

the usual way, using Lebesgue sums. It is then extended to unbounded functions and to infinite intervals, and compared with the Riemann integral. This development occupies about 70 pages. Approximately equal space is devoted to establishing the principal properties of this integral: convergence theorems, Vitali's theorem, differentiation of functions of bounded variation, absolute continuity, the completeness of both real and complex  $L^p$  spaces, Lusin's theorem, orthogonal expansions and the Riesz-Fischer theorem. The last 34 pages deal with plane measure, multiple integration, and the Riemann-Stieltjes integral.

All the material is standard and classical; the aim is to include "those portions of the theory which find immediate application in other fields, e.g. in probability theory or theoretical physics." The exposition is mostly self-sufficient, though the critical reader will occasionally find it necessary to expand a proof or justify a tacit assumption. For some reason the authors omit a proof that the positive and negative variations of a continuous function of bounded variation are continuous, although this fact is mentioned and later used. With greater justification, Féjèr's theorem is explicitly assumed without proof. No problems or exercises are included.

This book may be recommended, especially as preliminary or collateral reading in connection with a more intensive and sophisticated course in measure theory or real functions. The current fashion is to present measure and integration theory in a more general and postulational form. This is as it should be, but thereby important classical theorems are sometimes neglected. This book may help to fill the gap. It may also prove useful to an applied mathematician who wishes to learn quickly the essentials of Lebesgue theory, although it may be questioned whether a knowledge of only the ordinary Lebesgue integral is adequate nowadays for either probability theory or physics. According to the preface, "This book contains material which constitutes a good introduction to the theory of real functions. The subject matter comprises concepts and theorems . . . which every young mathematician ought to know." These claims are certainly justified, and since there are now so many things that a young mathematician needs to know it is useful to have available this brief exposition of the classical theories presented here.

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*Mathematical theory of compressible fluid flow.* By Richard von Mises. Completed by Hilda Geiringer and G. S. S. Ludford. Applied Mathematics and Mechanics, vol. 3. Academic Press, New York, 1958. 13+514 pp. \$15.00.

It is pointed out in the preface to this book that only the first three

chapters were written by von Mises, the remaining two chapters and appended notes having been prepared in cooperative effort by Hilda Geiringer and G. S. S. Ludford, after von Mises' death. So carefully has the original spirit been observed, however, that the change of authorship is hardly discernible.

The material is presented in a style characteristic of its original author, which is to say it is a service to science. General theorems and discussion of primarily mathematical interest are avoided, although suitable references are given where pertinent. Similarly, the mathematical foundations of the subject are only sketchily set forth. The interest centers in the development of methods for solving specific problems of gas dynamics, but it must be emphasized that this is no cookbook for practicing engineers, as a high level of sophistication prevails throughout the work. The author's approach to the subject displays a combination of physical intuition and geometrical insight which enable him to isolate the essential difficulty in a situation and to deal with it effectively.

Although a considerable amount of general flow theory is presented, the major emphasis is in the discussion of supersonic flows and shock phenomena. Here the material overlaps the earlier treatise on this subject by Courant and Friedrichs [Interscience, New York, 1948]. The presentation here differs somewhat in detail and in organization from that of the earlier work, and may be more accessible to people with engineering backgrounds. Of particular interest is the consistent use as a conceptual device of the "speedgraph," analogous to the hodograph of Chaplygin, for discussion of one-dimensional flows.

The text contains an abundance of carefully prepared illustrations, and also many examples of particular flows, which are chosen to clarify important points of the theory. There are over forty pages of notes and addenda, containing historical and bibliographical comments together with much suggestive side discussion on points of special interest.

Mention should be made also of the careful treatment of Chaplygin's integration method, and of its later development by Bergman, Lighthill, and others. Much attention has evidently been paid here to minimizing technical detail and to clarifying the essential ideas and connections among the various methods. This discussion leads naturally to a concluding section on transonic flow, in which various results and conjectures on this difficult but intriguing subject are outlined.

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