to δ-systems as "quasi-disjoint families". But this is hardly serious and is due simply to the fact that the authors are topologists.

As a set theorist, this reviewer has naturally required the assistance of a topologist during the preparation of this review and would here like to thank Frank Tall for some helpful discussions. All opinions expressed here, however, are due to the reviewer.

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BULLETIN (New Series) OF THE
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Linear and combinatorial optimization in ordered algebraic structures, by U. Zimmermann, Annals of Discrete Mathematics, vol. 10, North-Holland Publishing Company, Amsterdam, 1981, x + 380 pp., $61.00, Dfl. 125.00. ISBN 0-4448-6153-X

Many optimization problems arise in connection with systems which incorporate discrete structures for which the mathematics is combinatorial rather than continuous: one thinks of sequencing, scheduling and flow-problems and of the great variety of questions which can be reformulated as path-finding, circuit-finding or subgraph-finding problems on an abstract graph. To match the growing interest in such problems arising from, for example, operations research and systems theory, the past thirty years have witnessed a vigorous growth in the theory and practice of combinatorial optimization.

A related, but perhaps less well-known, development has been in the application of ordered algebraic structures to optimization problems. This application is made relevant by the fact that many optimization questions depend essentially on the presence of two features: an algebraic language within which a system can be modelled and an algorithm articulated; and an ordering among the elements which enables a significance to be given to the concept of minimization or maximization. A familiar example here is a well-known method of resolving degeneracy in linear programming which depends upon the fact that the simplex algorithm may be extended to linear programs in which values are taken in a certain ordered ring.

By adopting this algebraic point of view we can make useful reformulations: certain bottleneck problems become algebraic linear programs; certain machine-scheduling problems reduce to finding eigenvectors and eigenvalues of a matrix over a semiring; certain path-finding problems reduce to the solution of linear equations over an ordered structure.

Many combinatorial optimization problems assume, under such reformulation, the appearance of problems of linear algebra over an ordered system of scalars. Hence we may look to the highly-developed classical theory of linear algebra over the real field to give us hints as to how we might approach these problems, or, if appropriate adaptations of classical techniques cannot be found, we have a well-defined research program to elucidate the theory of linear algebra over such ordered structures, and to see how far the algorithms and duality principles, familiar to us from linear and combinatorial optimization over the real field, extend to more general structures.

These questions have stimulated a good deal of research over the last twenty-five years. From a few isolated publications by one or two researchers in the late 1950s, the subject has matured into an identifiable branch of applicable mathematics with an international following.

The author has made a comprehensive survey of this work, to which he himself has notably contributed. His book is divided into two sections. In the first, a systematic theory of ordered algebraic structures is presented; in the second, the subject of linear algebraic optimization is explored. The topics discussed are generally, though not exclusively, of one or two kinds: either they relate to the properties of matrices over such of the algebraic structures as are rich enough to permit matrix multiplication, or they analyse the extent to which analogues of familiar linear and combinatorial optimization problems may be formulated and algorithmically solved for general ordered algebraic structures.

Because it so comprehensively reviews a literature which is widely scattered throughout a great variety of journal articles, this book will be a valuable addition to the library of any researcher seriously interested in this field.

R. A. CUNINGHAME-GREEN

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Pseudo-differential operators, by Hitoshi Kumano-go, The MIT Press, Cambridge, Massachusetts, 1982, xviii + 455 pp., $60.00. ISBN 0-2621-1080-6.

Introduction to the Fourier transform and pseudo-differential operators, by Bent E. Petersen, Pitman Advanced Publishing Program, Boston, Massachusetts, 1983, xi + 356 pp., $48.00. ISBN 0-2730-8600-6.

Functions of a finite set of selfadjoint operators, commuting with each other, can be defined through spectral theory. Pseudo-differential operators come into action when the need to represent functions of noncommuting operators arises. More specifically, let us consider the Hilbert space $L^2(\mathbf{R}^n)$, and, for