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Fukaya categories and Picard-Lefschetz theory, by Paul Seidel, European Mathematical Society (EMS), Zürich, 2008, vii+326 pp., €46, ISBN 978-3-03719-063-0

One route to investigating the differential topology of real manifolds is through Morse theory. A smooth manifold M is decomposed into the level-sets $f^{-1}(x)$ of a smooth real-valued function $f : M \to \mathbb{R}$, and the global topology of M emerges from the descriptions of how these level-sets change. One can understand the whole homotopy type of M from this point of view [Mil69], or one can pass more quickly to algebra by defining the Morse complex, which has generators the critical points $\{x \in M \,|\, df(x) = 0\}$ of f and a differential counting downward gradient flow-lines $\gamma : \mathbb{R} \to M$ which solve $\gamma'(t) = -\nabla f(\gamma(t))$. This Morse complex computes the usual singular cohomology, and one immediate consequence is a nontrivial lower bound on the number of critical points of a generic smooth function on a closed manifold. For complex algebraic varieties, Picard-Lefschetz theory [La81] is a complexification of this picture, which studies a projective manifold through the level-sets of a *holomorphic* function $f : X \dashrightarrow \mathbb{C}$ (since holomorphic functions on compact complex manifolds are constant, this will be defined only away from some subvariety). The locus of critical values being finite in $\mathbb{C}$ and having connected complement, the inverse images of regular points $f^{-1}(x)$ will typically all be diffeomorphic; the global topology of X now emerges through the *monodromy*, which describes how these fibres are twisted globally in the family. Picard-Lefschetz theory provides a kind of dimensional-induction machine, in which one studies varieties through their hyperplane sections which, being of lower dimension, are presumed more tractable.

In a short but influential note [Arn95], V. I. Arnol'd pointed out that the monodromy transformations of Picard-Lefschetz theory have a basic connection to *symplectic* topology. The heart of this connection is straightforward to describe. Morse functions have isolated singularities modeled locally on nondegenerate quadratic forms. Over $\mathbb{C}$, the unique local model is the map $\pi : (z_1, \dots, z_n) \mapsto \sum_j z_j^2$. The general fibre $\pi^{-1}(1) \subset \mathbb{C}^n$ is symplectically diffeomorphic to the cotangent bundle of a sphere T^*S^{n-1}, which carries the canonical symplectic structure of classical mechanics, and Arnol'd's basic observation is that the monodromy of the family of varieties $\{\pi^{-1}(t) \,|\, t \in S^1\}$ is a *symplectic* diffeomorphism of T^*S^{n-1}. In the lowest nontrivial dimension $n = 2$, the general fibre is an annulus, the monodromy map is a Dehn twist in the obvious waist curve $\{(z_1, z_2) \in \mathbb{R}^2 \,|\, z_1^2 + z_2^2 = 1\}$, and the symplectic property asserts that this preserves area. In higher dimensions there is an analogous Dehn twist, which acts antipodally on the corresponding real sphere (i.e., the zero-section of T^*S^{n-1}) and is compactly supported in a neighbourhood of that sphere. Even in the local model, the symplectic structure keeps track of information not visible in classical differential topology: when $n = 3$, the Dehn twist on an affine quadric surface is of infinite order as a compactly supported symplectomorphism, but its square is differentiably isotopic to the identity.

It has become clear over the last decade that this change in perspective, from algebraic to symplectic topology in the context of Picard-Lefschetz theory, is actually rather profound, providing a key inroad into the symplectic natures of algebraic

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varieties. Many of the modern tools of symplectic topology arise from the pseudo-holomorphic curve methods pioneered by Gromov [Gr85] and Floer [Flo88]. The book under review is in some sense the fruition of the seed that Arnol'd's observation planted; it sets out a framework for the computation, by dimensional induction of the sort alluded to above, of a certain class of holomorphic curve invariants for affine algebraic varieties. The assembling of Seidel's beautiful machine involves large tracts of mathematics, and in particular of homological algebra, which are much more recent entrants into the symplectic topological arena. To set this wider stage, it may be helpful to give a very brief reminder of the basics of Lagrangian Floer cohomology.

A symplectic manifold (M^{2n}, ω) is a real manifold equipped with a closed non-degenerate 2-form. The prototypical examples are cotangent bundles T^*Q, smooth projective or affine varieties, or more generally Kähler or Stein manifolds. A *Lagrangian submanifold* is a half-dimensional submanifold $L^n \subset M$ for which the restriction of the symplectic form vanishes identically, $\omega|_L \equiv 0$; zero-sections of cotangent bundles and real loci in algebraic varieties provide examples. A central feature of symplectic topology is the tendency for Lagrangian submanifolds to intersect "more" than classical topology suggests. For instance, the Arnol'd conjecture (now a theorem of Fukaya-Ono and Liu-Tian) says that symplectic diffeomorphisms arising as the time-one maps of Hamiltonian flows have as many fixed points as the sum of the Betti numbers of the ambient manifold, rather than just the alternating sum, the bound provided by the classical Lefschetz fixed point theorem. These fixed points can be viewed as intersections between the Lagrangian diagonal $\Delta \subset (M \times M, -\omega \oplus \omega)$ and the Lagrangian graph of the given symplectic diffeomorphism. One of the central questions in modern symplectic topology is the description of (constraints on) the Lagrangian submanifolds of a given M. In general, this seems ferociously hard: we are far from knowing which smooth manifolds admit Lagrangian embeddings in projective space, or the classification up to Hamiltonian isotopy of Lagrangian tori $T^n \subset \mathbb{C}^n$, or more or less anything else. Nonetheless, there is steady progress, largely through various versions of Floer theory.

In its simplest form, Lagrangian Floer cohomology takes a pair of Lagrangian submanifolds $(L_0, L_1) \subset M$ of a symplectic manifold M, and associates to that pair a vector space $HF(L_0, L_1)$ invariant under Hamiltonian deformations of either L_i. This Floer homology is obtained from a chain complex, where the chain groups are generated by the (presumed transverse) intersections $L_0 \pitchfork L_1$, and where the differential counts solutions to a Cauchy-Riemann type equation

$$u : \mathbb{R} \times [0,1] \to M; \quad u(\mathbb{R} \times \{i\}) \subset L_i; \quad \partial_s u + J \partial_t u = 0, \tag{1}$$

which are asymptotic at the ends of the strip to prescribed intersection points $x^{\pm} \in L_0 \cap L_1$. Here J is an auxiliary choice of (time-dependent) almost complex structure on the strip chosen compatibly with the symplectic form. Formally, one can view this as a Morse-type chain complex for an action functional on the infinite-dimensional space of paths from L_0 to L_1; practically, the essential feature of the theory is the *ellipticity* of the Cauchy-Riemann equation, which makes counting the relevant flow-lines feasible in at least a range of situations. If one takes (L_0, L_1) to comprise the zero-section of a cotangent bundle $Q \subset T^*Q$ and its Hamiltonian image under the flow generated by a Morse function, one essentially recovers Morse theory on Q, and indeed the first application of Floer homology was to the Arnol'd conjecture by reduction to the Morse inequalities for numbers of critical points of smooth functions, where we began.

Lagrangian Floer homology has had numerous successes in a range of problems in symplectic topology. There is a large literature addressing partial computations and special situations. The Heegaard Floer homology of Ozsváth and Szabó [OS04] is a version of Lagrangian Floer homology (for certain Lagrangian tori in the symmetric product of a Riemann surface) and is expected to contain equivalent information to Seiberg-Witten theory; it has found remarkable applications in knot theory and 3-manifold theory, leading amongst other breakthroughs to an algorithm for computing the genus of a knot [MOS]. All that is in spite of the fact that to set Floer theory on a solid footing is itself a substantial task—indeed, a gargantuan task if the setting is made sufficiently general, as attested by the recent 1000-page opus of Fukaya, Oh, Ohta, and Ono [FO3], the aim of which is largely to understand when the above heuristics can be filled out even to *define* the Floer homology groups. The essential issue is that perturbations of the geometric or auxiliary data (the almost complex structure, inhomogeneous terms in the Cauchy-Riemann equation, Hamiltonian isotopies of the Lagrangians) are not collectively sufficient, in general, to achieve transversality for the moduli spaces of solutions to equation (1).

The gradual resolution of these issues has involved deep and delicate new ideas in analysis. However, Seidel sensibly confines himself to situations where most of the bugbears are absent: he focuses on *exact* symplectic manifolds and their *exact* Lagrangian submanifolds. Namely, he assumes the symplectic form $\omega = d\theta$ is globally exact, and if $L \subset M$ is a Lagrangian under consideration, moreover $\theta|_L = df$. Cotangent bundles and affine varieties belong to this class; crucially, however, *closed* symplectic manifolds can never be exact. The exactness means, first, that M contains no holomorphic spheres, and second that L bounds no holomorphic discs. These trivial consequences of Stokes' theorem sweep away the foundational headaches that have tormented the general development of the subject for a decade and which Fukaya-Oh-Ohta-Ono's Kuranishi spaces and the ongoing polyfold program of Hofer and his collaborators will eventually defeat. Nonetheless, Floer theory in the exact case is far from trivial: indeed, stripped of technical problems in its formulation, it becomes easier to appreciate its elegance and its power.

Much of this power comes from additional structure. The first versions of Floer theory, for action functionals on spaces of connexions arising in gauge theory, came with long exact sequences (Floer's eponymous exact triangle) relating the groups associated to 3-manifolds related by surgeries; the power of Heegaard Floer homology derives in good measure from its computability, which in turn hinges on the groups being bound together by exact sequences from surgeries and by functorially defined maps from cobordisms. Floer homology of the diagonal $HF(\Delta, \Delta) \cong QH^*(M)$ recovers quantum cohomology with its famous product structure. When viewed from the Lagrangian submanifold side, that product counts holomorphic triangles, and it was introduced by Donaldson who pointed out that one could make a category whose objects were Lagrangian submanifolds of M and whose morphism groups were Floer homology groups. The composition in the category is then provided by the holomorphic triangle product. The landscape was changed more drastically by the appreciation, partly derived from Kontsevich's *homological mirror symmetry* conjecture, that one should "algebraise" the situation much further still. Homological mirror symmetry is a huge undertaking that relates algebraic and symplectic geometry in a variety of settings. The important point for our current purposes is that the symplectic geometry which enters is precisely the geometry of Lagrangian submanifolds and their Floer homology groups. These enter not in isolation but gathered together into a much richer algebraic structure, the *Fukaya category*. To first approximation, the objects of the Fukaya category $\mathcal{F}(M)$ are the Lagrangian submanifolds of M, and the morphisms in the category are the chain complexes $CF(L_0, L_1)$ which underlie the Floer homology groups. Donaldson's category is recovered on passing to homology (in this context, called passing to the derived category). Not only are the morphism spaces in $\mathcal{F}(M)$ chain complexes rather than groups, but there are hierarchies of additional operations, in the form of chain-level products

$$\mu^k : CF(L_{k-1}, L_k) \otimes \cdots \otimes CF(L_0, L_1) \to CF(L_0, L_k)[2-k]$$

of degree $2-k$, for $k \geq 1$, for all collections $(L_0, \ldots, L_k)$ of objects. Here μ^1 is the Floer differential, counting holomorphic bigons $\mathbb{R} \times [0,1]$; μ^2 counts holomorphic triangles and serves to make the group $HF(L, L)$ into a unital ring; the higher μ^k, counting holomorphic $(k+1)$-gons, carry along the information of Massey-type products in Floer homology. Again in analogy with classical topology, just as we learn early on that homology groups are better invariants than Euler characteristics, so the algebra of differential forms or the singular cochain complex on a manifold is better still. The Fukaya category is the symplectic generalisation of this more inclusive package; for the cotangent bundle T^*Q, the Fukaya category is essentially equivalent data to the rational homotopy type of Q, seen through Morse theory.

Fukaya categories have a rather fearsome reputation: indeed, many practitioners of symplectic topology probably remain unconvinced that they exist, or at least shy away from them with nervousness. The very definition relies on combining triangulated A_∞-categories, not the most transparent corner of homological algebra, with the output of a theory based on elliptic partial differential equations and whose examples are drawn from diverse parts of algebraic geometry. The operations $\{\mu^k\}_{k>2}$ are *not* chain maps and do not descend to Floer homology; moreover, they are defined by counting pseudo-holomorphic polygons, and the counts depend critically on the choice of auxiliary data (including the almost complex structure). The Fukaya category is somewhat like an infinite rank matrix depending on many choices, for which none of the matrix coefficients is actually invariant: rather, the entire structure is invariant up to a suitable notion of quasi-equivalence. This is a much bleaker state of affairs than one encounters in traditional Gromov-Witten theory or quantum cohomology, where one counts only closed holomorphic curves and the individual counts are well defined; the added complication here is that moduli spaces of holomorphic polygons have *codimension one* boundary strata in their compactifications, and hence do not carry fundamental cycles, in contrast to the Deligne-Mumford moduli spaces of stable closed curves. (A part of this is that the Fukaya category is not strictly a category at all, but an A_∞-category.) All that notwithstanding, there are many simple ideas buried beneath the largely technical obscuring facades.

The first point to make is that restricting attention to a suitable class of non-compact symplectic manifolds has philosophical as well as technical advantages. To date, there is no way of telling whether a projective variety contains any Lagrangian submanifolds whose Floer cohomology is well defined whatsoever. By contrast, at least if one allows noncompact Lagrangian submanifolds whose behaviour at infinity is carefully constrained, such objects always exist on affine varieties. Indeed, if Y is affine and $w : Y \to \mathbb{C}$ is a Lefschetz pencil, then one can take the *Lefschetz thimble* of a critical point. In the local model $\pi : \mathbb{C}^n \to \mathbb{C}$ of the Morse singularity described above, this is just the real locus $\mathbb{R}^n \subset \mathbb{C}^n$, which can more invariantly be viewed as the set of points which emanate from the critical point at the origin under the natural symplectic flow defined by parallel transport over the positive real axis $\pi(\mathbb{R}^n) = \mathbb{R}_{\geq 0} \subset \mathbb{C}$. Any affine variety admits Lefschetz fibrations and so contains Lefschetz thimbles; although they reach infinity, the maximum principle applied to their images in $\mathbb{C}$ quickly shows that their pseudo-holomorphic curve theory is no more problematic than for closed exact Lagrangians. Thus, one has somewhere to start, and the Fukaya categories that Seidel considers admit these particular noncompact objects from the outset: indeed, they play a central role. (For cotangent bundles, we have shifted attention away from the zero-section to include the cotangent *fibre*.) One compelling contribution of the present book is a *split-generation* result, which says that the Fukaya category of closed Lagrangian submanifolds inside Y is in a sense completely determined by the subcategory obtained from a finite "distinguished basis" of Lefschetz thimbles, namely, those associated to a collection of paths in the base $\mathbb{C}$ of the Lefschetz fibration which are disjoint, one emanating from each critical value. The split-generation result is formally analogous to Beilinson's classical resolution of the diagonal for studying coherent sheaves on projective spaces; a more fanciful, but perhaps helpful, perspective would be that it provides a kind of "geometric Fourier theory", in which unknown Lagrangians are abstractly "resolved" in terms of a familiar basis.

Donaldson pointed out that Lagrangian "matching spheres" $S^n \cong L \subset Y$ can sometimes be swept out by families of Lagrangian spheres in the fibres $\{S^{n-1} \cong L_t \subset w^{-1}(t)\}_{t\in\gamma}$ over paths $\gamma \subset \mathbb{C}$ between critical values of the Lefschetz fibration w. The S^{n-1}'s degenerate to critical points of w over the ends of γ: a matching sphere is given by gluing together two Lefschetz thimbles along their boundaries. This construction "relativises" the idea of passing to a hyperplane section, so it applies in a suitable sense to Lagrangian spheres as well as the affine varieties containing them. Starting from this, Seidel shows the Fukaya category of an affine variety is "combinatorially computable", in an appropriate and rigorous sense, from the data of a collection of Lefschetz pencils: one on the total space, one on the fibre of the first pencil, one on the fibre of the next, and so forth. This dimensional-induction machine, which eventually reduces one to studying holomorphic curve theory in a punctured Riemann surface where one can appeal to the Riemann mapping theorem, is the culmination of the book. We should emphasise that, although in exact symplectic manifolds $HF(L, L) \cong H^*(L)$ is a topological object, there is in general no prediction for the rank of the Floer cohomology $HF(L_0, L_1)$ when $L_0 \neq L_1$, and its definition involves solutions to an elliptic partial differential equation that one has no expectation of writing down explicitly. To assemble even a partial or theoretical algorithm for extracting detailed information on these groups is qualitatively as well as quantitatively striking. We should also mention that one can hope to approach the symplectic topology of a closed variety X indirectly by virtue of the theory in $Y = X\backslash\Sigma_0$ and ideas from algebraic deformation theory [Sei02]. This strategy has been carried out successfully in two nontrivial cases—for the quartic surface [Sei03] and the genus two curve [Sei08].

The split-generation result for Fukaya categories of Lefschetz fibrations is the culmination of many other developments. At the heart of these is a basic relationship between geometry and algebra: *Dehn twists act via algebraic twists.* Recall that the local monodromy of a Lefschetz fibration is a Dehn twist in a Lagrangian sphere $S^{n-1} \cong L \subset M = w^{-1}(t) \subset Y$. There is also an *algebraic twist functor*, which is the cone on the canonical evaluation

$$\mathrm{Hom}_{\mathcal{F}(M)}(L, \bullet) \otimes L \xrightarrow{ev} \bullet. \tag{2}$$

The algebraic twist can be defined for any L; the theorem asserts that if L is represented by a Lagrangian sphere, then the Dehn twist τ_L and algebraic twist T_L act in the same way on the Fukaya category. This equivalence between algebraic and geometric twists was conjectured by Kontsevich and is proved here for the first time. One sees at once, in the left-hand side of (2), that we have allowed objects of the shape $V \otimes L$ for chain complexes V. This is one part of the long algebraisation process, involving enlarging the Fukaya category to include not just geometric objects (Lagrangian submanifolds) but things built out of those in ways that make homological algebra methods relevant and applicable (formal chain complexes of Lagrangian submanifolds, formal summands of such complexes). Although it may be counterintuitive, the power of the machinery is only realised because of this enlargement: it perhaps entails a loss of information, but the information retained is rendered more tractable.

A brief description of an application shows how the algebraic structure enters in practice. This is an unpublished result of Seidel himself and is an application of homological mirror symmetry for projective space. The mirror of $\mathbb{CP}^n$ is a certain Lefschetz fibration $w : (\mathbb{C}^*)^n \to \mathbb{C}$ (see [HV00]), and mirror symmetry asserts the equivalence between the bounded derived category of coherent sheaves $D^b(\mathbb{P}^n)$ and a Fukaya category generated by Lefschetz thimbles $D^\pi \mathcal{F}((\mathbb{C}^*)^n)$ as introduced and studied in the book under review. Suppose $L \subset (\mathbb{C}^*)^n$ is a closed connected exact Lagrangian submanifold which, in addition, carries a spin structure and has vanishing Maslov class. It defines an object of $\mathcal{F}((\mathbb{C}^*)^n)$ and hence a complex $\mathcal{E}_L \to \mathbb{P}^n$ of coherent sheaves. The compactness of L implies that this complex of sheaves is invariant under the Serre functor of the category, which is given on $\mathbb{P}^n$ by tensoring with the ample anticanonical bundle. Only sheaves with zero-dimensional support have this property: starting from here, one argues that up to a shift in grading the complex $\mathcal{E}_L$ is actually quasi-isomorphic to the skyscraper sheaf of a point, hence by a well-known computation has $\mathrm{Ext}^*(\mathcal{E}_L, \mathcal{E}_L) \cong \Lambda(\mathbb{C}^n)$. These Ext-groups correspond, under mirror symmetry, to the Floer homology groups $HF(L, L)$ of our original Lagrangian submanifold, which in the exact setting reproduce ordinary cohomology; so one infers that L has the same complex cohomology groups as the torus. This is a partial but highly nontrivial result towards Arnol'd's "nearby Lagrangian submanifold" problem, which would predict that L is Hamiltonian isotopic to $T^n \subset T^*T^n = (\mathbb{C}^*)^n$. This sketch, however superficial (and errors introduced by the reviewer notwithstanding), should immediately bring home the central tenet of the philosophy, namely that a single unknown Lagrangian might sometimes most profitably be studied by first obtaining a description of the entire derived Fukaya category. Away from mirror symmetry, one can use algebraic approximation and the Nash-Tognoli theorem to bring similar techniques to bear on Lagrangian submanifolds of arbitrary cotangent bundles, whence to questions arguably of interest in dynamics.

The book is in three parts. The first is a comprehensive albeit condensed introduction to A_∞-categories, twisted complexes, and idempotent completion. The second part, after a quick review of classical Lagrangian Floer theory (material which does not yet appear in any textbook), concerns the basic construction of the Fukaya category and its fundamental invariance properties. The third part of the book proves the relation between algebraic and geometric twists, derives the split-generation theorem, and builds the dimensional-induction machine. The book ends with one application: the Fukaya category of a punctured Riemann surface of genus at least two is proved to not be formal (not equivalent to a category with vanishing higher-order products). This showcases some of the algebra in action and has a deeper significance in view of the role played by surfaces in low-dimensional topology. The global structure of the book—first algebra generalities, with symplectic geometry making its appearance in the second part, and Lefschetz fibrations and Dehn twists defered to the third—hugely simplifies the task of teasing out the logical structure of any given argument and the set of prerequisites on which a particular piece of machinery rests. The exposition, whilst terse, is exemplary in precision.

Seidel has gone to great effort to lay out the foundations of Fukaya categories, in the simplest nontrivial setting, in a complete and accessible fashion. He has accomplished the remarkable and generous feat of variously developing, clarifying, and writing the basic theory in a way which gives a safe point of entry for anyone hoping to learn or deploy these methods. At least for geometers it deserves to become the standard (re)source for the material it covers. We should end by pointing out that the Fukaya category is only one of several algebraic frameworks which emerge from the Gromov-Floer theory of pseudo-holomorphic curves, for instance, there are the differential graded algebras of symplectic field theory and the integrable systems which lurk mysteriously behind Gromov-Witten theory. How these various strands will eventually tie together is unclear, and the topic of much current enquiry.

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