and visiting positions. We look forward to receiving your application or inquiry at any time. There are no deadlines.

Please send your resume to `mathjobs@tju.edu.cn`. For more information, please visit `www.cam.tju.edu.cn` or contact Ms. Erica Liu at `mathjobs@tju.edu.cn`, telephone: 86-22-2740-6039.

00001

KOREA

Korea Institute for Advanced Study (KIAS)
Postdoctoral Fellow in Computational Sciences at KIAS

The School of Computational Sciences at Korea Institute for Advanced Study (KIAS) invites applicants for the positions at the level of postdoctoral research fellow in Computational Sciences. The School performs research in a broad range of fields for which computational methods are appropriate or even necessary: quantum information science, combinatorics, discrete mathematics, bioinformatics, and the theoretical and computational aspects of physics, biophysics, material science, and artificial intelligence. Applicants are expected to have demonstrated exceptional research potential, including major contributions beyond or through the doctoral dissertation.

The annual salary starts from 50,000,000 Korean Won (approximately US$45,000 at current exchange rate). In addition, individual research funds of 10,000,000 Korean Won are available per year. The initial appointment for the position is for two years and is renewable once for up to two additional years, depending on research performance and the needs of the research program at KIAS.

Application materials (CV with a cover letter, a list of publications, a research plan, and three letters of recommendation) should be sent to `comp@kias.re.kr`.

00006

FELLOWSHIP AVAILABLE

Thesis-writing fellowship for students doing extraordinary teaching and outreach.

A $15,000 fellowship for PhD students graduating in the 2018–2019 academic year who have done extraordinary teaching and outreach during their time as graduate students especially during summers. Offered through Noah Snyder's NSF CAREER Grant DMS-1454767. Details at `pages.iu.edu/~nsnyder1/fellowship.html`.

00004

2018 Annual Meeting of the American Association for the Advancement of Science

The 2018 annual meeting of the American Association for the Advancement of Science will take place in Austin, Texas, February 15–19. The theme of the meeting is “Advancing Science Discovery to Application.”

The following session should be of special interest to mathematicians:

Hans Engler and Hans Kaper, both of Georgetown University, have organized a symposium on “Mathematics of Planet Earth: Superbugs, Storm Surges, and Ecosystem Change“ for Sunday, February 18, 2018, 3:30 PM–5:00 PM. This session highlights recent progress in the modeling and analysis of three major challenges in public health, environmental stewardship, and ecology. Mathematical modeling and computation, together with new methods of data gathering, can predict or reconstruct the spread of disease across populations, make detailed forecasts of the effects of storm surges or land loss on coastal landscapes, and explain the occurrence of shrubland patterns and fairy rings in savannahs. These same models can also help assess the effectiveness of vaccinations against disease or of evacuation plans in extreme weather events, and they can identify early warning signals for irreversible ecosystem changes such as desertification.

These are examples of collaborations across disciplines where mathematics and computational science play an essential role. Speakers share these as part of an emerging effort to develop the “Mathematics of Planet Earth.” Additional topics and speakers are:

- “Unseen Enemies: Surveilling, Predicting, and Controlling Epidemic Outbreaks”—**Glenn Webb**, Vanderbilt University, Nashville, TN
- “Spatial Self-Organization and Its Implications for Ecosystem Robustness”—**Corina Tarnita**, Princeton University, Princeton, NJ
- “Resilient and Sustainable Coasts: How Mathematics and Simulation Play a Role”—**Clint Dawson**, University of Texas, Austin, Austin, TX.

—Andy Magid
Secretary, Section A (Mathematics)
AAAS

Fan China Exchange Program

- Gives eminent mathematicians from the U.S. and Canada an opportunity to travel to China and interact with fellow researchers in the mathematical sciences community
- Allows Chinese scientists in the early stages of their careers to come to the U.S. and Canada for collaborative opportunities

Applications received before March 15 will be considered for the following academic year.

AMS AMERICAN MATHEMATICAL SOCIETY

For more information on the Fan China Exchange Program and application process see www.ams.org/employment/chinaexchange.html or contact the AMS Membership and Programs Department by telephone at 800-321-4267, ext. 4113 (U.S. and Canada), or 401-455-4113 (worldwide), or by email at chinaexchange@ams.org

General Information Regarding Meetings & Conferences of the AMS

Speakers and Organizers: The Council has decreed that no paper, whether invited or contributed, may be listed in the program of a meeting of the Society unless an abstract of the paper has been received in Providence prior to the deadline.

Special Sessions: The number of Special Sessions at an Annual Meeting are limited. Special Sessions at annual meetings are held under the supervision of the Program Committee for National Meetings and, for sectional meetings, under the supervision of each Section Program Committee. They are administered by the associate secretary in charge of that meeting with staff assistance from the Meetings and Conferences Department in Providence. (See the list of associate secretaries on page 90 of this issue.)

Each person selected to give an Invited Address is also invited to generate a Special Session, either by personally organizing one or by having it organized by others. Proposals to organize a Special Session are sometimes solicited either by a program committee or by the associate secretary. Other proposals should be submitted to the associate secretary in charge of that meeting (who is an *ex officio* member of the program committee) at the address listed on page 90 of this issue. These proposals must be in the hands of the associate secretary at least seven months (for sectional meetings) or nine months (for national meetings) prior to the meeting at which the Special Session is to be held in order that the committee may consider all the proposals for Special Sessions simultaneously. Special Sessions must be announced in the *Notices* in a timely fashion so that any Society member who so wishes may submit an abstract for consideration for presentation in the Special Session.

Talks in Special Sessions are usually limited to twenty minutes; however, organizers who wish to allocate more time to individual speakers may do so within certain limits. A great many of the papers presented in Special Sessions at meetings of the Society are invited papers, but any member of the Society who wishes to do so may submit an abstract for consideration for presentation in a Special Session. Contributors should know that there is a limit to the size of a single Special Session, so sometimes all places are filled by invitation. An author may speak by invitation in more than one Special Session at the same meeting. However, multiple talks given by the same author at a meeting must be distinct and have distinct abstracts. Papers submitted for consideration for inclusion in Special Sessions but not accepted will receive consideration for a contributed paper session, unless specific instructions to the contrary are given.

The Society reserves the right of first refusal for the publication of proceedings of any Special Session. If published by the AMS, these proceedings appear in the book series *Contemporary Mathematics*. For more detailed information on organizing a Special Session, see `www.ams.org/meetings/meet-specialsessionmanual.html`.

Contributed Papers: The Society also accepts abstracts for ten-minute contributed papers. These abstracts will be grouped by related Mathematical Reviews subject classifications into sessions to the extent possible. The title and author of each paper accepted and the time of presentation will be listed in the program of the meeting. Although an individual may present only one ten-minute contributed paper at a meeting, any combination of joint authorship may be accepted, provided no individual speaks more than once.

Other Sessions: In accordance with policy established by the AMS Committee on Meetings and Conferences, mathematicians interested in organizing a session (for either an annual or a sectional meeting) on employment opportunities inside or outside academia for young mathematicians should contact the associate secretary for the meeting with a proposal by the stated deadline. Also, potential organizers for poster sessions (for an annual meeting) on a topic of choice should contact the associate secretary before the deadline.

Abstracts: Abstracts for all papers must be received by the meeting coordinator in Providence by the stated deadline. Unfortunately, late papers cannot be accommodated.

Submission Procedures: Visit the Meetings and Conferences homepage on the Web at `www.ams.org/meetings` and select "Submit Abstracts."

Site Selection for Sectional Meetings

Sectional meeting sites are recommended by the associate secretary for the section and approved by the Secretariat. Recommendations are usually made eighteen to twenty-four months in advance. Host departments supply local information, ten to fifteen rooms with overhead projectors and a laptop projector for contributed paper sessions and Special Sessions, an auditorium with twin overhead projectors and a laptop projector for Invited Addresses, space for registration activities and an AMS book exhibit, and registration clerks. The Society partially reimburses for the rental of facilities and equipment needed to run these meetings successfully. For more information, contact the associate secretary for the section.

MEETINGS & CONFERENCES OF THE AMS

JANUARY TABLE OF CONTENTS

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at: **www.ams.org/meetings/**.

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to page 88 in the January 2018 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit **www.ams.org/cgi-bin/abstracts/abstract.pl/**. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

MEETINGS IN THIS ISSUE

See `www.ams.org/meetings/` *for the most up-to-date information on the meetings and conferences that we offer.*

ASSOCIATE SECRETARIES OF THE AMS

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; email: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; email: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, email: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; email: lapidus@math.ucr.edu; telephone: 951-827-5910.

AMERICAN MATHEMATICAL SOCIETY

CURRENT EVENTS BULLETIN

Friday, January 12, 2018, 1:00 PM to 4:45 PM

Room 6E, Upper Level, San Diego Convention Center
Joint Mathematics Meeting, San Diego, CA

1:00 PM **Richard D. James**, *University of Minnesota*

New materials from mathematics, real and imagined

From group theory to partial differential equations, why materials behave as they do—and how to change that.

2:00 PM **Craig L. Huneke**, *University of Virginia*

How complicated are polynomials in many variables?

Polynomials get more complicated as the number of variables increases... or do they?

3:00 PM **Isabelle Gallagher**, *Université Paris Diderot*

From Newton to Navier-Stokes, or how to connect fluid mechanics equations from microscopic to macroscopic scales

Navier-Stokes is supposed to describe fluid flow—how does (or doesn't) the behavior of atoms do that?

4:00 PM **Joshua A. Grochow**, *University of Colorado, Boulder*

The Cap Set Conjecture, the polynomial method, and applications (after Croot-Lev-Pach, Ellenberg-Gijswijt, and others)

The capset conjecture generalizes the question: how frustrating can the popular game of "Set" be—and involves some beautiful and serious mathematics.

AMS AMERICAN MATHEMATICAL SOCIETY

Organized by David Eisenbud, University of California, Berkeley

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See www.ams.org/meetings/.

Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL .

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday

Meeting #1135

Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Issue of *Abstracts*: Volume 39, Issue 1

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/national.html.

Joint Invited Addresses

Gunnar Carlsson, Stanford University, *Topological Modeling of Complex Data* (AMS-MAA Invited Address).

Moon Duchin, Tufts University, *Political Geometry: Voting districts, "compactness," and ideas about fairness* (MAA-AMS-SIAM Gerald and Judith Porter Public Lecture).

André Neves, University of Chicago, *Wow, so many minimal surfaces!* (AMS-MAA Invited Address).

Jill Pipher, Brown University, *Nonsmooth boundary value problems* (AWM-AMS Noether Lecture).

AMS Invited Addresses

Federico Ardila, San Francisco State University, *Algebraic structures on polytopes.*

Robert L. Bryant, Duke University, *The concept of Holonomy—its history and recent developments* (AMS Retiring Presidential Address).

Ruth Charney, Brandeis University, *Searching for hyperbolicity.*

Cynthia Dwork, Harvard University, *Privacy in the land of plenty* (AMS Josiah Willard Gibbs Lecture).

Dana Randall, Georgia Institute of Technology, *Emergent phenomena in random structures and algorithms.*

Edriss S. Titi, Texas A&M University; and The Weizmann Institute of Science, *The Navier-Stokes, Euler and related equations.*

Avi Wigderson, Institute for Advanced Study, *Alternate Minimization and Scaling algorithms: theory, applications and connections across mathematics and computer science* (AMS Colloquium Lectures: Lecture I).

Avi Wigderson, Institute for Advanced Study, *Proving algebraic identities* (AMS Colloquium Lectures: Lecture II).

Avi Wigderson, Institute for Advanced Study, *Proving analytic inequalities* (AMS Colloquium Lectures: Lecture III).

AMS Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at jointmathematicsmeetings.org/meetings/abstracts/abstract.pl?type=jmm.

Some sessions are cosponsored with other organizations. These are noted within the parenthesis at the end of each listing, where applicable.

A Showcase of Number Theory at Liberal Arts Colleges, **Adriana Salerno**, Bates College, and **Lola Thompson**, Oberlin College.

Accelerated Advances in Mathematical Fractional Programming, **Ram Verma**, International Publications USA, and **Alexander Zaslavski**, Israel Institute of Technology.

Advances in Applications of Differential Equations to Disease Modeling, **Libin Rong**, Oakland University, **Elissa Schwartz**, Washington State University, and **Naveen K. Vaidya**, San Diego State University.

Advances in Difference, Differential, and Dynamic Equations with Applications, **Elvan Akin**, Missouri University S&T, and **John Davis**, Baylor University.

Advances in Operator Algebras, **Marcel Bischoff**, Vanderbilt University, **Ian Charlesworth**, University of California, Los Angeles, **Brent Nelson**, University of California, Berkeley, and **Sarah Reznikoff**, Kansas State University.

Advances in Operator Theory, Operator Algebras, and Operator Semigroups, **Asuman G. Aksoy**, Claremont McKenna College, **Zair Ibragimov**, California State University, Fullerton, **Marat Markin**, California State University, Fresno, and **Ilya Spitkovsky**, New York University, Abu Dhabi.

Algebraic, Analytic, and Geometric Aspects of Integrable Systems, Painlevé Equations, and Random Matrices, **Vladimir Dragovic**, University of Texas at Dallas, **Anton Dzhamay**, University of Northern Colorado, and **Sevak Mkrtchyan**, University of Rochester.

Algebraic, Discrete, Topological and Stochastic Approaches to Modeling in Mathematical Biology, **Olcay Akman**, Illinois State University, **Timothy D. Comar**, Benedictine University, **Daniel Hrozencik**, Chicago State University, and **Raina Robeva**, Sweet Briar College.

Alternative Proofs in Mathematical Practice, **John W. Dawson, Jr.**, Pennsylvania State University, York.

Analysis of Fractional, Stochastic, and Hybrid Dynamic Systems, **John R. Graef**, University of Tennessee at Chattanooga, **Gangaram S. Ladde**, University of South Florida, and **Aghalaya S. Vatsala**, University of Louisiana at Lafayette.

Analysis of Nonlinear Partial Differential Equations and Applications, **Tarek M. Elgindi**, University of California, San Diego, and **Edriss S. Titi**, Texas A&M University and Weizmann Institute of Science.

Applied and Computational Combinatorics, **Torin Greenwood**, Georgia Institute of Technology, and **Jay Pantone**, Dartmouth College.

Arithmetic Dynamics, **Robert L. Benedetto**, Amherst College, **Benjamin Hutz**, Saint Louis University, **Jamie Juul**, Amherst College, and **Bianca Thompson**, Harvey Mudd College.

Beyond Planarity: Crossing Numbers of Graphs (a Mathematics Research Communities Session), **Axel Brandt**, Davidson College, **Garner Cochran**, University of South Carolina, and **Sarah Loeb**, College of William and Mary.

Bifurcations of Difference Equations and Discrete Dynamical Systems, **Arzu Bilgin** and **Toufik Khyat**, University of Rhode Island.

Boundaries for Groups and Spaces, **Joseph Maher**, CUNY College of Staten Island, and **Genevieve Walsh**, Tufts University.

Combinatorial Commutative Algebra and Polytopes, **Robert Davis**, Michigan State University, and **Liam Solus**, KTH Royal Institute of Technology.

Combinatorics and Geometry, **Federico Ardila**, San Francisco State University, **Anastasia Chavez**, MSRI and University of California, Davis, and **Laura Escobar**, University of Illinois Urbana Champaign.

Commutative Algebra in All Characteristics, **Neil Epstein**, George Mason University, **Karl Schwede**, University of Utah, and **Janet Vassilev**, University of New Mexico.

Computational Combinatorics and Number Theory, **Jeremy F. Alm**, Lamar University, and **David Andrews** and **Rob Hochberg**, University of Dallas.

Connections in Discrete Mathematics: Graphs, Hypergraphs, and Designs, **Amin Bahmanian**, Illinois State University, and **Theodore Molla**, University Illinois Urbana-Champaign.

Differential Geometry, **Vincent B. Bonini** and **Joseph E. Borzellino**, Cal Poly San Luis Obispo, **Bogdan D. Suceava**, California State University, Fullerton, and **Guofang Wei**, University of California, Santa Barbara.

Diophantine Approximation and Analytic Number Theory in Honor of Jeffrey Vaaler, **Shabnam Akhtari**, University of Oregon, **Lenny Fukshansky**, Claremont McKenna College, and **Clayton Petsche**, Oregon State University.

Discrete Dynamical Systems and Applications, **E. Cabral Balreira**, **Saber Elaydi**, and **Eddy Kwessi**, Trinity University.

Discrete Neural Networking and Applications, **Murat Adivar**, Fayetteville State University, **Michael A. Radin**, Rochester Institute of Technology, and **Youssef Raffoul**, University of Dayton.

Dynamical Algebraic Combinatorics, **James Propp**, University of Massachusetts, Lowell, **Tom Roby**, University of Connecticut, **Jessica Striker**, North Dakota State University, and **Nathan Williams**, University of California Santa Barbara.

Dynamical Systems with Applications to Mathematical Biology, **Guihong Fan**, Columbus State University, **Jing Li**, California State University Northridge, and **Chunhua Shan**, University of Toledo.

Dynamical Systems: Smooth, Symbolic, and Measurable (a Mathematics Research Communities Session), **Kathryn Lindsey**, Boston College, **Scott Schmieding**, Northwestern University, and **Kurt Vinhage**, University of Chicago.

Emergent Phenomena in Discrete Models, **Dana Randall**, Georgia Institute of Technology, and **Andrea Richa**, Arizona State University.

Emerging Topics in Graphs and Matrices, **Sudipta Mallik**, Northern Arizona University, **Keivan Hassani Monfared**, University of Calgary, and **Bryan Shader**, University of Wyoming.

Ergodic Theory and Dynamical Systems–to Celebrate the Work of Jane Hawkins, **Julia Barnes**, Western Carolina University, **Rachel Bayless**, Agnes Scott College, **Emily Burkhead**, Duke University, and **Lorelei Koss**, Dickinson College.

Extremal Problems in Approximations and Geometric Function Theory, **Ram Mohapatra**, University of Central Florida.

Financial Mathematics, Actuarial Sciences, and Related Fields, **Albert Cohen**, Michigan State University, **Nguyet Nguyen**, Youngstown State University, **Oana Mocioalca**, Kent State University, and **Thomas Wakefield**, Youngstown State University.

Fractional Difference Operators and Their Application, **Christopher S. Goodrich**, Creighton Preparatory School, and **Rajendra Dahal**, Coastal Carolina University.

Free Convexity and Free Analysis, **J. William Helton**, University of California, San Diego, and **Igor Klep**, University of Auckland.

Geometric Analysis, **Davi Maximo**, University of Pennsylvania, **Lu Wang**, University of Wisconsin-Madison, and **Xin Zhou**, University of California Santa Barbara.

Geometric Analysis and Geometric Flows, **David Glickenstein**, University of Arizona, and **Brett Kotschwar**, Arizona State University.

History of Mathematics, **Sloan Despeaux**, Western Carolina University, **Jemma Lorenat**, Pitzer College, **Clemency Montelle**, University of Canterbury, **Daniel Otero**, Xavier University, and **Adrian Rice**, Randolph-Macon College.

Homotopy Type Theory (a Mathematics Research Communities Session), **Simon Cho**, University of Michigan, **Liron Cohen**, Cornell University, and **Edward Morehouse**, Wesleyan University.

If You Build It They Will Come: Presentations by Scholars in the National Alliance for Doctoral Studies in the Mathematical Sciences, **David Goldberg**, Purdue University, and **Phil Kutzko**, University of Iowa.

Interactions of Inverse Problems, Signal Processing, and Imaging, **M. Zuhair Nashed**, University of Central Florida, **Willi Freeden**, University of Kaiserslautern, and **Otmar Scherzer**, University of Vienna.

Markov Chains, Markov Processes and Applications, **Alan Krinik** and **Randall J. Swift**, California State Polytechnic University.

Mathematical Analysis and Nonlinear Partial Differential Equations, **Hongjie Dong**, Brown University, **Peiyong Wang**, Wayne State University, and **Jiuyi Zhu**, Louisiana State University.

Mathematical Fluid Mechanics: Analysis and Applications, **Zachary Bradshaw** and **Aseel Farhat**, University of Virginia.

Mathematical Information in the Digital Age of Science, **Patrick Ion**, University of Michigan, **Olaf Teschke**, zbMath Berlin, and **Stephen Watt**, University of Waterloo.

Mathematical Modeling and Analysis of Infectious Diseases, **Kazuo Yamazaki**, University of Rochester.

Mathematical Modeling of Natural Resources, **Shandelle M. Henson**, Andrews University, and **Natali Hritonenko**, Prairie View A&M University.

Mathematical Modeling, Analysis and Applications in Population Biology, **Yu Jin**, University of Nebraska-Lincoln, and **Ying Zhou**, Lafayette College.

Mathematical Problems in Ocean Wave Modeling and Fluid Mechanics, **Christopher W. Curtis**, San Diego State University, and **Katie Oliveras**, Seattle University.

Mathematical Relativity and Geometric Analysis, **James Dilts** and **Michael Holst**, University of California, San Diego.

Mathematics Research from the SMALL Undergraduate Research Program, **Colin Adams**, **Frank Morgan**, and **Cesar E. Silva**, Williams College.

Mathematics of Gravitational Wave Science, **Andrew Gillette** and **Nikki Holtzer**, University of Arizona.

Mathematics of Quantum Computing and Topological Phases of Matter, **Paul Bruillard**, Pacific Northwest National Laboratory, **David Meyer**, University of California San Diego, and **Julia Plavnik**, Texas A&M University.

Metric Geometry and Topology, **Christine Escher**, Oregon State University, and **Catherine Searle**, Wichita State University.

Modeling in Differential Equations - High School, Two-Year College, Four-Year Institution, **Corban Harwood**, George Fox University, **William Skerbitz**, Wayzata High School, **Brian Winkel**, SIMIODE, and **Dina Yagodich**, Frederick Community College.

Multi-scale Modeling with PDEs in Computational Science and Engineering:Algorithms, Simulations, Analysis, and Applications, **Salim M. Haidar**, Grand Valley State University.

Network Science, **David Burstein**, Swarthmore College, **Franklin Kenter**, United States Naval Academy, and **Feng Shi**, University of North Carolina at Chapel Hill.

New Trends in Celestial Mechanics, **Richard Montgomery**, University of California Santa Cruz, and **Zhifu Xie**, University of Southern Mississippi.

Nilpotent and Solvable Geometry, **Michael Jablonski**, University of Oklahoma, **Megan Kerr**, Wellesley College, and **Tracy Payne**, Idaho State University.

Noncommutative Algebras and Noncommutative Invariant Theory, **Ellen Kirkman**, Wake Forest University, and **James Zhang**, University of Washington.

Nonlinear Evolution Equations of Quantum Physics and Their Topological Solutions, **Stephen Gustafson**, University of British Columbia, **Israel Michael Sigal**, University of Toronto, and **Avy Soffer**, Rutgers University.

Novel Methods of Enhancing Success in Mathematics Classes, **Ellina Grigorieva**, Texas Womans University, and **Natali Hritonenko**, Prairie View A&M University.

Open and Accessible Problems for Undergraduate Research, **Michael Dorff**, Brigham Young University, **Allison Henrich**, Seattle University, and **Nicholas Scoville**, Ursinus College.

Operators on Function Spaces in One and Several Variables, **Catherine Bénéteau**, University of South Florida, and **Matthew Fleeman** and **Constanze Liaw**, Baylor University.

Orthogonal Polynomials and Applications, **Abey Lopez-Garcia**, University of South Alabama, and **Xiang-Sheng Wang**, University of Louisiana at Lafayette.

Orthogonal Polynomials, Quantum Probability, and Stochastic Analysis, **Julius N. Esunge**, University of Mary Washington, and **Aurel I. Stan**, Ohio State University.

Quantum Link Invariants, Khovanov Homology, and Low-dimensional Manifolds, **Diana Hubbard**, University of Michigan, and **Christine Ruey Shan Lee**, University of Texas at Austin.

Quaternions, **Terrence Blackman**, Medgar Evers College, City University of New York, and **Johannes Familton** and **Chris McCarthy**, Borough of Manhattan Community College, City University of New York.

Recent Trends in Analysis of Numerical Methods of Partial Differential Equations, **Sara Pollock**, Wright State University, and **Leo Rebholz**, Clemson University.

Research by Postdocs of the Alliance for Diversity in Mathematics, **Aloysius Helminck**, University of Hawaii - Manoa, and **Michael Young**, Iowa State University.

Research from the Rocky Mountain-Great Plains Graduate Research Workshop in Combinatorics, **Michael Ferrara**, University of Colorado Denver, **Leslie Hogben**, Iowa State University, **Paul Horn**, University of Denver, and **Tyrrell McAllister**, University of Wyoming.

Research in Mathematics by Early Career Graduate Students, **Michael Bishop**, **Marat Markin**, **Khang Tran**, and **Oscar Vega**, California State University, Fresno.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs, **Tamas Forgacs**, CSU Fresno, **Darren A. Narayan**, Rochester Institute of Technology, and **Mark David Ward**, Purdue University (AMS-MAA-SIAM).

Set Theory, Logic and Ramsey Theory, **Andrés Caicedo**, Mathematical Reviews, and **José Mijares**, University of Colorado, Denver (AMS-ASL).

Set-theoretic Topology (Dedicated to Jack Porter in Honor of 50 Years of Dedicated Research), **Nathan Carlson**, California Lutheran University, **Jila Niknejad**, University of Kansas, and **Lynne Yengulalp**, University of Dayton.

Special Functions and Combinatorics (in honor of Dennis Stanton's 65th birthday), **Susanna Fishel**, Arizona State University, **Mourad Ismail**, University of Central Florida, and **Vic Reiner**, University of Minnesota.

Spectral Theory, Disorder and Quantum Physics, **Rajinder Mavi** and **Jeffery Schenker**, Michigan State University.

Stochastic Processes, Stochastic Optimization and Control, Numerics and Applications, **Hongwei Mei**, University of Central Florida, **Zhixin Yang** and **Quan Yuan**, Ball State University, and **Guangliang Zhao**, GE Global Research.

Strengthening Infrastructures to Increase Capacity Around K-20 Mathematics, **Brianna Donaldson**, American Institute of Mathematics, **William Jaco** and **Michael Oehrtman**, Oklahoma State University, and **Levi Patrick**, Oklahoma State Department of Education.

Structure and Representations of Hopf Algebras: a Session in Honor of Susan Montgomery, **Siu-Hung Ng**, Louisiana State University, and **Lance Small** and **Henry Tucker**, University of California, San Diego.

Theory, Practice, and Applications of Graph Clustering, **David Gleich**, Purdue University, and **Jennifer Webster** and **Stephen J. Young**, Pacific Northwest National Laboratory.

Topological Data Analysis, **Henry Adams**, Colorado State University, **Gunnar Carlsson**, Stanford University, and **Mikael Vejdemo-Johansson**, CUNY College of Staten Island.

Topological Graph Theory: Structure and Symmetry, **Jonathan L. Gross**, Columbia University, and **Thomas W. Tucker**, Colgate University.

Visualization in Mathematics: Perspectives of Mathematicians and Mathematics Educators, **Karen Allen Keene**, North Carolina State University, and **Mile Krajcevski**, University of South Florida.

Women in Symplectic and Contact Geometry and Topology, **Bahar Acu**, Northwestern University, **Ziva Myer**, Duke University, and **Yu Pan**, Massachusetts Institute of Technology (AMS-AWM).

Columbus, Ohio

Ohio State University

March 17–18, 2018

Saturday – Sunday

Meeting #1136

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: December 2017
Program first available on AMS website: January 31, 2018
Issue of *Abstracts*: Volume 39, Issue 2

Deadlines

For organizers: Expired
For abstracts: January 22, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Aaron Brown, University of Chicago, *Title to be announced.*

Tullia Dymarz, University of Wisconsin-Madison, *Title to be announced.*

June Huh, Institute for Advanced Study, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances in Integral and Differential Equations (Code: SS 26A), **Jeffrey T. Neugebauer**, Eastern Kentucky University, and **Min Wang**, Rowan University.

Algebraic Coding Theory and Applications (Code: SS 27A), **Heide Gluesing-Luerssen**, University of Kentucky, **Christine A. Kelley**, University of Nebraska-Lincoln, and **Steve Szabo**, Eastern Kentucky University.

Algebraic Combinatorics: Association Schemes, Finite Geometry, and Related Topics (Code: SS 15A), **Sung Y. Song**, Iowa State University, and **Bangteng Xu**, Eastern Kentucky University.

Algebraic Curves and Their Applications (Code: SS 17A), **Artur Elezi**, American University, **Monika Polak**, Maria Curie-Sklodowska University (Poland) and University of Information Science and Technology (Mac, and **Tony Shaska**, Oakland University.

Algebraic and Combinatorial Aspects of Tropical Geometry (Code: SS 11A), **Maria Angelica Cueto**, Ohio State University, **Yoav Len**, University of Waterloo, and **Martin Ulirsch**, University of Michigan.

Algebraic, Combinatorial, and Quantum Invariants of Knots and Manifolds (Code: SS 6A), **Cody Armond**, Ohio State University, Mansfield, **Micah Chrisman**, Monmouth University, and **Heather Dye**, McKendree University.

Analytical and Computational Advances in Mathematical Biology Across Scales (Code: SS 30A), **Veronica Ciocanel** and **Alexandria Volkening**, Mathematical Biosciences Institute.

Categorical, Homological and Combinatorial Methods in Algebra (Celebrating the 80th birthday of S. K. Jain) (Code: SS 28A), **Pedro A. Guil Asensio**, University of Murcia, **Ivo Herzog**, Ohio State University, **Andre Leroy**, University of Artois, and **Ashish K. Srivastava**, Saint Louis University.

Coherent Structures in Interfacial Flows (Code: SS 14A), **Benjamin Akers** and **Jonah Reeger**, Air Force Institute of Technology.

Commutative and Combinatorial Algebra (Code: SS 18A), **Jennifer Biermann**, Hobart and William Smith Colleges, and **Kuei-Nuan Lin**, Penn State University, Greater Allegheny.

Convex Bodies in Algebraic Geometry and Representation Theory (Code: SS 20A), **Dave Anderson**, Ohio State University, and **Kiumars Kaveh**, University of Pittsburgh.

Differential Equations and Applications (Code: SS 8A), **King-Yeung Lam** and **Yuan Lou**, Ohio State University, and **Qiliang Wu**, Michigan State University.

Function Spaces, Operator Theory, and Non-Linear Differential Operators (Code: SS 21A), **David Cruz-Uribe**, University of Alabama, and **Osvaldo Mendez**, University of Texas.

Geometric Methods in Shape Analysis (Code: SS 10A), **Sebastian Kurtek** and **Tom Needham**, Ohio State University.

Graph Theory (Code: SS 5A), **John Maharry**, Ohio State University, **Yue Zhao**, University of Central Florida, and **Xiangqian Zhou**, Wright State University.

Homological Algebra (Code: SS 4A), **Ela Celikbas** and **Olgur Celikbas**, West Virginia University.

Homotopy Theory (Code: SS 29A), **Ernest Fontes**, **John E. Harper**, **Crichton Ogle**, and **Gabriel Valenzuela**, Ohio State University.

Lefschetz Properties (Code: SS 24A), **Juan Migliore**, University of Notre Dame, and **Uwe Nagel**, University of Kentucky.

Mathematical Modeling of Neuronal Networks (Code: SS 36A), **Janet Best**, Ohio State University, **Alicia Prieto Langarica**, Youngstown State University, and **Pamela B. Pyzza**, Ohio Wesleyan University.

Multiplicative Ideal Theory and Factorization (in honor of Tom Lucas retirement) (Code: SS 7A), **Evan Houston**, University of North Carolina, Charlotte, and **Alan Loper**, Ohio State University.

Noncommutative Algebra and Noncommutative Algebraic Geometry (Code: SS 16A), **Jason Gaddis**, Miami University, and **Robert Won**, Wake Forest University.

Nonlinear Evolution Equations (Code: SS 9A), **John Holmes** and **Feride Tiglay**, Ohio State University.

Nonlinear Waves and Patterns (Code: SS 19A), **Anna Ghazaryan**, Miami University, **Stephane Lafortune**, College of Charleston, and **Vahagn Manukian** and **Alin Pogan**, Miami University.

Parameter Analysis and Estimation in Applied Dynamical Systems (Code: SS 35A), **Adriana Dawes**, The Ohio State University, and **Reginald L. McGee**, Mathematical Biosciences Institute.

Probabilistic and Extremal Graph Theory (Code: SS 32A), **Louis DeBiasio** and **Tao Jiang**, Miami University.

Probability in Convexity and Convexity in Probability (Code: SS 2A), **Elizabeth Meckes**, **Mark Meckes**, and **Elisabeth Werner**, Case Western Reserve University.

Quantum Symmetries (Code: SS 3A), **David Penneys**, The Ohio State University, and **Julia Plavnik**, Texas A & M University.

Recent Advances in Approximation Theory and Operator Theory (Code: SS 1A), **Jan Lang** and **Paul Nevai**, The Ohio State University.

Recent Advances in Finite Element Methods for Partial Differential Equations (Code: SS 31A), **Ching-shan Chou**, **Yukun Li**, and **Yulong Xing**, The Ohio State University.

Recent Advances in Packing (Code: SS 23A), **Joseph W. Iverson**, University of Maryland, **John Jasper**, South Dakota State University, and **Dustin G. Mixon**, The Ohio State University.

Recent Development of Nonlinear Geometric PDEs (Code: SS 12A), **Bo Guan**, Ohio State University, **Qun Li**, Wright State University, **Xiangwen Zhang**, University of California, Irvine, and **Fangyang Zheng**, Ohio State University.

Several Complex Variables (Code: SS 13A), **Liwei Chen**, **Kenneth Koenig**, and **Liz Vivas**, Ohio State University.

Stochastic Analysis in Infinite Dimensions (Code: SS 22A), **Parisa Fatheddin**, Air Force Institute of Technology, and **Arnab Ganguly**, Louisiana State University.

Structure and Representation Theory of Finite Groups (Code: SS 33A), **Justin Lynd**, University of Louisiana at Lafayette, and **Hung Ngoc Nguyen**, University of Akron.

Symmetry in Differential Geometry (Code: SS 34A), **Samuel Lin**, Dartmouth College, **Barry Minemyer**, Bloomsburg University, and **Ben Schmidt**, Michigan State University.

The Mathematics of Phylogenetics (Code: SS 25A), **Colby Long**, Mathematical Biosciences Institute.

Topology and Geometry in Data Analysis (Code: SS 37A), **Sanjeevi Krishnan** and **Facundo Memoli**, Ohio State University.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include the Ohio state hotel tax (17.5%), local taxes and hotel fees may apply. Participants must state that they are with the **American Mathematical Society's (AMS) Spring Central Sectional Meeting** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

Courtyard by Marriott Columbus Downtown, 35 West Spring St, Columbus, OH 43215; (614) 228-3200; `www.marriott.com/hotels/travel/cmhcy-courtyard-columbus-downtown`. Rates are **US$119** per night for a room. Amenities include free Wi-Fi in guest rooms; breakfast, dinner, *Starbucks*® & evening cocktails; enjoy on-site dining at *The Bistro*; indoor pool, whirlpool and fitness center. Valet parking is available for a fee of **US$23** daily. This property is located about 3 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 14, 2018.**

Courtyard by Marriott Columbus/OSU, 780 Yard St, Columbus, OH 43212; (614) 453-4420; `www.marriott.com/hotels/fact-sheet/travel/cmhwg-courtyard-columbus-osu`. Rates are **US$154** per night for a standard guest room. Group rate includes one complimentary welcome water per guest booked in the room block. Amenities include *The Bistro* for breakfast, drinks and dinner during the evening; free Wi-Fi throughout; the latest news, weather and airport conditions via *GoBoard*® technology; fitness center and indoor pool; and complimentary on-site parking. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. This property is located minutes from campus. The deadline for reservations at a reduced rate is by **February 23, 2018.**

Courtyard Columbus Dublin, 5175 Post Rd, Dublin, OH 43017; (614) 764-9393; `www.marriott.com/hotels/travel/cmhdb-courtyard-columbus-dublin`. Rates are **US$79** per night for standard guest rooms. Amenities include free Wi-Fi; on-site restaurant, *The Bistro*, serving breakfast; fitness center with cardio equipment and free weights; complimentary self parking and on-site laundry facilities. This property is located about a 17 minute drive from the campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at a reduced rate is **February 23, 2018.**

Courtyard Columbus Worthington, 7411 Vantage Dr, Columbus, OH 43235; (614) 436-7070; `www.marriott.com/hotels/fact-sheet/travel/cmhcw-courtyard-columbus-worthington`. Rates are **US$84** per night for a standard guest room. Amenities includes complimentary high-speed Wi-Fi access; *The Bistro,* open daily for breakfast and dinner; specialty *Starbucks* and evening cocktails; fitness center and indoor pool and on-site laundry facilities. This property is located approximately a 15 minute drive from campus. Complimentary on-site parking is available. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 23, 2018.**

Crowne Plaza Columbus Downtown , 33 East Nationwide Blvd, Columbus, OH 43215; (614) 461-4100; `www.ihg.com/crowneplaza/hotels/us/en/columbus/cmhoc/hoteldetail`. Rates are **US$159** per night for a standard room with double beds. Amenities include fitness room and free Wi-Fi. Self-parking rate is **US$28** per day. This property is located about a 10 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 14, 2018.**

Drury Inn and Suites Columbus Convention Center, 88 East Nationwide Blvd, Columbus, OH 43215; (614) 221-7008; `www.druryhotels.com/locations/columbus-oh/drury-inn-and-suites-columbus-convention-center`. Rates are **US$125** per night for single or double occupancy in a non-smoking two queen beds deluxe room. Group room rate includes free hot breakfast served daily; free evening drinks and snacks from served daily from 5:30pm to 7:00pm; free Wi-Fi throughout the hotel and free soft drinks and popcorn in the lobby served daily from 3:00pm to 10:00pm. Amenities include business center, guest laundry facilities, fitness center and pool. On-site covered parking is available at a charge of **US$12** per parking spot per day. This property is located about a 10 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 16, 2018.**

Fairfield Inn and Suites Columbus OSU by Marriott, 3031 Olentangy River Rd, Columbus, OH 43202; (614) 267-1111; `www.marriott.com/hotels/travel/cmhsu-fairfield-inn-and-suites-columbus-osu`. Rates are **US$139** per night for a room. Amenities include complimentary Wi-Fi, indoor pool, fitness room, free daily breakfast and complimentary self parking. This property is located just one mile from The Ohio State University. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 14, 2018.**

Hampton Inn and Suites Columbus University Area, 3160 Olentangy River Rd, Columbus, OH 43202; (614)

268-8700; hamptoninn3.hilton.com/en/hotels/ohio/hampton-inn-and-suites-columbus-university-area-CMHUNHX/index.html. Rates are **US$159** for a room**.** Amenities include complimentary hot breakfast served every morning, complimentary Wi-Fi access throughout the hotel, indoor swimming pool and 24-hour fitness center, 24-hour business center and complimentary shuttle service within a 3-mile radius of the hotel. This property is located about 1 mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 23, 2018.**

Hampton Inn and Suites Downtown Columbus, 501 North High St, Columbus, OH 43215; (614) 559-2000; hamptoninn3.hilton.com/en/hotels/ohio/hampton-inn-and-suites-columbus-downtown-CMHH-SHX/index.html. Rates are **US$124** for a standard single room, **US$134** for a standard double room and **US$144** for a king suite room. Amenities include complimentary hot breakfast, complimentary Wi-Fi access and swimming pool and 24-hour fitness center. This property is located about a 10 minute drive from campus. Valet parking is available for **US$25** a night. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 16, 2018.**

Hyatt Regency Columbus, 350 North High St, Columbus, OH 43215; (614) 463-1234; columbus.regency.hyatt.com/en/hotel/home.html. Rates are **US$139** for a standard king room or a standard two double bed room. Amenities include indoor pool, sun deck, *StayFit* fitness facility and complimentary Wi-Fi access. This property is located about a 10 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 16, 2018.**

Red Roof PLUS+ Columbus OSU, 441 Ackerman Rd, Columbus, OH 43202; (614) 267-9941; columbusosu.redroof.com. Rates are **US$124** for a non-smoking room with two queen beds. This property is located about a 6 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 16, 2018.**

Renaissance Columbus Downtown Hotel, 50 North Third St, Columbus, OH 43215; (614) 228-5050; www.marriott.com/hotels/travel/cmhbr-renaissance-columbus-downtown-hotel. Rates are **US$151** for a standard king room. Amenities include *Latitude 41* restaurant, *Bar 41,* fitness center, seasonal rooftop pool, whirlpool and sauna. This property is located about a 6 minute drive from campus. Valet parking is available for a fee of **US$26** a day. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 23, 2018.**

Residence Inn Columbus Downtown, 36 East Gay St, Columbus, OH 43215; (614) 222-2610; www.marriott.com/hotels/travel/cmhrd-residence-inn-columbus-downtown. Rates are **US$119** for a studio suite room. Amenities include fully equipped kitchens in every suite, free Wi-Fi, complimentary hot breakfast buffet and on-site bar and restaurant, *Buckeye Bourbon House.* This property is located about a 10 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 16, 2018.**

Varsity Inn, 1445 Olentangy River Rd, Columbus, OH 43212; (614) 291-2983; www.varsityinn.com. Rates are **US$89** for a room. Amenities include outdoor pool and free hotel-wide Wi-Fi. This property is located about a 2 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 20, 2018.**

Food Services

On Campus: The below restaurants are scheduled to be open on Saturdays and Sundays. Please visit dining.osu.edu/ for a list of university restaurants and hours.

12th Ave. Bread Company, 251 W. 12th Ave (Kennedy Commons); open noon to 8:00pm on Saturdays and Sundays; serves sandwiches, soups, salads, and morning quiche and pastries. Indoor and outdoor seating is available.

EspressOH, 1739 North High St (The Ohio Union); open 9:00am to 8:00pm on Saturday and 10:00am to 9:00pm on Sunday; serves specialty coffee and homemade gelato.

Connecting Grounds, 160 West Woodruff Ave; open 10:00am to 8:00pm on Saturdays and Sundays; serves espresso, specialty coffees, bagels, muffins and pastries.

Courtside Cafe, 337 Annie and John Glenn Ave (RPAC); open 11:00am to 8:00pm on Saturday and noon to 10:00pm on Sunday; serves sandwiches, pastas, wraps, pizza, salad, soup, sushi, yogurt and fruit.

Street Sweets and *Marketplace,* 1578 Neil Ave; open 10:00am to 7:00pm on Saturday and 10:00am to 8:00pm on Sunday; the *Marketplace* serves a variety of entrees and *StreetSweets* features an espresso bar at the entrance.

Berry Cafe, 1858 Neil Ave Mall (William Oxley Thompson Library); open 8:00am to 8:00pm on Saturday and 11:00am to 11:00pm on Sunday; serves grab 'n go sandwiches, yogurt parfaits, muffins, hummus, edamame, salads and coffee.

Sloopy's Diner, 1739 North High St (The Ohio Union); open 9:00am to 2:00am on Saturday and 10:00am to midnight on Sunday; serves breakfast favorites, diner-style classics and sandwiches in a 1950's-inspired diner.

Union Market, 1739 North High St (The Ohio Union); open 11:00am to 8:00pm on Saturday and 11:00am to 10:00pm on Sunday; offers classic favorites at the *Fired Up! Grill*, fresh salads at *Across the Field*, international fare at *Buckeye Passports*, and fresh deli sandwiches and wraps at *Dough-HIO.*

Woody's Tavern, 1739 North High St (The Ohio Union); open 1:00pm to 10:00pm on Saturday and Sunday; serves pizza, beer and wine, popcorn and good old-fashioned root beer.

Off Campus: *Buckeye Donuts* has been a favorite stop since it opened in 1969. The old school donut shop serves breakfast sandwiches and gyros, too. *The Gateway* on campus is home to good eats, too, like big eateries and bars such as *Mad Mex, Ugly Tuna Saloona* and *World of Beer*. Or there's always-favorite *Hounddog's Pizza*, with their famous Smokin' Joes crusts. *Ethyl & Tank*, located behind *Newport Music Hall*, serves as a coffee shop, a bar, a lunch and dinner spot, and a weekend brunch haven. More information on restaurants and local attractions in the Columbus area can be found at www.experiencecolumbus.com/restaurants/near-osu.

Some options for coffee include:

- *Buckeye Donuts,* corner of 18th and High St, Columbus; alwaysopen.buckeye-donuts.com/; old school donut shop open 24 hours and serving breakfast sandwiches and gyros, too.
- *KAFE Kerouac,* 2250 N. High St, Columbus; (614) 299-2672; kafekerouac.com/; coffee house with locally roasted fair trade beans, local and imported beers and wine.
- *Starbucks*, 1782 N High St, Columbus; (614) 291-5692; Seattle-based coffeehouse chain known for its signature roasts, light bites and Wi-Fi availability.
- Some options for dining in the University District include:
- *Mad Mex,* South Campus Gateway, 1542 N. High St, Columbus; (614) 586-4007; www.madmex.com; California-Mexican food, microbrews and unique artwork.
- *Ugly Tuna Saloona,* South Campus Gateway, 1546 N. High St, Columbus; (614) 297-8862; www.uglytunasaloona.com; fresh fish and local music with 2nd floor patio.
- *World of Beer*, 1556 North High St, Columbus; (614) 403-3483; www.worldofbeer.com; serving a collection of dishes and tavern classics with craft beer.
- *Hounddog's Three Degree Pizza*, 2657 N High St, Columbus; (614) 261-4686; www.hounddogspizza.com; creative pizzas and classic subs about a mile from campus.
- *Ethyl & Tank,* 19 E. 13th Ave, Columbus; (614) 947-0140; www.ethylandtank.com; informal Southwestern meals & craft beers in a relaxed, brick-walled space with a video game arcade.
- *Bistro 2110 at the Blackwell*, 2110 Tuttle Park Pl, Columbus; (614) 247-2110; www.theblackwell.com/bistro-2110.html; hotel restaurant offering New American meals, plus brunch & lunch buffets, in an upscale setting.
- *Varsity Club Restaurant & Bar*, 278 W Lane Ave, Columbus; (614) 299-6269; www.varsityclubrestaurant-columbusoh.com; old-school sports bar with pub grub with TVs, patio seating and free Wi-Fi.

Registration and Meeting Information

Advance Registration: Advance registration for this meeting opens on **January 30, 2018.** Advance registration fees will be **US$61** for AMS members, **US$90** for nonmembers, and **US$10** for students, unemployed mathematicians, and emeritus members. Participants may cancel registrations made in advance by emailing mmsb@ams.org. The deadline to cancel is the first day of the meeting.

On-site Information and Registration: The registration desk, AMS book exhibit, and coffee service will be located in the lobby of Hitchcock Hall. The Invited Address lectures will be located in room 131 of Hitchcock Hall. Special Sessions and Contributed Paper Sessions will take place in the nearby classrooms. Please look for additional information about specific session room locations on the web and in the printed program. For further information on building locations, a campus map is available at www.osu.edu/map.

The registration desk will be open on Saturday, March 17, 7:30am to 4:00pm and Sunday, March 18, 8:00am to 12:00pm. The same fees listed above apply for on-site registration and are payable with cash, check or credit card.

Other Activities

Book Sales: Stop by the on-site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on-site orders! AMS members receive 40 percent off list price. Nonmembers receive a 25 percent discount. AMS Members receive additional discounts on books purchased at meetings, subscriptions to *Notices* and *Bulletin*, discounted registration for world-class meetings and conferences, and more!

Complimentary Coffee will be served courtesy of the AMS Membership Department.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

Special Needs

It is the goal of the AMS to ensure that its conferences are accessible to all, regardless of disability. The AMS will strive, unless it is not practicable, to choose venues that are fully accessible to the physically handicapped.

If special needs accommodations are necessary in order for you to participate in an AMS Sectional Meeting, please communicate your needs in advance to the AMS Meetings Department by:

- Registering early for the meeting

- Checking the appropriate box on the registration form, and

- Sending an email request to the AMS Meetings Department at mmsb@ams.org or meet@ams.org.

AMS Policy on a Welcoming Environment

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. In all its activities, the AMS seeks to foster an atmosphere that encourages the free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity, or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

Local Information and Maps

This meeting will take place on the campus of The Ohio State University. A campus map can be found at `www.osu.edu/map`. Information about The Ohio State University Mathematics Department can be found at `math.osu.edu`. Please visit the university website at `www.osu.edu` for additional information on the campus.

Please watch the AMS website at `www.ams.org/meetings/sectional/sectional.html` for additional information on this meeting.

Parking

Visitors and guests to The Ohio State University are required to purchase parking any time they park on campus. All parking on campus is allocated and signed for specific uses, and **parking regulations are enforced 24/7**. Visitor parking is available in parking garages, prevalent in the dense Academic Core and Wexner Medical Center areas, or surface lots, typically located in less dense areas along the campus perimeter.

CampusParc provides several garage options for visitor parking. Hourly parking rates are posted at the entrance of each pay facility and can also be found at `osu.campusparc.com/home/visitors-patients/visitor-parking/garage-parking`. All garage fees are due upon exit. Credit cards are accepted.

Most visitor garages are equipped with pay-on-foot machines. Customers receive a white entry ticket and keep it with them to make payment prior to returning to their vehicle, reducing backups at the exit gate. Once payment is made, customers receive a returned paid/validated ticket for use at the exit gate. Pay-on-foot machines accept cash, credit, and vouchers. *NOTE: Cash is not accepted at the exit gate.*

In garages not equipped with pay-on-foot machines, payment is made at the exit gate. Upon arrival at the exit gate, the customer should insert the white entry ticket into the pay-in-lane machine and pay the displayed amount due. Pay-in-lane machines accept cash, credit, and vouchers. Credit cards are the preferred method of payment to expedite exit. *NOTE: Pay-in-lane machines give change in coin only.*

Receipts for paid parking are offered from payment machines at the time of payment. If a receipt is not printed, customers can email `osuinfo@campusparc.com` with the time, date, garage location, and amount charged to receive a copy of their receipt via email.

Surface lot payment options are valid in surface lots only; if you park in a garage, all hourly fees must be paid upon exit. All parking rules are enforced 24/7. Surface lot parking is restricted on football Saturdays.

Surface lot payment options include single-space meters placed in high-demand areas to facilitate short-term parking. Meters are available to anyone (CampusParc permit holders or visitors) on campus provided that the meter fees are paid. All meters take US quarters, dimes, and nickels only; the maximum time allowed is posted on each meter.

Some surface lots include Pay-by-Plate machines located throughout campus to allow visitors to purchase parking. To pay for parking, walk to the nearest Pay-by-Plate machine (clearly marked by directional signs). Enter your license plate number and select the form of payment and amount of parking time desired, as indicated by printed instructions on the Pay-by-Plate machine. Since verification of payment is done via license plate, it is imperative to enter your license plate information correctly. Please verify all information before finalizing your transaction. Machines do not provide change or refunds. Check the campus parking map for Pay-by-Plate locations at `osu.campusparc.com/docs/default-source/maps/campus-parking-map.pdf?sfvrsn=67`.

Some surface lots allow *Parkmobile* as a payment option that uses pay-by-phone and smartphone applications. Customers can create a *Parkmobile* account online at `www.parkmobile.com`, via the Parkmobile app, or via their automated phone system at (877) 727-5009. After entering the required information and setting up an account, customers can begin "parking sessions" by accessing their account (via one of the three methods available) and confirming their license plate number, parking zone number, and intended duration of stay. Parking sessions can be terminated early or extended via the same method used to start the parking session.

Surface lot visitor permits may be purchased at the CampusParc Customer Service Center. Central Campus visitor permits are valid in any unrestricted surface lot spaces. Paid hourly parking is required for vehicles parked in marked hourly spaces, at single-space meters, or in parking garages. Permits are available in daily increments which are good for one calendar day. Vehicles parked overnight require the purchase of multiple day permits or paid hourly parking.

A campus parking map is available at `osu.campusparc.com/docs/default-source/maps/campus-parking-map.pdf?sfvrsn=67`. Additional information about parking on campus can be found at `osu.campusparc.com/home/visitors-patients/visitor-parking/surface-lot-parking`. Please call the CampusParc customer service line at (614) 688-0000 with parking inquiries.

A list of university parking rates are available at `osu.campusparc.com/docs/default-source/documents/ratetable.pdf?sfvrsn=22`.

Travel

This meeting will take place on the main campus of The Ohio State University located in Columbus, Ohio.

By Air: The John Glenn Columbus International Airport (CMH) is about 20 minutes from the Ohio State University and is the most convenient air travel choice. Please note, the drive could take longer during rush hour traffic. Please visit the airport web site for a list of airlines and lists of cities with daily direct flights; `flycolumbus.com`.

There are several options available for transportation to and from the airport.

The Central Ohio Transit Authority (COTA) provides direct bus service between the airport and downtown for only **US$2.75**. It's called *AirConnect,* an affordable and easy way to reach downtown hotels and the Greater Columbus Convention Center. Purchase your ride pass with a credit card at the bus stop, which is just to the right of the taxi station. COTA's *AirConnect* service runs every 30 minutes, 7 days a week. Once downtown, COTA operates dozens of bus lines, including the free CBUS that connects the Brewery District, downtown and the Short North together. More details and a map of downtown Columbus *AirConnect* stops are available at `www.cota.com/how-to-ride/airconnect`. The COTA bus now provides real time updates on Google Maps and the COTA transit app.

Taxis and pick-up locations for ridesharing services are available on the ground transportation level of the terminal 24 hours a day. The electronic meter will compute your fare from John Glenn International to your destination. The approximate fare to travel to downtown Columbus is **US$25**. For a rate chart of taxi fares visit `flycolumbus.com/getting-to-from/taxis-rideshare`.

There are eight car rental agencies located at the airport. Check-in counters are located on the ground floor of the parking garage and include *Alamo, Avis, Budget, Dollar, Enterprise, Hertz, National* and *Thrifty*. *car2go* members can pick up or drop off a car at *The Parking Spot*. For more information on *car2go* membership visit `https://www.car2go.com/US/en`.

Several companies provide shuttle and limo services in the area. A list of companies with phone numbers is available at `flycolumbus.com/getting-to-from/shuttles-limos`.

By Bus: The Columbus Greyhound Station is located at 111 E. Town Street in Columbus, about 5 miles from campus. For tickets and travel information visit their website at `www.greyhound.com/default.aspx`.

By Car: From John Glenn Columbus International Airport: From International Gateway follow signs for Interstate 670 W/US 62 W/Columbus. Take exit 2B to merge onto OH-315 N and take the Medical Cntr Dr exit toward King Ave.

From Ohio State University Airport: Follow W Case Rd and Godown Rd to Bethel Rd. Take OH-315 S to Ackerman Rd (signs for Ackerman Road/Dodridge St).

Car Rental: Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at `www.hertz.com`, click on the box "I have a discount", and type in our convention number (CV): CV#04N30008. You can also call Hertz directly at 800-654-2240 (US and Canada) or 1-405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

For directions to campus, inquire at your rental car counter.

Local Transportation

Walking, biking and personal cars are recommended to get around campus and Columbus.

By Bus: *The Campus Area Bus Service (CABS)* is a free transit service provided by The Ohio State University Transportation and Traffic Management. *CABS* is dedicated to providing clean, reliable, and hassle-free transportation on and around Ohio State's Columbus Campus. For schedules and routes visit `ttm.osu.edu/cabs.`

Bike Share: This program offers university students, faculty and staff members, and campus visitors an alternative option to traveling across campus and support the "park once" philosophy. Visitors can choose from the public annual rate of **US$75** or choose a 24-hour rental at **US$6** a day. Payment options include BuckID and credit card. Ride up to 2 hours at a time during the week; 3 hours at a time on weekends. Visit `ttm.osu.edu/bikeshare` to sign up.

Other options: Other local transportation options include *Uber*; `www.uber.com` and *Lyft;* `www.lyft.com`.

Weather:

The average weather in March in Columbus, Ohio is characterized by rapidly rising daily high temperatures, with daily highs increasing by 12°F, from 45°F to 57°F over the course of the month, and rarely exceeding 73°F or dropping below 31°F. Attendees are advised to wear coats and layered clothing. The weather can be unpredictable so umbrellas are also recommended.

Social Networking:

Attendees and speakers are encouraged to tweet about the meeting using the hashtag #AMSmtg.

Information for International Participants:

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at `travel.state.gov/content/travel/en.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `cro@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence.

Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the US host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Nashville, Tennessee

Vanderbilt University

April 14–15, 2018

Saturday – Sunday

Meeting #1138

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: January 2018
Program first available on AMS website: February 22, 2018
Issue of *Abstracts*: Volume 39, Issue 2

Deadlines

For organizers: Expired
For abstracts: February 13, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Andrea Bertozzi, University of California Los Angeles, *Title to be announced* (Erdős Memorial Lecture).

J. M. Landsberg, Texas A & M University, *Title to be announced.*

Jennifer Morse, University of Virginia, *Title to be announced.*

Kirsten Wickelgren, Georgia Institute of Technology, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Operator Algebras (Code: SS 7A), **Scott Atkinson**, **Dietmar Bisch**, **Vaughan Jones**, and **Jesse Peterson**, Vanderbilt University.

Algebraic Geometry, Representation Theory, and Applications (Code: SS 21A), **Shrawan Kumar**, University of North Carolina at Chapel Hill, **J. M. Landsberg**, Texas A&M University, and **Luke Oeding**, Auburn University.

Boundaries and Non-positive Curvature in Group Theory (Code: SS 15A), **Spencer Dowdall** and **Matthew Hallmark**, Vanderbilt University, and **Michael Hull**, University of Florida.

Commutative Algebra (Code: SS 8A), **Florian Enescu** and **Yongwei Yao**, Georgia State University.

Difference Equations and Applications (Code: SS 2A), **Michael A. Radin**, Rochester Institute of Technology, and **Youssef Raffoul**, University of Dayton, Ohio.

Evolution Equations and Applications (Code: SS 14A), **Marcelo Disconzi**, **Chenyun Luo**, **Giusy Mazzone**, and **Gieri Simonett**, Vanderbilt University.

Function Spaces and Operator Theory (Code: SS 9A), **Cheng Chu** and **Dechao Zheng**, Vanderbilt University.

Harmonic Analysis, Functional Analysis, and Their Applications (Code: SS 11A), **Akram Aldroubi** and **Keaton Hamm**, Vanderbilt University, **Michael Worthington**, Georgia Institute of Technology, and **Alex Powell**, Vanderbilt University.

Hermitian Geometry (Code: SS 18A), **Mehdi Lejmi**, Bronx Community College of CUNY, and **Rares Rasdeaconu** and **Ioana Suvaina**, Vanderbilt University.

Interactions between Geometry, Group Theory and Dynamics (Code: SS 13A), **Jayadev Athreya**, University of Washington, and **Caglar Uyanik** and **Grace Work**, Vanderbilt University.

Macdonald Polynomials and Related Structures (Code: SS 23A), **Jennifer Morse**, University of Virginia, and **Dan Orr** and **Mark Shimozono**, Virginia Polytechnic Institute and State University.

Mathematical Chemistry (Code: SS 10A), **Hua Wang**, Georgia Southern University.

Matroids and Related Structures (Code: SS 5A), **Carolyn Chun**, United States Naval Academy, **Deborah Chun** and **Tyler Moss**, West Virginia University Institute of Technology, and **Jakayla Robbins**, Vanderbilt University.

Partial Differential Equations and New Perspective of Variational Methods (Code: SS 16A), **Abbas Moameni**, Carleton University, **Futoshi Takahashi**, Osaka City University, **Michinori Ishiwata**, Osaka University, and **Craig Cowen**, University of Manitoba.

Probabilistic Models in Mathematical Physics (Code: SS 6A), **Robert Buckingham**, University of Cincinnati, **Seung-Yeop Lee**, University of South Florida, and **Karl Liechty**, DePaul University.

Quantization for Probability Distributions and Dynamical Systems (Code: SS 1A), **Mrinal Kanti Roychowdhury**, University of Texas Rio Grande Valley.

Random Discrete Structures (Code: SS 22A), **Lutz P Warnke**, Georgia Institute of Technology, and **Xavier Pérez-Giménez**, University of Nebraska-Lincoln.

Recent Advances in Mathematical Biology (Code: SS 12A), **Glenn Webb** and **Yixiang Wu**, Vanderbilt University.

Recent Advances on Complex Bio-systems and Their Applications (Code: SS 17A), **Pengcheng Xiao**, University of Evansville.

Recent Progress and New Directions in Homotopy Theory (Code: SS 20A), **Anna Marie Bohmann**, Vanderbilt University, and **Kirsten Wickelgren**, Georgia Institute of Technology.

Selected Topics in Graph Theory (Code: SS 3A), **Songling Shan**, Vanderbilt University, and **David Chris Stephens** and **Dong Ye**, Middle Tennessee State University.

Structural Graph Theory (Code: SS 4A), **Joshua Fallon**, Louisiana State University, and **Emily Marshall**, Arcadia University.

Tensor Categories and Diagrammatic Methods (Code: SS 19A), **Marcel Bischoff**, Ohio University, and **Henry Tucker**, University of California San Diego.

Portland, Oregon

Portland State University

April 14–15, 2018

Saturday – Sunday

Meeting #1137

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: January 2018
Program first available on AMS website: February 15, 2018
Issue of *Abstracts*: Volume 39, Issue 2

Deadlines

For organizers: Expired
For abstracts: February 6, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Sándor Kovács, University of Washington, Seattle, *Title to be announced.*

Elena Mantovan, California Institute of Technology, *Title to be announced.*

Dimitri Shlyakhtenko, University of California, Los Angeles, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Geometry and its Connections (Code: SS 9A), **Sándor Kovács**, University of Washington, Seattle, and **Karl Schwede**, University of Utah, Salt Lake City.

Algebraic Topology (Code: SS 23A), **Angélica Osorno**, Reed College, and **Dev Sinha**, University of Oregon.

Algebraic and Combinatorial Structures in Knot Theory (Code: SS 3A), **Allison Henrich**, Seattle University, **Inga Johnson**, Willamette University, and **Sam Nelson**, Claremont McKenna College.

Automorphisms of Riemann Surfaces and Related Topics (Code: SS 14A), **S. Allen Broughton**, Rose-Hulman Institute of Technology, **Mariela Carvacho**, Universidad Tecnica Federico Santa Maria, **Anthony Weaver**, Bronx Community College, the City University of New York, and **Aaron Wootton**, University of Portland.

Biomathematics - Progress and Future Directions (Code: SS 4A), **Hannah Callender Highlander**, University of Portland, **Peter Hinow**, University of Wisconsin - Milwaukee, and **Deena Schmidt**, University of Nevada, Reno.

Commutative Algebra (Code: SS 5A), **Adam Boocher**, University of Utah, and **Irena Swanson**, Reed College.

Complex Analysis and Applications (Code: SS 11A), **Malik Younsi**, University of Hawaii Manoa.

Differential Geometry (Code: SS 19A), **Christine Escher**, Oregon State University, and **Catherine Searle**, Wichita State University.

Forest Modeling (Code: SS 20A), **Gatziolis Demetrios**, Pacific Northwest Research Station, US Forest Service, and **Nikolay Strigul**, Washington State University, Vancouver.

General Relativity and Geometric Analysis (Code: SS 13A), **Paul T. Allen**, Lewis & Clark College, **Jeffrey Jauregui**, Union College, and **Iva Stavrov Allen**, Lewis & Clark College.

Geometric Measure Theory and Partial Differential Equations (Code: SS 12A), **Mark Allen**, Brigham Young University, and **Spencer Becker-Kahn** and **Mariana Smit Vega Garcia**, University of Washington.

Inverse Problems (Code: SS 2A), **Hanna Makaruk**, Los Alamos National Laboratory (LANL), and **Robert Owczarek**, University of New Mexico, Albuquerque & Los Alamos.

Mock Modular and Quantum Modular Forms (Code: SS 24A), **Holly Swisher**, Oregon State University, and **Stephanie Treneer**, Western Washington University.

Modeling, Analysis, and Simulation of PDEs with Multiple Scales, Interfaces, and Coupled Phenomena (Code: SS 17A), **Yekaterina Epshteyn**, University of Utah, and **Malgorzata Peszynska**, Oregon State University.

Moduli Spaces (Code: SS 21A), **Renzo Cavalieri**, Colorado State University, and **Damiano Fulghesu**, Minnesota State University Moorhead.

Motivic homotopy theory (Code: SS 6A), **Daniel Dugger**, University of Oregon, and **Kyle Ormsby**, Reed College.

Noncommutative Algebraic Geometry and Related Topics (Code: SS 16A), **Jesse Levitt**, University of Southern California, **Hans Nordstrom**, University of Portland, and **Xinting Wang**, Temple University.

Nonsmooth Optimization and Applications(Dedicated to Prof. B. S. Mordukhovich on the occasion of his 70th birthday) (Code: SS 7A), **Mau Nam Nguyen**, Portland State University, **Hung M. Phan**, University of Massachusetts Lowell, and **Shawn Xianfu Wang**, University of British Columbia.

Numerical Methods for Partial Differential Equations (Code: SS 22A), **Brittany A. Erickson** and **Jeffrey S. Ovall**, Portland State University.

Pattern Formation in Crowds, Flocks, and Traffic (Code: SS 1A), **J. J. P. Veerman**, Portland State University, **Alethea Barbaro**, Case Western Reserve University, and **Bassam Bamieh**, UC Santa Barbara.

Recent Advances in Actuarial Mathematics (Code: SS 18A), **Sooie-Hoe Loke**, Central Washington University, and **Enrique Thomann**, Oregon State University.

Spectral Theory (Code: SS 8A), **Jake Fillman**, Virginia Tech, and **Milivoje Lukic**, Rice University.

Teaching and Learning in Undergraduate Mathematics (Code: SS 15A), **Natalie LF Hobson**, Sonoma State University, and **Elise Lockwood**, Oregon State University.

Wavelets, Frames, and Related Expansions (Code: SS 10A), **Marcin Bownik**, University of Oregon, and **Darrin Speegle**, Saint Louis University.

Boston, Massachusetts

Northeastern University

April 21–22, 2018

Saturday – Sunday

Meeting #1139

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: January 2018
Program first available on AMS website: March 1, 2018
Issue of *Abstracts*: Volume 39, Issue 2

Deadlines

For organizers: Expired
For abstracts: February 20, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Jian Ding, University of Pennsylvania, *Random walk, random media and random geometry*.

Edward Frenkel, University of California, Berkeley, *Imagination and knowledge* (Einstein Public Lecture in Mathematics).

Valentino Tosatti, Northwestern University, *Metric limits of Calabi-Yau manifolds*.

Maryna Viazovska, École Polytechnique Fédérale de Lausanne, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Number Theory (Code: SS 35A), **Michael Bush**, Washington and Lee University, **Farshid Hajir**, University of Massachusetts, and **Christian Maire**, Université Bourgogne Franche-Comté.

Algebraic Statistics (Code: SS 33A), **Kaie Kubjas** and **Elina Robeva**, Massachusetts Institute of Technology.

Algebraic, Geometric, and Topological Methods in Combinatorics (Code: SS 21A), **Florian Frick**, Cornell University, and **Pablo Soberón**, Northeastern University.

Algorithmic Group Theory and Applications (Code: SS 26A), **Delaram Kahrobaei**, City University of New York, and **Antonio Tortora**, University of Salerno.

Analysis and Geometry in Non-smooth Spaces (Code: SS 5A), **Nageswari Shanmugalingam** and **Gareth Speight**, University of Cincinnati.

Arithmetic Dynamics (Code: SS 1A), **Jacqueline M. Anderson**, Bridgewater State University, **Robert Benedetto**, Amherst College, and **Joseph H. Silverman**, Brown University.

Arrangements of Hypersurfaces (Code: SS 2A), **Graham Denham**, University of Western Ontario, and **Alexander I. Suciu**, Northeastern University.

Combinatorial Aspects of Nilpotent Orbits (Code: SS 15A), **Anthony Iarrobino**, Northeastern University, **Leila Khatami**, Union College, and **Juliana Tymoczko**, Smith College.

Combinatorial Representation Theory (Code: SS 41A), **Laura Colmenarejo**, York University, **Ricky Liu**, North Carolina State University, and **Rosa Orellana**, Dartmouth College.

Connections Between Trisections of 4-manifolds and Low-dimensional Topology (Code: SS 32A), **Jeffrey Meier**, University of Georgia, and **Juanita Pinzon-Caicedó**, North Carolina State University.

Discretization in Geometry and Dynamics (Code: SS 36A), **Richard Kenyon**, **Wai Yeung Lam**, and **Richard Schwartz**, Brown University.

Dynamical systems, Geometric Structures and Special Functions (Code: SS 23A), **Alessandro Arsie**, University of Toledo, and **Oksana Bihun**, University of Colorado, Colorado Springs.

Effective Behavior in Random Environments (Code: SS 25A), **Jessica Lin**, McGill University, and **Charles Smart**, University of Chicago.

Ergodic Theory and Dynamics in Combinatorial Number Theory (Code: SS 7A), **Stanley Eigen** and **Daniel Glasscock**, Northeastern University, and **Vidhu Prasad**, University of Massachusetts, Lowell.

Extremal Graph Theory and Quantum Walks on Graphs (Code: SS 13A), **Sebastian Cioabă**, University of Delaware, **Mark Kempton**, Harvard University, **Gabor Lippner**, Northeastern University, and **Michael Tait**, Carnegie Mellon University.

Facets of Symplectic Geometry and Topology (Code: SS 3A), **Tara Holm**, Cornell University, **Jo Nelson**, Columbia University, and **Jonathan Weitsman**, Northeastern University.

Geometries Defined by Differential Forms (Code: SS 34A), **Mahir Bilen Can**, Tulane University, **Sergey Grigorian**, University of Texas Rio Grande Valley, and **Sema Salur**, University of Rochester.

Geometry and Analysis of Fluid Equations (Code: SS 28A), **Robert McOwen** and **Peter Topalov**, Northeastern University.

Geometry of Moduli Spaces (Code: SS 10A), **Ana-Marie Castravet** and **Emanuele Macrí**, Northeastern University, **Benjamin Schmidt**, University of Texas, and **Xiaolei Zhao**, Northeastern University.

Global Dynamics of Real Discrete Dynamical Systems (Code: SS 30A), **M. R. S. Kulenović** and **O. Merino**, University of Rhode Island.

Harmonic Analysis and Partial Differential Equations (Code: SS 29A), **Donatella Danielli**, Purdue University, and **Irina Mitrea**, Temple University.

Homological Commutative Algebra (Code: SS 11A), **Sean Sather-Wagstaff**, Clemson University, and **Oana Veliche**, Northeastern University.

Hopf Algebras, Tensor Categories, and Homological Algebra (Code: SS 8A), **Cris Negron**, Massachusetts Institute of Technology, **Julia Plavnik**, Texas A&M, and **Sarah Witherspoon**, Texas A&M University.

Mathematical Perspectives in Quantum Information Theory (Code: SS 24A), **Aram Harrow**, Massachusetts Institute of Technology, and **Christopher King**, Northeastern University.

Mathematical Problems of Relativistic Physics: Classical and Quantum (Code: SS 37A), **Michael Kiessling** and **A. Shadi Tahvildar-Zadeh**, Rutgers University.

Modeling of Biological Processes (Code: SS 38A), **Simone Cassani** and **Sarah Olson**, Worcester Polytechnic Institute.

New Developments in Inverse Problems and Imaging (Code: SS 9A), **Ru-Yu Lai**, University of Minnesota, and **Ting Zhou**, Northeastern University.

Noncommutative Algebra and Representation Theory (Code: SS 22A), **Van C. Nguyen**, Hood College, and **Alex Martsinkovsky** and **Gordana Todorov**, Northeastern University.

Nonlinear Reaction-Diffusion Equations and Their Applications (Code: SS 31A), **Nsoki Mavinga** and **Quinn Morris**, Swarthmore College.

Nonlinear and Stochastic Partial Differential Equations and Applications (Code: SS 19A), **Nathan Glatt-Holtz** and **Vincent Martinez**, Tulane University, and **Cecilia Mondaini**, Texas A&M University.

Numerical Methods and Applications (Code: SS 16A), **Vera Babenko**, Ithaca College.

Optimization Under Uncertainty (Code: SS 40A), **Yingdong Lu** and **Mark S. Squillante**, IBM Research.

Polytopes and Discrete Geometry (Code: SS 6A), **Gabriel Cunningham**, University of Massachusetts, Boston, **Mark Mixer**, Wentworth Institute of Technology, and **Egon Schulte**, Northeastern University.

Regularity of PDEs on Rough Domains (Code: SS 14A), **Murat Akman**, University of Connecticut, and **Max Engelstein**, Massachusetts Institute of Technology.

Relations Between the History and Pedagogy of Mathematics (Code: SS 20A), **Amy Ackerberg-Hastings**, and **David L. Roberts**, Prince George's Community College.

Singularities of Spaces and Maps (Code: SS 4A), **Terence Gaffney** and **David Massey**, Northeastern University.

The Analysis of Dispersive Equations (Code: SS 39A), **Marius Beceanu**, University at Albany, and **Andrew Lawrie**, Massachusetts Institute of Technology.

The Gaussian Free Field and Random Geometry (Code: SS 12A), **Jian Ding**, University of Pennsylvania, and **Vadim Gorin**, Massachusetts Institute of Technology.

Topics in Qualitative Properties of Partial Differential Equations (Code: SS 27A), **Changfeng Gui**, University of Texas at San Antonio, **Changyou Wang**, Purdue University, and **Jiuyi Zhu**, Louisiana State University.

Topics in Toric Geometry (Code: SS 17A), **Ivan Martino**, Northeastern University, and **Emanuele Ventura**, Texas A&M University.

Topology of Biopolymers (Code: SS 18A), **Erica Flapan**, Pomona College, and **Helen Wong**, Carleton College.

Shanghai, People's Republic of China

Fudan University

June 11–14, 2018

Monday – Thursday

Meeting #1140

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: April 2018
Program first available on AMS website: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Yu-Hong Dai, Academy of Mathematics and System Sciences, *Title to be announced.*

Kenneth A. Ribet, University of California, Berkeley, *Title to be announced.*

Richard M. Schoen, University of California, Irvine, *Title to be announced.*

Sijue Wu, University of Michigan, *Title to be announced.*

Chenyang Xu, Peking University, *Title to be announced.*

Jiangong You, Nankai University, *Title to be announced.*

Newark, Delaware

University of Delaware

September 29–30, 2018

Saturday – Sunday

Meeting #1141

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June 2018
Program first available on AMS website: August 9, 2018
Issue of *Abstracts*: Volume 39, Issue 3

Deadlines

For organizers: February 28, 2018
For abstracts: July 31, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Leslie Greengard, New York University, *Title to be announced.*

Elisenda Grigsby, Boston College, *Title to be announced.*

Davesh Maulik, Massachusetts Institute of Technology, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Applied Algebraic Topology (Code: SS 2A), **Chad Giusti**, University of Delaware, and **Gregory Henselman**, Princeton University.

Convex Geometry and Functional Inequalities (Code: SS 3A), **Mokshay Madiman**, University of Delaware, **Elisabeth Werner**, Case Western Reserve University, and **Artem Zvavitch**, Kent State University.

Operator and Function Theory (Code: SS 4A), **Kelly Bickel**, Bucknell University, **Michael Hartz**, Washington University, St. Louis, **Constanze Liaw**, University of Delaware, and **Alan Sola**, Stockholm University.

Recent Advances in Nonlinear Schrödinger Equations (Code: SS 1A), **Alexander Pankov**, Morgan State University, **Junping Shi**, College of William and Mary, and **Jun Wang**, Jiangsu University.

Fayetteville, Arkansas

University of Arkansas

November 3–4, 2018

Saturday – Sunday

Meeting #1142

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: July 2018
Program first available on AMS website: August 16, 2018
Issue of *Abstracts*: Volume 39, Issue 3

Deadlines

For organizers: April 3, 2018
For abstracts: September 4, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Mihalis Dafermos, Princeton University, *Title to be announced.*

Jonathan Hauenstein, University of Notre Dame, *Title to be announced.*

Kathryn Mann, University of California Berkeley, *Title to be announced.*

Ann Arbor, Michigan

University of Michigan, Ann Arbor

October 20–21, 2018

Saturday – Sunday

Meeting #1143

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: July 2018
Program first available on AMS website: August 30, 2018
Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: March 20, 2018
For abstracts: August 21, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Elena Fuchs, University of Illinois Urbana-Champaign, *Title to be announced.*

Andrew Putman, University of Notre Dame, *Title to be announced.*

Charles Smart, University of Chicago, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

From Hyperelliptic to Superelliptic Curves (Code: SS 6A), **Tony Shaska**, Oakland University, **Nicola Tarasca**, Rutgers University, and **Yuri Zarhin**, Pennsylvania State University.

Geometry of Submanifolds, in Honor of Bang-Yen Chens 75th Birthday (Code: SS 1A), **Alfonso Carriazo**, University of Sevilla, **Ivko Dimitric**, Penn State Fayette, **Yun Myung Oh**, Andrews University, **Bogdan D. Suceava**, California State University, Fullerton, **Joeri Van der Veken**, University of Leuven, and **Luc Vrancken**, Universite de Valenciennes.

Interactions between Algebra, Machine Learning and Data Privacy (Code: SS 3A), **Jonathan Gryak**, University of Michigan, **Kelsey Horan**, CUNY Graduate Center, **Delaram Kahrobaei**, CUNY Graduate Center and New York University, **Kayvan Najarian** and **Reza Soroushmehr**, University of Michigan, and **Alexander Wood**, CUNY Graduate Center.

Probabilistic Methods in Combinatorics (Code: SS 7A), **Patrick Bennett** and **Andrzej Dudek**, Western Michigan University, and **David Galvin**, University of Notre Dame.

Random Matrix Theory Beyond Wigner and Wishart (Code: SS 2A), **Elizabeth Meckes** and **Mark Meckes**, Case Western Reserve University, and **Mark Rudelson**, University of Michigan.

Self-similarity and Long-range Dependence in Stochastic Processes (Code: SS 4A), **Takashi Owada**, Purdue University, **Yi Shen**, University of Waterloo, and **Yizao Wang**, University of Cincinnati.

Structured Homotopy Theory (Code: SS 5A), **Thomas Fiore**, University of Michigan, Dearborn, **Po Hu** and **Dan Isaksen**, Wayne State University, and **Igor Kriz**, University of Michigan.

San Francisco, California

San Francisco State University

October 27–28, 2018
Saturday – Sunday

Meeting #1144

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: July 2018
Program first available on AMS website: September 6, 2018
Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: March 27, 2018
For abstracts: August 28, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Srikanth B. Iyengar, University of Utah, *Title to be announced.*

Sarah Witherspoon, Texas A&M University, *Title to be announced.*

Abdul-Aziz Yakubu, Howard University, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Coupling in Probability and Related Fields (Code: SS 3A), **Sayan Banerjee**, University of North Carolina, Chapel Hill, and **Terry Soo**, University of Kansas.

Homological Aspects of Noncommutative Algebra and Geometry (Code: SS 2A), **Dan Rogalski**, University of California San Diego, **Sarah Witherspoon**, Texas A&M University, and **James Zhang**, University of Washington, Seattle.

Mathematical Biology with a focus on Modeling, Analysis, and Simulation (Code: SS 1A), **Jim Cushing**, The University of Arizona, **Saber Elaydi**, Trinity University, **Suzanne Sindi**, University of California, Merced, and **Abdul-Aziz Yakubu**, Howard University.

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019
Wednesday – Saturday

Meeting #1145

Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2018
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2018
For abstracts: To be announced

Auburn, Alabama

Auburn University

March 15–17, 2019
Friday – Sunday

Meeting #1146

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Honolulu, Hawaii

University of Hawaii at Manoa

March 22–24, 2019
Friday – Sunday

Meeting #1147
Central Section
Associate secretaries: Georgia Benkart and Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: May 15, 2018
For abstracts: January 22, 2019

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Barry Mazur, Harvard University, *Title to be announced* (Einstein Public Lecture in Mathematics).

Aaron Naber, Northwestern University, *Title to be announced*.

Deanna Needell, University of California, Los Angeles, *Title to be announced*.

Katherine Stange, University of Colorado, Boulder, *Title to be announced*.

Andrew Suk, University of Illinois at Chicago, *Title to be announced*.

Hartford, Connecticut

University of Connecticut Hartford (Hartford Regional Campus)

April 13–14, 2019
Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: September 13, 2018
For abstracts: To be announced

Quy Nhon City, Vietnam

Quy Nhon University

June 10–13, 2019
Monday – Thursday
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Binghamton, New York

Binghamton University

October 12–13, 2019
Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: March 12, 2019
For abstracts: To be announced

Gainesville, Florida

University of Florida

November 2–3, 2019
Saturday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Denver, Colorado

Colorado Convention Center

January 15–18, 2020
Wednesday – Saturday
Joint Mathematics Meetings, including the 126th Annual Meeting of the AMS, 103rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM)
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2019
Program first available on AMS website: November 1, 2019
Issue of *Abstracts*: To be announced

Deadlines
For organizers: April 1, 2019
For abstracts: To be announced

Washington, District of Columbia

Walter E. Washington Convention Center

January 6–9, 2021
Wednesday – Saturday
Joint Mathematics Meetings, including the 127th Annual Meeting of the AMS, 104th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: October 2020
Program first available on AMS website: November 1, 2020
Issue of *Abstracts*: To be announced

Deadlines
For organizers: April 1, 2020
For abstracts: To be announced

American Mathematical Society

Who Wants to Be a Mathematician

High School Students Compete for $10,000

Photos by Kate Awtrey, Atlanta Convention Photography

2018 Championship Contest

Students from the US, UK, and Canada

Saturday, January 13

1:00–2:30 pm

San Diego Convention Center 6C (Upper Level)

Mathemati-Con #JMM2018

The 2018 *Who Wants to Be a Mathematician* Championship is part of Mathemati–Con events at the meeting, open to the public on Saturday from 9:00 am to 4:00 pm, featuring the 2018 Mathematical Art Exhibition, James Tanton, Math Circles, Matt Parker, Math Wrangle, the Porter Lecture by Moon Duchin, and more.

Photo by Mihaela Bodlovic

Photo by Laura McHugh

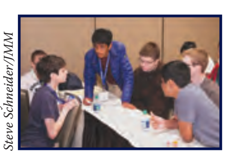
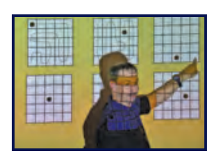


Photo by Steve Schneider/JMM



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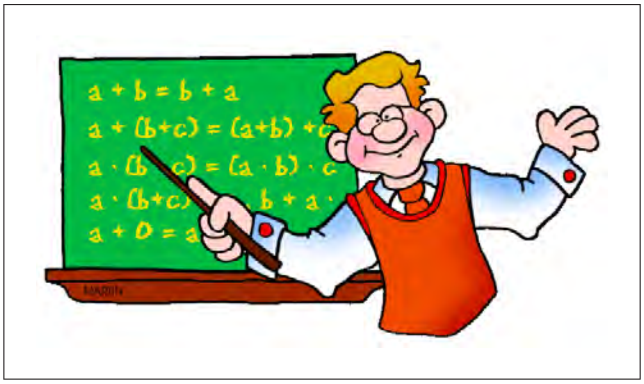
www.ams.org/wwtbam

THE BACK PAGE

The October Contest Winner Is...

R. B. Killgrove, who receives our book award.



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FIELD DAY.

The January 2018 Caption Contest:

What's the Caption?

Photo by Peter Mercer.

Submit your entry to `captions@ams.org` by January 25. The winning entry will be posted here in the April 2018 issue.

Various rational values for pi have been "proved" recently in peer-reviewed journals, according to a blog post by David Bailey `mathscholar.org/pi-and-the-collapse-of-peer-review`.

—Submitted by Nelson H. F. Beebe

Send in your favorite image of a math-related license plate. If *Notices* uses it, we'll send you an AMS frame. For more, see `https:tinyurl.com/y7jgsrp5`.

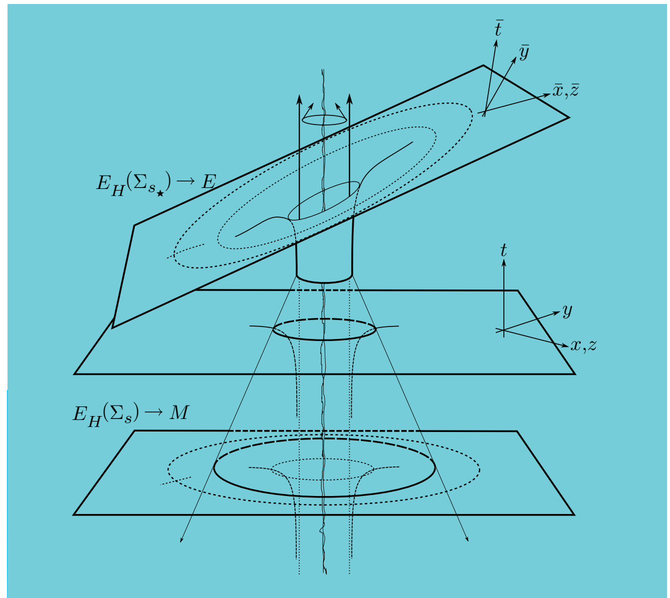
What crazy things happen to you? Readers are invited to submit original short amusing stories, math jokes, cartoons, and other material to: **`noti-backpage@ams.org`**.

IN THE NEXT ISSUE OF NOTICES

FEBRUARY 2018

In celebration of Black History Month, February Guest Editor Talithia Williams presents:

- Arlie O. Petters, "Belonging"
- Candice Price, Shelby Wilson, Erica Graham, and Raegan Higgins, "The 'Mathematically Gifted and Black' Website"
- David Goldberg and Phil Kutzko, "The Math Alliance and Its Roots in the African American Community"
- Talitha M. Washington "Behind Every Successful Woman, There Are a Few Good Men"
- Asamoah Nkwanta and Janet E. Barber, "Episodes in the Life of a Genius: J. Ernest Wilkins Jr."
- Johnny L. Houston, "Ten African American Pioneers and Mathematicians Who Inspired Me"
- Edray Herber Goins, "Three Questions: The Journey of One Black Mathematician"
- Vernon R. Morris and Talitha M. Washington, "The Role of Professional Societies in STEM Diversity"



Null geometry is counterintuitive in a number of ways. Hubert L. Bray and Henri P. Roesch's February feature article explores the recent **Proof of a Null Geometry Penrose Conjecture**.

American Mathematical Society
Distribution Center

35 Monticello Place,
Pawtucket, RI 02861 USA

A PREVIEW OF THE TITLES ON DISPLAY AT THE 2018 JOINT MATHEMATICS MEETINGS

An Illustrated Theory of Numbers

Martin H. Weissman, University of California, Santa Cruz

This comprehensive introduction to number theory, with complete proofs, worked examples, and exercises, reflects the most recent scholarship in mathematics and its history and includes historical notes that curate primary sources and secondary scholarship to trace the development of number theory within and outside the Western tradition.

2017; 323 pages; Hardcover; ISBN: 978-1-4704-3493-9; List US$69; AMS members US$55.20; Order code MBK/105

A Mathematical Gallery

Lisl Gaal

This playful mathematical tour, aided by Lisl Gaal's illustrations of familiar scenes and whimsical triggers for the imagination, is a book to read and revisit, gaining new insights each time.

2017; 64 pages; Softcover; ISBN: 978-1-4704-4159-3; List US$25; AMS members US$20; Order code MBK/111

A Conversational Introduction to Algebraic Number Theory

Arithmetic Beyond $\mathbb{Z}$

Paul Pollack, University of Georgia, Athens

Written in a conversational style, this introduction to algebraic number theory lays out the three classical "fundamental theorems": unique factorization of ideals, finiteness of the class number, and Dirichlet's unit theorem, while also frequently alluding to recent developments within the field.

Student Mathematical Library, Volume 84; 2017; 312 pages; Softcover; ISBN: 978-1-4704-3653-7; List US$52; AMS members US$41.60; Order code STML/84

From Groups to Geometry and Back

Vaughn Climenhaga, University of Houston *and* Anatole Katok, Pennsylvania State University, University Park

While exploring the connections between group theory and geometry, this book introduces some of the main ideas of transformation groups, algebraic topology, and geometric group theory.

Student Mathematical Library, Volume 81; 2017; 420 pages; Softcover; ISBN: 978-1-4704-3479-3; List US$58; All individuals US$46.40; Order code STML/81

A First Course in Sobolev Spaces

Second Edition

Giovanni Leoni, Carnegie Mellon University, Pittsburgh, PA

The second edition of this book about differentiation of functions focuses more on higher order derivatives and has the potential to be used as two separate textbooks, one for an advanced undergraduate course in functions of one variable and one for a graduate course on Sobolev functions.

Graduate Studies in Mathematics, Volume 181; 2017; 734 pages; Hardcover; ISBN: 978-1-4704-2921-8; List US$94; AMS members US$75.20; Order code GSM/181

Chaos on the Interval

Sylvie Ruette, Université Paris-Sud, Orsay, France

The aim of this book is to survey the relations between the various kinds of chaos and related notions for continuous interval maps from a topological point of view.

University Lecture Series, Volume 67; 2017; 215 pages; Softcover; ISBN: 978-1-4704-2956-0; List US$54; AMS members US$43.20; Order code ULECT/67

◆ =Textbook ▯ =Available in eBook format

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