RESEARCH PROBLEMS

26. P. T. Church and E. Hemmingsen: *Topology.*

Let f: $M \to N$ be continuous, where M and N are n-manifolds, $n \geqq 3$. Let the branch set B_f of f be the set of points at which f is not a local homeomorphism. Can $\dim(B_f) = 0$?

If it is also assumed that $\dim(f(B_f)) < n$, the question can be reduced (by appropriate restriction of the map) to the following: Does there exist such an f for which $f|f^{-1}(f(B_f))$ (restriction) is one-to-one, $f|M - f^{-1}(f(B_f))$ is a k-to-one covering map for some natural number k, $N = E^n$, and $M \subset E^n$? (Cf. Duke Math. J. **27** (1960), 527–536; an example of a three-to-one map f: $S^n \to S^n$ for which B_f has [some] point components appears in the same journal **28** (1961).) (Received October 19, 1961.)

27. Richard Bellman: *Calculus of variations—differential equations.*

The theory of stability of solutions of differential equations is concerned with the invariance of properties of solutions under structural changes in the equations. In particular, a great deal of effort has gone into the study of boundedness of solutions. Frequently, however, precise bounds have not been obtained. Towards this end, it would be worthwhile to obtain solutions to the following questions:

Given the linear differential equation $u'' + (1 + f(t))u = 0$, $u(0) = 1$, $u'(0) = 0$, and the associated functional $J(f) = \max_{0 \leq t \leq T} |u(t)|$, determine the maximum of $J(f)$ over all f subject to the constraints

$$\text{(a)} \qquad |f(t)| \leqq b_1 < 1, \qquad 0 \leqq t \leqq T,$$

or

$$\text{(b)} \qquad \int_0^T |f(t)|\, dt \leqq b_2 < \infty,$$

or

$$\text{(c)} \qquad \int_0^T |f(t)|^p dt \leqq b_3 < \infty, \qquad p > 1.$$

Similarly, we would like to determine the maximum of

$$J_1(f) = \int_0^T |u(t)|^q dt, \qquad q \geqq 1,$$

and the asymptotic behavior of these maxima as $T \to \infty$.

Most important, of course, would be the development of a systematic technique for solving questions of this nature. There are many multidimensional versions of the foregoing problems of greater difficulty, which arise very naturally in the modern theory of control processes. (Received October 20, 1961.)

28. Richard Bellman: *Differential equations of threshold type.*

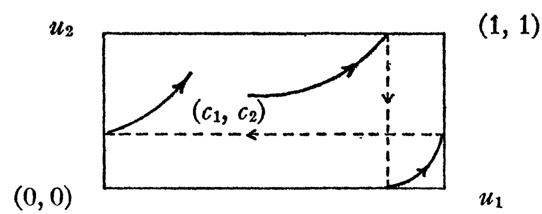
The study of the firing of nerves leads to equations of the following type:

$$\frac{du_1}{dt} = a_{11}u_1 + a_{12}u_2, \qquad u_1(0) = c_1,$$

$$\frac{du_2}{dt} = a_{21}u_1 + a_{22}u_2, \qquad u_2(0) = c_2,$$

for $0 \leqq u_1, u_2 < 1$, where $0 \leqq c_1, c_2 \leqq 1$, $a_{ij} \geqq 0$. As soon as either u_1 or u_2 attain the value 1, it instantaneously returns to zero value, leaving the other value unchanged, and the above equation takes over again.

Graphically, the solution may take the form



What are the periodic and ergodic properties of solutions of equations of this type? Are there simple analytic expressions for the solution at time t?

The foregoing problem can be generalized in several ways. We can first of all consider the multidimensional versions, and secondly take account of more complex transformations when a boundary (or firing threshold) is reached. (Received November 27, 1961.)