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The logic of quantum mechanics, by Enrico G. Beltrametti and Gianni Cassinelli, Encyclopedia of Mathematics and its Applications, vol. 15, Addison-Wesley, Reading, Mass., 1981, xxvi + 305 pp., $31.50.

By their very nature, scientific theories cannot be proved. No matter how successful a theory has been in explaining the Universe, there always exists the possibility, however remote, that this particular theory is not the only one that can explain the given phenomena. There conceivably could exist another theory that could do just as well—if not better. This possibility is not as remote as it may seem. In the past, very few physical theories have lasted more than a century without being discarded or substantially modified.

Quantum theory was brought about at the turn of the century by the failure of classical physics to explain the results of more accurate experiments which could measure atomic phenomena. The success of the theory was overwhelming, and currently its acceptance among scientists is unquestioned. In the beginning the theory consisted of statements concerning physical quantities, but later writers attempted to axiomatize it and divorce it from concepts of classical mechanics. The final result was a Hilbert space formalism which can be described briefly as follows. To each physical system there is assigned a separable Hilbert space $\mathcal{H}$ over the complex field, and to each physical quantity there is associated a linear, selfadjoint operator on $\mathcal{H}$. Contributions to this process were made by Bohr, Born, Dirac, Heisenberg, Schrödinger and von Neumann.

In [**1**] Mackey presents eight axioms from which he deduces the theory. His starting point is a collection $\mathbb{S}$ of "states" and a collection $\mathcal{O}$ of "observables" (physical quantities). To each A in $\mathcal{O}$, each α in $\mathbb{S}$ and each Borel set E there is a real number $p(A, \alpha, E)$ representing the probability that a measurement of A in the state α, will give a value in E. Observables that have only two possible outcomes are called questions (propositions). One of Mackey's axioms is

AXIOM VII. The partially ordered set of all questions in quantum mechanics is isomorphic to the partially ordered set of all closed subspaces of a separable infinite-dimensional Hilbert space.

He then says "This axiom has rather a different character from Axioms I through VI. These all had some degree of physical naturalness and plausibility. Axiom VII seems entirely ad hoc. Why do we make it?... We make it because it "works", that is, it leads to a theory which explains physical phenomena and successfully predicts the results of experiments. It is conceivable that a quite different assumption would do likewise but this is a possibility that no one seems to have explored."

It is to this question that many writers have directed their energies. Is the Hilbert space structure necessary, or can it be replaced by another system. The present book describes the progress to date of the attempt to replace Mackey's Axiom VII with others that are more "physically plausible" and then show that they lead to the Hilbert space structure. Previous books addressing this problem include Jauch [**2**] and Varadarajan [**3**].

Beltrametti and Cassinelli divide their book into three parts. The first part describes the Hilbert space formalism. The second describes some of the basic structures found in the formalism and the fundamental concepts of quantum theory. The third part shows how much of the Hilbert space structure can be recovered if one begins by assuming only those principles described in the second part that have physical foundations. In particular, they assume that the propositions of a quantum system form a complete, orthomodular, atomic, irreducible lattice having the covering property. It is shown that this lattice $\mathcal{L}$ admits a vector space coordinatization, i.e., it is isomorphic to some lattice of subspaces of a vector space. The rub is that the field K over which the vector space is defined is unknown. It certainly can be the complex, real or quarternion fields since Hilbert spaces over these fields produce lattices having all of the properties of $\mathcal{L}$. It is unknown if other fields are possible. However, it is shown that in the case of these fields continuity of an involution produces the usual Hilbert space quantum mechanics.

The book is well written in that it expains the physical and mathematical concepts clearly. However, little is actually proved in the text. Most results are quoted from other sources. Moreover, most of the material contained in the text can be found in other books. But it certainly does well as a readable survey.

The title of the book is taken from the 1936 paper [**4**] by Birkhoff and von Neumann, which gave the impetus for much of the research in quantum mechanics. The word "logic" in the title refers to the mathematical foundations of quantum mechanics and not to quantum logic which is mentioned only briefly in the book.

A person unfamiliar with quantum theory will have difficulty reading the book. The authors might have pleased more readers by restructuring the book and including more background material. But the majority of readers will consider it a worthwhile addition to the literature.

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Representations of real reductive Lie groups, by David A. Vogan, Jr., Progress in Mathematics, vol. 15, Birkhäuser, Boston, Basel, Stuttgart, 1981, xvii + 754 pp., $35.00.

1. Introductory. The representation theory of Lie groups is a vast and imposing edifice. Its foundations were laid by E. Cartan in 1913. He gave a classification of the finite-dimensional irreducible representations of a complex semisimple Lie algebra $\mathfrak{g}$ [**2**]. This is the "infinitesimal" version of the classification of finite-dimensional irreducible representations of a semisimple Lie group G. It was realized by H. Weyl in the twenties [**9**] that if G is compact (and connected and simply connected in the topological sense) any continuous irreducible finite-dimensional complex representation of G can be obtained by "integration" from a similar representation of the complexification $\mathfrak{g}$ of the Lie algebra of G. He also showed that any representation of G is equivalent to a unitary one.

Around the same time Peter and Weyl [**6**] showed that these irreducible unitary representations are fundamental objects for noncommutative Fourier analysis on the compact Lie group G. The representation theory of Lie groups is thus tied up with Fourier analysis.

Any attempt at a straightforward generalization of these elegant results to the case of noncompact Lie groups breaks down. To develop Fourier analysis on noncompact Lie groups one needs infinite-dimensional representations of a Lie group G, more precisely continuous representations π of G by bounded operators in a Hilbert space H. Such a representation π is irreducible if no closed nontrivial subspace of H is invariant under all $\pi(x)$ $(x \in G)$.