## BOOK REVIEWS

*Mathematische Statistik.* By B. L. van der Waerden. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1957. 9+360 pp. DM 46. Bound DM 49.60.

The aim of this book is clearly stated in the preface. Since his student days the author has been consulted on statistical questions arising from various fields of application. By reflection and study of the literature he has learned better and better methods to deal with them. These methods are to be presented with realistic examples taken from natural and social sciences. Thus it is hoped that the reader will be spared some groping in the dark.

The style of the book is set with this object in view. A concrete and practical problem is posed. It is at once precisely and clearly formulated in mathematical terms. Frequently it is reduced to the calculation of a certain probability, that of the rejection of the null hypothesis when it is true. Whenever possible, the exact combinatorial solution is derived. Approximations are then given, either by Stirling's formula or expansion in series. This is sometimes helped out by graphing, by tabulating, and by reports on actual computations the author has undertaken. If a statistic is used in testing, its mean and variance are calculated and its asymptotic properties which are proved in the literature are cited. If a novel mathematical concept is involved, this is dragged out into the open and probed. When several methods are available, their merits are compared theoretically and/or empirically. For instance, at the end of the book it is concluded that Spearman's rank correlation is to be preferred to the newer one of M. G. Kendall.

This manner of proceeding makes it easy for the reader who wishes to learn about a particular topic with dispatch or the general layman who is discouraged by dross and jargon. Further pedagogic devices are used to facilitate the use of the book: some repetitions are allowed to make for greater independence between the chapters; each chapter begins with a summary of the required preparation; an index of examples classified according to subject matter; an English vocabulary (a tribute to the British-American school which coined the statistical nomenclature). The informal, sometimes personal style also helps. On p. 175, e.g., there is the suggestion that the company, between the choice of two statistics, may be more interested in saving money than improving its product. Read in German, where "money" is "Geld," I found this truism rather amusing. The mathematical level

is somewhat below that of Cramér's Mathematical Methods of Statistics. For example, most asymptotic results are stated without proof. The first few chapters, alternating probability background with statistical applications, instead of piling up all mathematical tools at the beginning, succeed in breaking the usual monotony.

The contents will be given with a somewhat random sampling of particulars. Chapter 1, *General foundations* (with the subheading: "The study of this chapter is indispensable"). Of course it is the rigorous set-theoretic foundation that is given. Chapter 2, *Probability and frequency*: including test of the true $p$, hypergeometric distribution, the chi-square test. Chapter 3, *Mathematical tools*: including orthogonal transformation, the beta and gamma functions. Chapter 4, *Empiric determination of distribution, mean and variance.* This begins with a childhood reminiscence of picking 89 willow leaves to form a Quetelet curve; includes the recent result of Birnbaum and Tingey, a brief discussion of the Kolmogorov test, and references to the literature on order statistics. Chapter 5, *Fourier integral and limit theorems.* It is a good feature of the book that the second limit theorem (convergence of moments), the usefulness of which is amply illustrated both here and elsewhere, is given in addition to the Lévy-Cramér theorem. Chapter 6, *Gauss's theory of errors and Student's test.* Chapter 7, *The method of least squares*: with an example from the author's study of a Byzantine solar chart. Chapter 8, *Estimation of unknown parameters*: including the maximum likelihood method; Fréchet inequality (listed in the English vocabulary as Cramér-Rao, also as "information inequality"); sufficient statistic. Chapter 9, *Evaluation of observed frequencies.* Chapter 10, *Bio-assay*: including graphic methods and the recent stochastic approximation method of Robbins and Monro. Chapter 11, *Testing hypothesis*: including analysis of variance and the Neyman-Pearson theory. Chapter 12, *Order tests.* The Smirnov test is briefly discussed; Wilcoxon's test and the author's $X$ test are discussed at some length. Chapter 13, *Correlation.* Chapter 14, *Tables.* There are 13 covering 19 pages.

One may of course question the choice of material. The emphasis on methods dealing with practical problems may have cut down the discussion of more stilted (and sometimes stultified) topics. It is explicitly stated in the Preface that completeness is not striven for. "Many important theories such as those of sequential tests, decision functions and stochastic processes are completely omitted." One may yet wonder if some brief introduction to these theories could not be included, leading from simple examples to some of the basic ideas behind them, with references to the literature for details, just as is

done for other topics favored in the book. It would seem that some of the minor problems and methods included might gracefully make room for this. Even the utilitarian reader would have profited by some contact with these modern notions and been spared the labor of tackling the larger works to which the author deferred.

The reviewer takes exception to the remark on p. 98 that the strong law of large numbers "scarcely plays a role in mathematical statistics." This is like saying e.g. that Dedekind cut scarcely plays a role in numerical analysis (or dynamics). The point is, even if the strong law is meaningless in a final statistical statement it may well enter into an argument or proof which is essential to the statistical conclusion, just as the real number system is surely at the back of many calculations although the IBM machines yield nothing but terminating decimals. To cite one concrete example, the asymptotic normality result mentioned on p. 220 has been recently extended (albeit slightly) by using convergence with probability one.

Finally, irrelevantly but inevitably, a reader of Professor van der Waerden's new book cannot help recalling his well-known volumes on *Moderne Algebra*. The format is there complete with the graphic guide; the masterful exposition is there; and the various pedagogic devices mentioned above are there. If the total impression is different this is due more to the subject matter than to the treatment. Mathematical Statistics, being a branch of fiercely applied mathematics with a relatively short history, does not have nor perhaps even care for the idealism and formalism of Algebra. Indeed, the criteria of excellence are somewhat different in these two fields. Statistics is primarily concerned with utility, not beauty; nevertheless there is no lack of neat things in this volume, and a good deal more in the field, as there always will be when competent hands work with Mathematics.

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*Convexity*. By H. G. Eggleston. Cambridge Tracts in Mathematics and Mathematical Physics, no. 47, Cambridge University Press, 1958. 8+136 pp. $4.00.

This tract provides a brief and clear introduction to the theory of convex sets in $E^n$ on an elementary level. It is not intended for the specialist, because it covers in the main only topics found in Bonnesen and Fenchel's *Theorie der konvexen Körper*. There are, of course, innovations in methods and proofs. As examples we mention the greater use of duality in $E^n$ and the proof that the mixed volumes are non-negative.