the beauty and unity of mathematics, we must broaden our perspectives and hone our intellectual curiosity. Perhaps algebraic topologists should learn some mathematical physics, and mathematical physicists some algebraic topology; not because it is useful, but because it is interesting! It seems that Lefschetz agreed.

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Geometry of Banach spaces–Selected topics, by Joseph Diestel, Lecture Notes in Mathematics, no. 485, Springer-Verlag, Berlin, Heidelberg, New York, 1975, xi + 282 pp., $11.50.

Beginning with Banach's *Operations lineaires* the study of Banach spaces has been a pursuit of classification. This can be in terms of the classically important spaces as $C(K)$ or $L_p(\mu)$, or it can be in terms of certain desirable internal conditions on norm and specified elements such as smoothness and convexity properties, or it can be in terms of external conditions, for example, on dual spaces, subspaces, or factorization of operators. An elegant example of the first is the Bohnenblust-Kakutani result that a Banach lattice is linearly isometric to $L_p(\mu)$ or a sublattice of $C(K)$ if and only if $\|x + y\|^p = \|x\|^p + \|y\|^p$ or $\|x + y\| = \max(\|x\|, \|y\|)$ whenever $x \wedge y = 0$. Smoothness refers to the existence of unique supporting hyperplanes to points on the surface of the unit ball and is usually phrased in terms of the differentiability of the norm. Uniform convexity describes the shape of the surface of the unit ball. These concepts are important to minimization problems in optimization theory and P.D.E. among others. These conditions are "geometric" in nature and are not, in general, preserved under isomorphisms. However, deep studies have been made into various Banach space properties which imply smoothness or convexity conditions under some equivalent norm. Perhaps the deepest and most important of these is the one obtained by P. Enflo which states that a Banach space X has an equivalent norm under which it is uniformly convex if and only if it is superreflexive (i.e., every Banach space Y which has the property that every finite dimensional subspace of Y is almost isometrically embeddable into X is itself reflexive). This is an elegant blending of the internal geometry (uniform convexity) with the external geometry (superreflexivity). There are many examples of the third type mentioned above. For example, the theory of $\mathcal{L}_p$ spaces (roughly, a Banach space is a $\mathcal{L}_p$ space if it is the union of an upwards directed family of finite dimensional spaces each uniformly equivalent to an $l_p(n)$, $1 \leqslant p \leqslant \infty$). Thus there is the elegant Lindenstrauss-Pełczyński-Rosenthal result that X is a $\mathcal{L}_p$ space or a Hilbert space if and only if it is isomorphic to a complemented subspace of an $L_p(\mu)$ space, $1 \leqslant p < \infty$. In duality there is the Lindenstrauss-Rosenthal result that X is a $\mathcal{L}_p$ space if and only if X^* is a $\mathcal{L}_{p'}$ space, $1 \leqslant p \leqslant \infty$. In factorization of operators, there is the beautiful theorem of Davis, Figil, Johnson and Pełczyński that a weakly compact operator factors through a reflexive space. One also has the deeply significant work of many authors (Lindenstrauss, Pełczyński, Nikišin, Stein, Rosenthal, Maurey, and others) on absolutely p-

summing operators, Grothendieck's inequality, and factorization of operators through L_p spaces. This work has deep and important ramifications to the theory of Banach spaces and harmonic analysis.

In the work under review the author has isolated on the aspects of the theory which are germane to the study of the Radon-Nikodym property in Banach spaces. This, however, turns out to include a significant amout of material from the internal geometry of Banach spaces such as smoothness, differentiability of the norm, uniform and local uniform convexity, and renorming techniques to improve the smoothness or convexity of the norm. He presents this in the first third of the book together with clear and concise proofs written in a relaxed style. He uses, whenever possible, two central theorems: The first by Bishop and Phelps states that every Banach space X has the property that the collection of bounded linear functionals on X which attain their norm is norm dense in X. The second by James is the important characterization of weak compact sets in a Banach space (and hence of reflexive spaces) to wit: that a convex set C in X is weakly compact if and only if every continuous linear functional on X attains its sup on C. The middle third of the text is concerned with results about weak compact sets including the concept of a weakly completely generated (WCG) Banach space (i.e., one which is the closure of the space of a weakly compact set), Eberlein compact spaces (i.e., compact Hausdorff spaces homeomorphic to a weakly compact subset of some Banach space), and the above mentioned factorization theorem for weakly compact operators. Thus he presents the basic results due to Lindenstrauss and Amir that if X is a WCG Banach space, then there is a 1-1 bounded linear operator of X into $c_0(\Gamma)$ for some Γ and that a weakly compact subset of a Banach space is "linearly" homeomorphic to a weakly compact subset of $c_0(\Gamma)$ for some Γ. He also presents Trojanski's theorem that a WCG Banach space admits an equivalent locally uniformly convex norm together with several important corollaries. In addition, he includes Rosenthal's characterization of Eberlein's compact spaces in terms of the existence of a sequence of point-finite families of open $\mathcal{F}_\sigma$ sets satisfying certain conditions. Finally, he presents many examples and results concerning stability (or lack of it) in WCG spaces.

The final third of the book is concerned with one of the newest entries into the geometric theory of Banach spaces, the Radon-Nikodym property. Several results presented here were quite recently obtained. Briefly, a Banach space X has the *Radon-Nikodym property* (RNP) if every X-valued countably additive measure possessing a finite variation is differentiable (in the Bochner sense) with respect to its variation. The reason that the RNP is geometric in nature is that it is intimately tied to such notions as dentability (a set D in X is *dentable* if for each $\varepsilon > 0$ there is an x_ε in D which is not contained in the closed convex hull of D minus the ε-ball around x_ε), and the Kreĭn-Milman property (KMP) (which is that every closed bounded convex set in X is the closed convex hull of its extreme points). Thus, for example, he presents the Huff-Maynard-Phelps theorem that if X has RNP, then every bounded set in X is dentable. The Lindenstrauss theorem that RNP implies the KMP is proved together with the corresponding converse by Huff and Morris for dual Banach spaces. He also includes Edgar's theorem concerning integral representation of points in closed bounded convex separable subspaces of spaces with the RNP and Phelp's theorem that X possesses the RNP if and only if each nonempty closed bounded convex set in X is the closed convex hull of its strongly exposed points.

In sum, the book is a valuable source to workers in the area of Banach spaces. It is full of details and proofs which are concisely and clearly presented. It is a welcome addition to the growing number of books on Banach spaces.

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Statistical prediction analysis, by J. Aitchison and I. R. Dunsmore, Cambridge Univ. Press, London, New York, Melbourne, 1975, xi + 273 pp., $24.50.

No doubt future statisticians will find it remarkable that not before the last quarter of the 20th century, did a textbook on what Statistics is really about, finally appear. Too long has the estimation of parameters dominated statistical theory and consequently warped and cluttered up the methodology–the raison d'être of the field, while the prediction of observables, which should have been preeminent, receded into the background. There are several reasons why this occurred. Chief among them is the tremendous preoccupation that theoreticians have had analysing the logical distinctions inherent in the various so-called modes of Inference, i.e., Bayesian (Jefferies, de Finetti, Savage), Frequentist (Neyman-Pearson), Fiducial (Fisher), Likelihood (Fisher, Barnard), etc., rather than what should be the proper subject for statistical analysis–parameters or potential observables. In the early history of Statistics there was no sharp distinction drawn between statistics (functions of observables) and hypothetical parameters, resulting in the tendency for the issue to be obscured. R. A. Fisher correctly made the sharp distinction necessary for the advance of thinking in this area. But since then and apparently through no fault of Fisher's, mathematical statisticians became so enamoured of those artificial constructs–parameters, that all their work tended to be framed and executed parametrically. Oddly enough even in that branch of Statistics which is often referred to as Non-Parametric Inference, developers and practitioners also attempt, to this day, to orient their work towards the estimation of parameters–so ingrained is the habit. Some, perhaps realizing the paradox, even altered the taxonomy by referring to this branch as Distribution-Free Inference.

One must also realize that the parametric approach has advantages, though illusory. Mathematical statisticians using any mode were often seduced by the niceties of the mathematics of parametric structures. Making precise statements about unobservables, i.e., parameters, also serves applied statisticians very well in that it is virtually impossible to contradict them by observation. Of course a predictivist, who by definition is in the business of making statements about potential observables, lacks such security. His statements, to