Wiener integral expansion mentioned above. This conjecture was settled affirmatively only in 1980 [**1**]. However, this point can be avoided by making a Girsanov transformation after which the observation process Z_t is a brownian motion, and by using a stopping time argument.

There have been a number of interesting recent developments in nonlinear filtering theory, which are beyond the scope of Kallianpur's book. One direction concerns the theory of "robust" or "pathwise" solutions to the filtering equations [**4**]. The objective is to obtain $\hat{s}_t$ for all possible observation trajectories $Z.$, not just for a set of probability 1, in such a way that $\hat{s}_t$ depends continuously on $Z.$ in the uniform norm. Another direction of recent research is to explain the structure of the optimal filter by studying a certain Lie algebra associated with it [**3**]. A related problem is to find finite-dimensional nonlinear filters, in other words, filters whose evolution in time is described by a finite number of stochastic differential equations [**2**].

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 7, Number 2, September 1982

0273-0979/82/0000-0225/$02.50

Perturbation methods in applied mathematics, by J. Kevorkian and J. D. Cole, Applied Mathematical Sciences, vol. 34, Springer-Verlag, Berlin and New York, 1981, x + 558 pp., $42.00.

Introduction to perturbation techniques, by Ali Hasan Nayfeh, Wiley, New York, 1981, xiv + 519 pp., $29.95.

Singular perturbations, in 1982, is a maturing mathematical subject with a fairly long history and a strong promise for continued important applications throughout science. Though the basic intuitive ideas involving local patching of solutions can be found in early work by Laplace, Kirchhoff, and others, Prandtl's paper at the 1904 Leipzig Mathematical Congress began the study of fluid dynamical boundary layers by analyzing viscous incompressible flow past an object as the Reynolds number becomes infinite (Prandtl [**1905**]). The distinguishing feature of singular perturbation problems occurs, viz. a thin region near the solid boundary where the velocity changes from zero (as required by the no-slip condition) to an outer flow which is essentially inviscid. Expressed mathematically, the solution converges nonuniformly in the domain as a parameter $\varepsilon = \mathrm{Re}^{-1}$ tends to zero. In the first part of this century, analysis of asymptotic solutions to linear ordinary differential equations progressed through the work of Birkhoff [**1908**], Langer [**1931**], and others, with significant work on turning point problems being done by physicists (Wentzel [**1926**], Kramers [**1926**], Brillouin [**1926**], and the survey by McHugh [**1971**]). Friedrichs and his student Wasow seem to be the first mathematicians to initiate a prolonged study of the asymptotic solution of singularly perturbed boundary value problems (cf. Wasow [**1941, 1944a, 1944b**], Friedrichs and Wasow [**1946**], Friedrichs [**1953, 1955**]; noting that Tschen [**1935**], Nagumo [**1938**], and Rothe [**1939**] certainly preceded them). Their work was motivated by an analysis of the edge effect for buckled plates (Friedrichs and Stoker [**1941**] and Stoker [**1942**]); they first used the term singular perturbations in print in the title of Friedrichs and Wasow [**1946**]. Other mathematicians, including Levinson and Tikhonov, began studying related problems soon afterwards (Levinson [**1950a**], [**1950b**], Tikhonov [**1948**], and Vasil'eva and Volosov [**1967**]). Levinson began the study of a wide spectrum of important topics in asymptotics and made definitive contributions to singular perturbations (before the mid-1950's) together with a number of promising students and young collaborators including Aronson, Coddington, Davis, Flatto, Haber, and Levin. The Russian school also did outstanding work on many subjects including boundary layer methods (Vishik and Lyusternik [**1957**] and Vasil'eva [**1963**]), relaxation oscillators (Pontryagin [**1961**] and Mishchenko and Rozov [**1980**]), and the method of averaging (Bogoliubov and Mitropolsky [**1955**] and Volosov [**1962**]).

From around 1950, fluid dynamicists solved some very interesting physical problems like the linoleum-rolling problem (Carrier [**1953**]) and low Reynolds number flow past bodies (Proudman and Pearson [**1957**] and Kaplun [**1957**]). At Caltech's Guggenheim Aeronautical Laboratory, Lagerstrom, Cole, Latta, Van Dyke, Kaplun and others became equally involved in asymptotic expansion procedures for more general singular perturbation problems. Many would claim that the most significant work was done by Kaplun (see the posthumous book, Kaplun [**1967**]), though numerous engineers who later successfully used perturbation methods would probably not understand his Extension Principle.

A (possibly oversimplified) matching procedure was presented in the book of Van Dyke [**1964**]. The straightforward recipe he provided made it easy for a tremendous variety of scientists to learn the rudiments of matching and to solve important problems in their own disciplines. The basic idea, much as in Friedrich's early and Erdélyi's current lectures (Erdélyi [**1961**]), involved an asymptotic matching of the inner and outer expansions at the edge of the boundary layer (where they should both be appropriate). A uniformly valid composite asymptotic approximation might be obtained by adding the inner and outer expansions (to an appropriate number of terms) and subtracting their common part. (The novice can roughly figure out the procedure by determining the limiting behavior of the solution to the sample problem $\varepsilon y'' + y' = 1$ on $0 \leq x \leq 1$, with $y(0)$ and $y(1)$ prescribed, as $\varepsilon \to 0^+$.) Fraenkel [**1969**] and Lagerstrom [**1976**] questioned the universality and orthodoxy of Van Dyke's rules, while Cole [**1968**] instead stressed "limit process expansions" and "two-timing" in a context far broader than fluid mechanics. Important new applications had arisen including geometrical optics (cf. Keller [**1958** and **1978**]), enzyme kinetics (e.g., Bowen, Acrivos, and Oppenheim [**1963**]), shell theory (Gol'denveizer [**1961**]), and stiff differential equations (Dahlquist [**1969**]). Indeed, results obtained through matching generally coincided with those known through the intuitive folkways of the various fields.

Wasow's [**1965**] book placed singular perturbations in the context of the analytic theory of differential equations, including singular point and turning point theory, and presented much recent research of the author, Sibuya, and others. It stimulated much more research by mathematicians, both pure and applied, since it was becoming clear that much important formal work could be proven to be asymptotically valid. Certainly numerous theses on related topics were appearing in Cambridge, Madison, Minneapolis, New York, Pasadena, Stanford, and elsewhere throughout the world.

By 1970, courses in perturbation methods became common in engineering and applied mathematics departments, and inevitably a string of textbooks and higher level monographs began to appear. They included Nayfeh [**1973**], which presented numerous specialized methods and referenced hundreds of applied papers; Lions [**1973**] which emphasized partial differential equations, with applications to control, from a modern point of view; Eckhaus [**1973**] (with a more complete book in 1979) which examined the basis of matching and included much Dutch work involving boundary value problems for linear elliptic and nonlinear ordinary differential equations; Vasil'eva and Butuzov [**1973**], which primarily surveyed expansion techniques from the extensive Soviet literature, though mostly for ordinary differential equations; and O'Malley [**1974**], which also emphasized ordinary differential equations and applications via boundary layer corrections, instead of the traditional matching. Carrier's various presentations (Carrier [**1974**]) emphasized the use of intuitive singular perturbation concepts when more elementary approaches break down. Recently, several introductory texts include chapters on singular perturbations. (One might well argue that perturbation methods will help most sophomores more than a few days exposure to the Laplace transform or power series about regular singular points.) Bender and Orszag [**1978**] emphasized more general asymptotic techniques for ODE's. It has an especially good collection of examples (with many solution portraits) and some very challenging exercises. Among more specialized monographs, Bensoussan et al. [**1978**] uses a two-time analysis to develop a theory of homogenization; Schuss [**1980**] shows how to use singular perturbation concepts to analyze solutions to stochastic differential equations; and Miranker [**1981**] shows some ways to use asymptotic results to develop numerical algorithms for stiff differential equations. (My jaundiced opinions of Nayfeh [**1973**], Eckhaus [**1973**], and Schuss [**1980**] are, incidentally, noted after those references.) Numerous conference proceedings (note, especially, Meyer and Parter [**1980**], Miller [**1980**], and Eckhaus and deJager [**1982**]) and special topic and review articles (note, e.g., Howes [**1978**] and O'Malley [**1978**]) attest to the present vitality of the field. Primary motivation for mathematical research still comes from important new applications (e.g., Buckmaster and Ludford [**1982**] which would teach combustion experts asymptotics and vice-versa). New books by Lagerstrom and Boa, Chang and Howes, Smith, Kapila, and others will continue to enliven and enrich the subject for some time to come.

As suggested above, Cole's 1968 book was important because it reemphasized the point that the art of singular perturbation techniques could be successfully applied in a variety of fields. Little attempt was made to solve general classes of problems or to justify (in the sense of pure mathematics) the approaches taken, but the reader who worked his or her way through the sequence of challenging model and physical problems gained the perspective and experience to enable him to attack new problems requiring modelling, skill, perseverance, and understanding. (I am still not completely satisfied with my solution of the two-point (dust jacket) problem for $\varepsilon y'' + yy' - y = 0$ or of the relaxation oscillations for van der Pol's equation, but I learn more about them and the subject every time I try to work through the material.) The book benefitted greatly from Cole's long involvement with the Caltech community. It also included the first textbook presentation of the two-variable expansion procedure which was highly developed through the combined efforts of Kevorkian and Cole (Kevorkian [**1962**]). An appreciation of that method can be gained by examining the linear oscillator with small damping. On a long time scale, the cumulative effect of the damping can be accounted for through use of a slow time εt together with a fast time of the form $(1 + \varepsilon\omega_1 + \varepsilon^2\omega_2 + \cdots)t$ for appropriate ω_i's. Here, the approach is more related to averaging than to the layer methods discussed previously. It is also natural to use two-timing for analyzing equations with slowly-varying coefficients.

The new Kevorkian and Cole contains revised presentations of virtually all the topics found in Cole [**1968**], with somewhat expanded discussions and a generally more readable text. This is especially welcome, since the original Cole has been out of print for some time. Except for a couple of PDE models (MHD pipe flow and viscous boundary layers for rotating fluids), all the favorite examples are still there, plus more, with stimulating exercises. Considerably more material has been added relating to Kevorkian's continuing involvement with multi-variable expansion procedures and their application to the motion of satellites. The approach throughout is personal, reflecting the authors' expert viewpoints. In particular, the book does not generally survey recent related work of other writers. AMS members will generally be glad to know, as one would expect from the efforts of talented senior applied mathematicians, that much of the work can be extended to more general contexts, and much can be shown to be rigorous (Greenlee and Snow [**1975**] and Sanders and Verhulst [**1981**], for example, give theory appropriate for two-timing and averaging). The difficult problem of passage through resonance (for systems with slowly-varying coefficients) has occupied Kevorkian for many years and it is discussed in some detail. Alternatives to multi-timing are also summarized. The chapter on partial differential equations now includes a discussion of Burgers' equation $\varepsilon u_{xx} = u_t + uu_x$ with various initial and boundary conditions, a potential problem from cell physiology which Cole has worked on, and applications to nonlinear dispersive and weakly nonlinear waves. The final chapter now discusses weakly nonlinear one-dimensional acoustics, small amplitude waves on shallow water, and more material on thin airfoil theory. The book is important to mathematicians, since it shows how mathematics can be applied by those who have a physical understanding and a command of the appropriate mathematical tools, including perturbation methods.

Nayfeh's new book is not a revision of his 1973 *Perturbation methods* or his more recent *Nonlinear oscillations* (written with Mook). (Nayfeh, incidentally, was a student of Van Dyke at Stanford and has since done a variety of applied asymptotics.) This book, instead, seeks to teach a diverse audience of advanced undergraduates and beginning graduate students about perturbation techniques and asymptotics in general. He elaborately presents numerous examples, so that readers can follow the calculations step-by-step. This is important because, until symbol manipulators become available to all, few students will be able to do the necessary and laborious operations correctly. The subjects presented are useful ones that many engineers and all applied mathematicians should be acquainted with, to wit: asymptotic expansions, solutions of algebraic equations with a small parameter (the implicit function theorem and the Newton polygon method should be added for some audiences), asymptotic expansions of integrals, perturbation schemes in nonlinear oscillations, etc. Most of the topics are also covered in Bender and Orszag [**1978**], but at a generally higher level which would not be reachable by the broader clientele who could still benefit from learning elementary techniques. More experienced readers will find the amount of detail overwhelming, but the book is not intended for them. The numerous calculations are generally well done; I would, however, quibble with the decision to play down the trivial singular solution to Cole's dust cover problem since this involves a complicated algebraic matching at one or two interior locations. One hopes that textbooks such as Nayfeh's will make the elementary concepts of singular and regular perturbation theory part of the undergraduate study of most scientists and engineers.

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