

The monograph is intended as an introduction to the theory of minimal models. Anyone who wishes to learn about the theory will find this book a very helpful and enlightening one. There are plenty of examples, illustrations, diagrams and exercises. The material is developed with patience and clarity. Efforts are made to avoid generalities and technicalities that may distract the reader or obscure the main theme. The theory and its power are elegantly presented. This is an excellent monograph.

It is pointed out that this is a revised and corrected version of a set of informal notes from a summer course taught by the authors together with Eric Friedlander in the summer of 1972. Regarding the origin of these notes, there is an acknowledgement to Dennis Sullivan.

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*Real variable methods in Fourier analysis*, by M. de Guzmán, Mathematics Studies, no. 46, Notas de Matemática, North-Holland Publishing Company, Amsterdam, 1981, xiii + 392 pp., \$44.00. ISBN 0-4448-6124-6

*Interpolation of operators and singular integrals, an introduction to harmonic analysis*, by Cora Sadosky, Pure and Applied Mathematics, no. 53, Marcel Dekker, New York, 1979, xi + 375 pp., \$35.00. ISBN 0-8247-6883-3

In 1948 Anthony Zygmund went to Buenos Aires at the invitation of Gonzales Dominquez. This event has had a number of remarkable consequences, the first of which was Zygmund's meeting A. P. Calderón, and Misha Cotlar. The second, and concomitant consequence was the flowering of classical hard analysis and the problems associated with the Polish school of mathematics in a number of Spanish speaking countries. The two books under review are some of the most recent blooms.

Both Miguel de Guzmán and Cora Sadosky were students of Professor Zygmund in Chicago in the middle sixties. de Guzmán is now the center of a group of young mathematicians working in a variety of hard classical problems in differentiation theory at the Universidad Complutense de Madrid. Sadosky was at universities in Argentina, Uruguay, and Venezuela working on various problems of weights and singular integrals, and is now at Howard University.

A colleague walked into our office, and saw de Guzmán's book on a desk, and made the remark "Another book about those damn little rectangles! Why do people do such things?" This attitude is understandable because many of the theorems about little rectangles are quite messy; it is unfortunate since understanding the geometric interactions of the little rectangles is at the heart of some of the most important results in real analysis. Just as the Vitali covering lemma is at the heart of the proof of Lebesgue's theorem on the differentiation of integrals, so too, many more recondite covering lemmas and

differentiation bases lie at the heart of more visible (and famous) theorems. An earlier work [3] focused on the “damn little rectangles”. This one mentions those rascals but focuses on some of the new methods that have been developed with that knowledge and contains some of the more famous modern results obtained using these tools by Córdoba, both Feffermans, Stein, Wainger and many others. There is no better introduction to the recent work in “hard” Fourier analysis.

It is largely because of the work of de Guzmán that the current generation of Fourier analysts is aware of some of the mysteries of the Perron tree, the Besicovitch set, and the Nikodym set. The earlier work [3] contains what is unquestionably the clearest, best organized, best written exposition of this material—and many other topics in this area that were studied by the Polish school in the twenties and thirties but had faded from currency. About one third of the material in this new book originated in [3]. Some of this material is just duplicated (with an occasional *gaffe*—Theorem 6.6.3 has no conclusion) but more frequently it is amplified to include whatever new results are appropriate. The book is not intended to be an encyclopedia of the subject, but a guide to a family of modern techniques showing both their origins and some of their consequences.

But what techniques? The following is a brief description. First, the proof of the pointwise convergence of a sequence of operators by proving the finiteness of an appropriate maximal operator. Second, the control of a maximal operator by the proof of a weak type inequality. Third, a survey of the ways the Hardy-Littlewood maximal operator can be used to control other operators. Fourth, the application of the Fourier transform to differentiation theory. Fifth, and finally, the application of differentiation theory to the theory of multipliers (including a lovely exposition of C. Fefferman’s Theorem on the disk being a multiplier for  $L^2$  only).

Another problem in hard analysis that was much studied by the Polish school of mathematics was the development of what should be called real variable techniques for the study of the conjugate function. The very famous lemma of Calderón and Zygmund [1] was the key to moving from a one-dimensional theory to an  $n$ -dimensional theory, and the theory of singular integral operators (“ $n$ -dimensional generalizations of the conjugate function”) has become one of the most powerful tools in a number of quite disparate fields—from partial differential equations to representation theory.

There are a number of excellent introductions to singular integrals. For example Zygmund’s Paris notes [7] or Neri’s excellent write up of Calderón’s and Zygmund’s courses at Chicago on the subject [4]. The basic material on complex interpolation is well presented in the book of Stein and Weiss [5]. There is even an excellent book at the introductory level, for a student who wants to learn the basic measure theory and other analytical tools necessary for the above topics (Wheeden and Zygmund [6]). All of these books presume a certain taste and background in hard analysis. For the student who is more comfortable with Banach algebras than (s)he is with spherical harmonics, Professor Sadosky has written a friendly guide to the subject. The background material is carefully presented, and the proofs are given in great detail.

The book begins with basic facts about Banach spaces, convolution structures, approximate identities and then carefully prepares three of the most useful tools of modern analysis—the Fourier transform, harmonic functions, and the interpolation of operators. The penultimate chapter is an introduction to the maximal function and an application of these ideas to ergodic theory (in the style of Cotlar's famous paper [2]). The final chapter begins with conjugate functions, Riesz transforms, and then goes on to simpler sorts of Calderón—Zygmund kernels (they satisfy the Dini condition).

This book grew out of a series of lectures Sadosky gave at the Universidad Central de Venezuela and the presentation of many of the topics shows the influence of the texts mentioned above. Unfortunately for the student, the material is given only a formal motivation (study A in order to learn A), and there are no exercises.

One final remark. Both books are produced from camera-ready copy. Even though the manuscripts were very carefully prepared on high-quality typewriters, we should think that for the price, the reader could expect the much more readable and attractive typesetting.

#### REFERENCES

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*Function theory of several complex variables*, by Steven G. Krantz, John Wiley & Sons, New York, 1982, xiii + 437 pp., \$39.95. ISBN 0-4710-9324-6

**1. Levi problem.** Many interesting results in complex analysis are concerned with the existence of analytic functions with prescribed properties. Most of these apply to “pseudoconvex” domains: this is a holomorphically invariant geometric criterion generalizing convexity. One such result is the solution of the so-called Levi problem: *Every pseudoconvex domain  $\Omega \subset \mathbb{C}^n$  is a domain of*