

Book Reviews

Edited by David S. Watkins

Featured Review: The Princeton Companion to Applied Mathematics. Edited by Nicholas J. Higham. Princeton University Press, Princeton, NJ, 2015. xx+988 pp., hardcover. ISBN 978-0-6911-5039-0.

What is applied mathematics? How does it relate to pure mathematics, or should we simply say, to mathematics? With the appearance of the *Princeton Companion to Applied Mathematics*, we have two magnificent data points, 1000 pages each, to help us reflect on these questions.

The first thing one feels on looking at this volume is, quite simply, pleasure. Seven years ago *The Princeton Companion to Mathematics* was published to wide acclaim, and it was clear that a similar work on applied mathematics might be a good idea. Now it has appeared. The look and feel are the same, and this is highly satisfying. Figure 1 shows maps of the two volumes, which I will have more to say about in a moment. In each case the editors divided the collection into eight parts featuring pieces of differing lengths and flavors. The formatting and the typesetting are closely matched, and the new *Companion* is a perfect companion to the earlier one.

PCM was masterminded by Timothy Gowers of the University of Cambridge, and *PCAM* has been created by Nick Higham of the University of Manchester. These must be two of the most capable editors on earth. Anyone who knows Gowers and Higham will be aware of their combination of broad mathematical vision with phenomenal attention to detail. Higham, the Richardson Professor of Applied Mathematics at Manchester and a Fellow of the Royal Society, is celebrated not just for his research in numerical analysis but also for his outreach activities, including his *Handbook of Writing for the Mathematical Sciences*, the *MATLAB Guide* (coauthored with brother Des), and his blog. Like Gowers, he is a leader who cares deeply about his field. Princeton's appointment of him as editor was the perfect choice.

Of course, it takes a village. Like *PCM* before it, *PCAM* has a board of associate editors who helped shape the volume and contributed some of the articles: Mark Dennis, Paul Glendinning, Paul Martin, Fadil Santosa, and Jared Tanner. (I am pleased to note that five of the six editors are connected with England.) It has 165 authors, experts in their topics, many of them very eminent. A key person in the back office was Sam Clark of T&T Productions Ltd, who as project manager for both *PCM* and *PCAM* was involved in all the details and deserves much of the credit for making them such a comfortable pair.

So, what can you do with a book so big that its weight is measured in kilograms? With a million words of first-rate applied mathematics?

One possibility is to read it cover to cover. I more or less did that, but I doubt you will. (I did it as much to learn about myself as to learn about applied mathematics, which brings us to our first difference between mathematics and applied mathematics. I could not have read *PCM* cover to cover.)

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PCM (mathematics)		PCAM (applied mathematics)	
4 × 19.0	I. Introduction	I. Introduction	6 × 13.3
7 × 11.4	II. History	II. Concepts	36 × 1.6
99 × 1.6	III. Concepts	III. Equations	31 × 1.2
26 × 14.1	IV. Areas	IV. Areas	40 × 10.5
35 × 1.5	V. Problems	V. Modeling	21 × 6.9
96 × 1.0	VI. People	VI. Examples	18 × 2.8
14 × 9.1	VII. Applications	VII. Applications	25 × 4.6
7 × 8.6	VIII. Essays	VIII. Essays	15 × 4.4

Fig. 1 The eight parts of the two Princeton Companions, with shortened names. Annotations indicate numbers and average lengths of articles. For example, PCAM has 18 articles in Part VI, on Example Problems, and their average length is 2.8 pages.

Another approach is to dip in and out at whim, as Higham et al. encourage.

Despite the careful organization, the editors expect that many readers will flick through the book to find something interesting, start reading, and by following cross-references navigate the book in an unpredictable fashion. This approach is perfectly reasonable.

You bet it's reasonable! Your eye will be caught by Barbara Keyfitz on conservation laws, by Berry and Howls on divergent series, or by Jane Wang on insect flight. Maybe by Phil Holmes on dynamical systems, or Jack Dongarra on high-performance computing, or Andreas Griewank on automatic differentiation. The treasures go on and on.

I loved some of the accounts of things I hadn't known about. Ken Golden on the mathematics of sea ice, showing us how percolation theory applies to actual percolation—fascinating. Villani and Mouhot's article on kinetic theory—idiosyncratic and thoughtful, including a summary of 50 important papers in the field from 1912 to 2013. Donald Saari's beautifully simple explanation of why physicists believe the universe is full of dark matter. Doug Arnold's flight of a golf ball, a perfect example of how, by focusing on something small, we can see things that are big.

A third approach to this book is to use it for reference on smaller subjects or serious learning of bigger ones. I think the potential here is very great. All of us have areas we've touched upon but not immersed ourselves in properly, and some of these

pieces offer outstanding opportunities for taking that next step. For example, I was grabbed by David Tong's article on classical mechanics, whose clarity is illustrated by its opening lines.

Classical mechanics is an ambitious subject. Its purpose is to predict the future and reconstruct the past, to determine the history of every particle in the universe.

There are dozens of truly deep and expert survey articles in *PCAM*, such as Brian Davies on spectral theory, Stephen Wright on continuous optimization, Hairer and Lubich on the numerical solution of ODEs, David Griffiths on quantum mechanics, and Emily Shuckburgh on the dynamics of the Earth's ocean and atmosphere. Graduate students and established researchers will be profitably reading these articles for many years.

But *eight parts??* What's going on here?

Those of us of a certain age remember when the *Encyclopedia Britannica* raised eyebrows with its 15th edition in the 1970s. Instead of the traditional flat collection of articles, they brought out 28 volumes divided into the *Propedia*, the *Macropedia*, and the *Micropedia*. Was this controversial organizational principle a success?

If you take a look at the eight parts of *PCAM*, charted in Figure 1, the presence of some of them seems self-explanatory. Of course there is going to be an Introduction to Applied Mathematics, which here consists of six articles on foundational material. (Five are by Higham, with a generally numerical viewpoint, but this lack of diversity in the opening 55 pages is unrepresentative. Overall the book is very balanced, not at all dominated by numerics.) And following *PCM's* successful model, it is satisfying to find a final Part VIII of Final Perspectives. This assortment begins with opinion pieces by Gowers and Higham themselves and moves on to Ian Stewart, David Donoho and Victoria Stodden, David Bailey and Jonathan Borwein, Heather Mendick, David Acheson, Peter Turner, Gil Strang, Rachel Levy, Ya-xiang Yuan, Maria Esteban, Jim Crowley, and Alistair Fitt. One of my favorites is Stewart's zestful essay on "How to Write a General Interest Mathematics Book."

I have called mathematics the Cinderella science. It does all the hard work but never gets to go to the ball.

PCAM's organizational confusion lies in the middle 800 pages, Parts II to VII, which are devoted to Concepts, Equations, Areas, Modeling, Examples, and Applications. Though one can only admire the impulse to classify, I am afraid it is very difficult to keep these six headings straight. Why is Benford's Law an Equation and the traveling salesman problem an Example? Why does modern optics belong to Modeling and control theory to Areas of Applied Mathematics? It all feels rather arbitrary.

Naturally you get curious and wonder, were the middle six parts of the earlier *PCM* equally hard to keep straight? On inspection it turns out that no, they were not, because three of them had specially memorable themes. Part II was on history, a category that *PCAM* does not repeat. Part VI was on mathematicians, 96 one-page mini-biographies from Pythagoras to Bourbaki, which *PCAM* also does not repeat. (Too many of the best applied mathematicians had already been covered as mathematicians, I suspect, though Bourbaki, I hasten to add, was certainly not applied, quite apart from the question of existence.) And Part VII of the original *Companion* was also special, being devoted to applications ("The Influence of Mathematics"). Perplexingly, this is a category that *PCAM* *does* repeat, giving us effectively Applications of Applied Mathematics. (One muses about GNU's Not Unix and turtles all the way down.)

So the organization of *PCAM* is unconvincing, and Princeton could have maintained the *PCM* look and feel with five parts instead of eight, but to tell the truth, it doesn't matter. No *Britannica* reader worried much about the *Propedia* back in the 1970s, and nobody's going to lose sleep over *PCAM*'s structure today. The gold is in the individual pieces, not their organization.

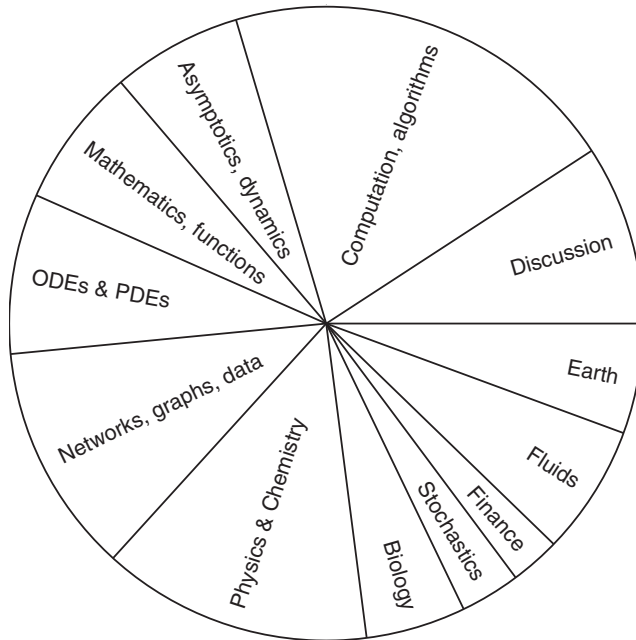


Fig. 2 Rough breakdown of the pages of *PCAM*.

Which brings us back to our opening question. What is applied mathematics? What view of the field does *PCAM* convey? To explore these matters it may be helpful to consider Figure 2, which outlines roughly where *PCAM* spends its pages.

One thing you notice is that there is a lot of physics and fluid mechanics here. These are the oldest, best established parts of applied mathematics, as important as ever, and their strength is powerfully displayed in articles on magnetohydrodynamics, quantum mechanics, optics, gravitation, and many other topics. Newer fields like finance, network theory, and biology are also well represented, although one can't help noting that whereas an article on a physics topic like kinetic theory or solid mechanics, say, will most likely be found in Areas of Applied Mathematics, a biological topic like physiology or biomechanics is more likely to appear in Modeling. It would seem that applied mathematics has terrain it has conquered and terrain it is still exploring. A century from now, will the boundaries have shifted?

It is striking that applied mathematics as displayed in *PCAM* does not define itself by its relationship to (pure?) mathematics, not at all. The book robustly stands on its own, and the proverbial Martian, if he stepped out of his flying saucer and read this volume of mathematics, would not suspect that earthlings knew any other kind.

PCAM shows us that applied mathematics is *vast* and it is *confident*. We see here a discipline engaged in every corner of the human enterprise, from cosmology to the

spread of infectious diseases, from pattern formation to aircraft design, from financial portfolio optimization to the ranking of movie preferences. As Strang writes, “Our subject is extremely large!”

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