characteristic roots of $A\bar{A}'$, Browne has shown that $\rho_1 \geqq |\lambda| \geqq \rho_n$. In this paper it is shown that for any complex number μ such that $\rho_1 \geqq |\mu| \geqq \rho_n$ there exists a matrix B which has the characteristic root μ and is such that $B\bar{B}' = A\bar{A}'$. Consideration is also given to the distribution of the roots of B within the annular region. (Received August 10, 1945.)

207. Edward Rosenthall: *On the sums of two squares and the sum of cubes.*

In this paper the complete rational integer solution of each of the diophantine systems $x_1^2+y_1^2=x_2^2+y_2^2= \cdots =x_n^2+y_n^2$ and $(x_1^2+y_1^2)(x_2^2+y_2^2) \cdots (x_n^2+y_n^2)=(u_1^2+v_1^2) \cdots (u_m^2+v_m^2)$ is obtained in terms of integral parameters. Also, a method is given for finding all the sets of rational integers satisfying the diophantine equation $x_1^3+x_2^3 + \cdots +x_{2n}^3=0$. This method reduces the resolution of this equation to a system of linear homogeneous equations in which the number of unknowns always exceeds the number of equations. The above results are deduced from the complete integer solution in the quadratic fields $Ra(i)$, $Ra((-3)^{1/2})$ of certain connected simple and extended multiplicative equations. (Received September 4, 1945.)

ANALYSIS

208. R. P. Agnew: *A simple sufficient condition that a method of summability be stronger than convergence.*

Let $\sigma_n = \sum_{k=1}^{\infty} a_{nk}s_k$ be a regular Silverman-Toeplitz transformation by which a sequence $s_1, s_2, \cdots$ is summable to σ if $\sigma_n \to \sigma$ as $n \to \infty$. It is shown that if $a_{nk} \to 0$ as $n, k \to \infty$, then some divergent sequences of zeros and ones are summable. A more general theorem applies to transformations which are not necessarily regular. If (i) $\sum |a_{nk}| < \infty$ for each fixed $n=1, 2, \cdots$ and if (ii) as $n \to \infty$, the maximum for $k=1, 2, \cdots$ of $|a_{nk}|$ converges to 0, then some divergent series of zeros and ones are summable. The theorems furnish criteria for determination of relations among methods of summability. (Received August 20, 1945.)

209. R. P. Agnew: *Characterization of methods of summability effective for power series inside circles of convergence.*

A matrix b_{nk} of real or complex constants determines a series-to-sequence transformation $\sigma_n = \sum_{k=0}^{\infty} b_{nk}u_k$ by means of which a series $\sum u_n$ is summable B to σ if $\sigma_0, \sigma_1, \cdots$ exist and $\lim \sigma_n = \sigma$. In order that the matrix b_{nk} be such that $\sum u_n$ is summable B whenever $\sum u_n z^n$ has radius of convergence greater than 1, it is necessary and sufficient that (1) constants $\beta_0, \beta_1, \cdots$ exist such that $\lim_{n\to\infty} b_{nk} = \beta_k$ when $k=0, 1, 2, \cdots$ and (2) to each number θ in the interval $0<\theta<1$ corresponds a constand $M(\theta)$ such that $|b_{nk}\theta^k| < M(\theta)$ when $n, k=0, 1, 2, \cdots$. If (1) and (2) hold and $\sum u_n z^n$ has radius of convergence greater than 1, then $\sum \beta_n u_n$ converges absolutely and the number $B\ \{\sum u_n\}$ to which $\sum u_n$ is summable B is $\sum \beta_n u_n$. In order that $B\{\sum u_n\} = \sum u_n$ whenever $\sum u_n$ has radius of convergence greater than 1, it is necessary and sufficient that (1) and (2) hold with $\beta_k = 1$ for each k. (Received August 3, 1945.)

210. Joshua Barlaz: *On some triangular summability methods.*

A class of triangular sequence-to-sequence summability methods is given by the transform $t_n = e^{-x_n} \sum_{\nu=0}^{\infty} s_\nu x_n^\nu / \nu!$, $n=0, 1, 2, \cdots$, where $\{x_n\}$ is a sequence of numbers tending to infinity. The transform is obtained by applying to Borel's exponential limit a process which Professor Szász applied to Abel summability in a recent paper (Ann. of Math. vol. 43). A necessary and sufficient condition for the regularity of the transform is that $(x_n-n)n^{-1/2}\to-\infty$ as $n\to\infty$. Imposing other conditions on the sequence $\{x_n\}$, the author makes an investigation of some of the properties of the class of summability methods. For instance if $x_n=o(n)$, it is shown that the transform is as powerful as Borel's method when applied to power series with a positive radius of convergence. It is also shown that the method sums the sequence $s_\nu=(-1)^\nu$ for all sequences $\{x_n\}$, $x_n\to\infty$. Finally it is shown that for no sequence $\{x_n\}$ does the transform include the method of arithmetic means. (Received August 10, 1945.)

211. R. P. Boas: *Some complete sets of functions.*

The following generalization of a theorem of W. H. J. Fuchs (Proc. Cambridge Philos. Soc. vol. 40 (1944) pp. 189–197) is proved. Let $\{\lambda_n\}$ be a sequence of positive numbers, and let $n(r)$ denote the number of $\lambda_n\leqq r$. Then the set $\{e^{-t}t^{\lambda_n}\}$ is complete in $L^p(0, \infty)$, $1\leqq p<\infty$, if $n(r)-r/2\geqq -r\delta(r)$, where, as $r\to\infty$, $\delta(r)\downarrow 0$, $r^{-1}\int_0^r\delta(u)du=0(\delta(r))$, and $\int^\infty r^{-1}\delta(r)dr<\infty$. Fuchs also proved that, if the set of all positive integers is divided into two complementary sets $\{\lambda_n\}$ and $\{\mu_n\}$, then at least one of $\{e^{-t}t^{\lambda_n}\}$ and $\{e^{-t}t^{\mu_n}\}$ is complete. This result is extended to subsets of a set $\{a_n\}$ whose function $n(r)$ satisfies $|n(r)-r|\leqq r\delta(r)$, with the same conditions on $\delta(r)$. Applications are given to singularities of power series with gaps. (Received August 9, 1945.)

212. Paul Civin: *Fourier coefficients of dominant functions.*

Let $f(x)$ be a complex integrable function of period 2π and let $F(x)$ be an integrable function of period 2π which dominates $f(x)$, that is, $F(x)\geqq|f(x)|$. If $h(x)\sim\sum_{-\infty}^{\infty}d_me^{mix}$, denote $(\sum_{-\infty}^{\infty}|d_m|^r)^{1/r}$ by $N_r(h)$ and $(\int_{-\pi}^{\pi}|h(x)|^rdx)^{1/r}$ by $A_r(h)$. The following theorem is proved. For $q=2, 4, 6, \cdots$, $N_q(f)\leqq N_q(F)$, while for $p=q/(q-1)$ there is some dominant function $F_p(x)$ of each $f(x)$ such that $N_p(F_p)\leqq N_p(f)$. This is the dual of a problem of Hardy and Littlewood (Quart. J. Math. Oxford Ser. vol. 6 (1935) pp. 304–315) who consider functions $F(x)$ whose Fourier series are majorants of the Fourier series of $f(x)$, and consider relations of inequality between $A_q(f)$ and $A_q(F)$. (Received September 18, 1945.)

213. J. B. Díaz: *On a class of partial differential equations of even order.*

Partial differential equations of the form $L^Nu=0$, where $u=u(x, y)$ and L is a certain linear partial differential operator of second order with variable coefficients (the most important special case being $Lu=\Delta u+\phi(x)u_x+\psi(y)u_y$), are studied by the method of hypercomplex variables, which involves replacing $L^Nu=0$ by a system of $2N$ partial differential equations of first order denoted by $E(\Sigma, N)$, where Σ is the matrix of the coefficients of L. $E(\Sigma, N)$ plays the role of the Cauchy-Riemann equations in the study of Laplace's equation. A function theory of $E(\Sigma, N)$ is constructed by introducing hypercomplex-valued functions, $f(x, y)=\sum_{i=0}^{2N-1}a_i(x, y)j_N^i$ (where the hypercomplex unit j_N satisfies $(1+j_N^2)^N=0$), such that the real functions a_i, $i=0, 1, \cdots, 2N-1$, satisfy $E(\Sigma, N)$. The processes of (Σ, N)-integration and (Σ, N)-differentiation are defined, and several analogues of known results for analytic functions are proved. (The system $E(\Sigma, 1)$ has been discussed by Bers and Gelbart, and L. Sobrero has studied $\Delta^2u=0$ employing hypercomplex variables.) There is given a sequence of particular solutions of $L^Nu=0$, obtained by quadratures from the coefficients of L, and such that every solution u of $L^N u=0$ may be expanded in a uniformly and absolutely convergent series of these particular solutions. (Received August 10, 1945.)

214. Nelson Dunford: *Boolean algebras of projections.*

The problem of embedding a bounded Boolean algebra of projections on a Banach space X in a complete Boolean algebra of projections in X is investigated. In particular, it is shown that if X is reflexive, this can always be accomplished. (Received August 11, 1945.)

215. Bernard Friedman: *Two theorems on simple functions.*

Let $f(z)=z+a_2z^2+a_3z^3+\cdots$ be a function regular and simple ("schlicht") in the unit circle. It is well known that $|a_2|<2$, $|a_3|<3$. With the help of an inequality due to Prawitz (Arkiv för Matematik, Astronomi och Fysik vol. 20 (1927–1928)) it is proved that $|a_4|<4.16$. It is also proved that if all the a_n are integers, then $f(z)$ must be one of the following: z, $z/(1\pm z^2)$, $z/(1\pm z+z^2)$, $z/(1\pm z)$, $z/(1\pm z)^2$. (Received August 10, 1945.)

216. Leonard Greenstone: *Some properties of conformal mappings of multiply connected domains.* Preliminary report.

The author investigates the Gaussian curvature of a non-Euclidean metric which is invariant with respect to conformal mappings of multiply connected domains. Employing Bergman's method of the minimum integral (*Partial differential equations, advanced topics*, Publication of Brown University), one can represent this curvature in terms of certain minima. Normal family methods, in conjunction with certain properties of these minima, are then employed in order to prove that if $\{B_n\}$ is a sequence of increasing non-null domains, all B_n lying in the unit circle, which sequence converges in the sense of Carathéodory to a domain B, then the curvature (of the metric) of B_n converges uniformly to the curvature (of the metric) of B. Let C be a circle which together with its boundary Γ is contained in a multiply connected domain B. The author obtains inequalities in C for the value of a function closely associated with the curvature (of the metric) of B, in terms of the values this function assumes on Γ. (Received August 11, 1945.)

217. Einar Hille: *A note on semi-groups analytic in a sector.*

Let $\mathfrak{S}=\{T(\zeta)\}$ be a semi-group of linear bounded transformations on a complex B-space $\mathfrak{X}$ to itself. Let $T(\zeta)$ be holomorphic in $-\pi/2\leqq\Phi_1<\arg\zeta<\Phi_2\leqq\pi/2$ and $\|T(re^{i\phi})\|\leqq B(\phi)$, $0<r<1$, where $B(\phi)$ is bounded for $\Phi_1+\epsilon\leqq\phi\leqq\Phi_2-\epsilon$. Then $\sigma(\phi)=\lim_{r\to\infty}r^{-1}\log\|T(re^{i\phi})\|$ exists; either $\sigma(\phi)\equiv-\infty$ or $\sigma(\phi)$ is continuous and is the function of support of a closed convex unbounded region D. If $R(\lambda; A)$ is the resolvent of the infinitesimal generator of $\mathfrak{S}$, then $R(\lambda; A)$ is holomorphic outside of $\overline{D}$, the conjugate of D in the complex λ-plane, and all extremal points of $\overline{D}$ are spectral points of A. Conversely, if A is an unbounded linear operator on $\mathfrak{X}$ to itself with domain dense in $\mathfrak{X}$, if $R(\lambda; A)$ is holomorphic outside of $\overline{D}$, all the extremal points of which are spectral points of A, if $\delta(\lambda)$ is the distance from λ to D, and if $\delta(\lambda)\|R(\lambda; A)\|$ is bounded in a certain qualified sense outside of $\overline{D}$, then A is the infinitesimal generator of a semi-group $\mathfrak{S}=\{T(\zeta)\}$, holomorphic in a sector and bounded in interior sectors near the origin, and the growth function $\sigma(\phi)$ of $T(\zeta)$ is the function of support of D. (Received September 26, 1945.)

218. Einar Hille: *On uniform ergodic theorems.*

Let $\mathfrak{S}=\{T(\xi)\}$ be a semi-group of linear bounded transformations on a complex B-space $\mathfrak{X}$ to itself. Let $T(\xi)$ be uniformly measurable for $\xi>0$, $\int_0^\infty e^{-\lambda\xi}\|T(\xi)\|d\xi$ exist for $\lambda>0$, and $(1/\eta)\int_0^\eta T(\xi)xd\xi\to x$ for all x when $\eta\to 0$. Then $T(\xi)$ is uniformly continuous for $\xi>0$ and $R(\lambda; A)=\int_0^\infty e^{-\lambda\xi}T(\xi)d\xi$ is holomorphic for $\Re(\lambda)>0$. The following assertions are equivalent: (1) $T(\xi)$ is uniformly Abel ergodic for $\xi\to\infty$, that is, $\lambda R(\lambda; A)\to P$ when $\lambda\to+0$. (2) $\lambda=0$ is a simple pole of $R(\lambda; A)$ with residue P. (3) $\lim_{\lambda\to+0}\lambda^2 R(\lambda; A)x=0$ for all x and $\mathfrak{X}=\mathfrak{R}[A]\oplus\mathfrak{Z}[A]$, $\mathfrak{R}[A]$ being closed. Further $\mathfrak{R}[A^k]=\mathfrak{R}[A]$ for all positive integers k. Here A is the infinitesimal generator of $\mathfrak{S}$, $\mathfrak{R}[A]$ the range of A and $\mathfrak{Z}[A]$ the manifold annihilated by A. If for some $\alpha\geqq 0$, $T_\alpha(\xi)$ is bounded, $0<\xi<\infty$, and (1) holds, then $T(\xi)$ is also uniformly (C, β) ergodic for $\beta>\alpha$, that is, $\lim_{\xi\to\infty}T_\beta(\xi)=P$. Here $T_0(\xi)=T(\xi)$, $T_\alpha(\xi)=\alpha\xi^{-\alpha}\int_0^\xi(\xi-\tau)^{\alpha-1}T(\xi)d\xi$, $\alpha>0$. If $T_\alpha(\xi)$ is feebly oscillating when $\xi\to\infty$, the (C, α) limit also exists. The proof uses the fact that solutions of the resolvent equation, having an n-tuple pole, are characterized by n simple limit conditions. (Received September 26, 1945.)

219. W. H. Ingram: *A boundary value problem and its Stieltjes integral equations.*

The problem $y'=(A(x)+\mu B(x))y$, $y(x_i+)-y(x_i-)=(A^*(x_i)+\mu B^*(x_i))y(x_i)$, $a\leqq x\leqq b$, $a\leqq x_i\leqq b$, $i=1, 2, \cdots$, $2y(x_i)=y(x_i+)+y(x_i-)$, $Ly(a)+Ry(b)=0$, contains as a special case the problem treated by Birkhoff and Langer, Bliss, and Reid. The matrices $A(x)$, $B(x)$ are of bounded variation on $[ab]$ but need not be defined over the set $x_1, x_2, \cdots$. The matrices $A^*(x_i)$, $B^*(x_i)$ are zero everywhere except on the set $x_1, x_2, \cdots$ and $\sum|A^*(x_i)|<\infty$, $\sum|B^*(x_i)|<\infty$. With accommodations as to form and the substitution of the Riemann-Stieltjes for the Riemann integral, the theorems on the number, distribution and reality of the critical values of the parameter μ, for a self-adjoint system as defined after Bliss, the relation between the nullity of the characteristic matrix and the multiplicity of μ, and the expansion theorem are generally preserved under extensions of the various definitions of definiteness of Bliss and Reid. (Received September 17, 1945.)

220. Hans Rademacher: *On a function related to Riemann's zeta-function.*

The function in question is $\eta(w)=\sum \exp(\rho w)$, where the sum is extended over the complex roots of $\zeta(s)$ of positive imaginary part. The sum is convergent in the upper w-halfplane and defines there an analytic function. The function $\eta(w)$ can be continued analytically and turns out to be meromorphic on the logarithmic Riemann surface. All its poles are of the first order. Their locations and residues can be fully determined. (Received August 10, 1945.)

221. Tibor Radó: *On continuous mappings of Peano spaces.*

The purpose of this paper is to discuss certain topological questions that arise in the theory of length and area. The notion of a cyclic decomposition of a mapping on a general Peano space, corresponding to the cyclic element decomposition of the middle space in a monotone-light factorization of the mapping, is first introduced and studied. Particular results are then developed concerning mappings of 2-spheres and 2-cells, the cases of especial significance in the theory of area. Finally, a general theorem on cyclic additivity of a general function $F(T)$ of a mapping T, which includes both Lebesgue area $A(T)$ and lower area $\alpha(T)$ in the sense of Geöcze, is formulated and proved. (Received August 8, 1945.)

222. W. T. Reid: *Definitely self-adjoint differential systems.*

This paper contains an analysis of the conditions that have been used by Bliss (Trans. Amer. Math. Soc. vol. 44 (1938) pp. 413–428) in defining a definitely self-adjoint boundary value problem consisting of a system of first-order linear ordinary differential equations and two-point boundary conditions. In particular, it is shown that if a differential system satisfies the conditions of definite self-adjointness with the exception of the normality condition, then there exists an "equivalent" system which is definitely self-adjoint. (Received August 10, 1945.)

223. W. T. Reid: *Integral criteria for solutions of linear differential equations.*

The following theorem is one of a set of related results established for linear ordinary differential equations. Suppose that $a_i(x)$ $(i=0, 1, \cdots, n)$ is of class $C^{(i)}$, and $a_n(x) \neq 0$ on (a, b), while $g(x)$ is integrable on this interval. If $f(x)$ is integrable on each closed sub-interval (α, β) interior to (a, b), then there exists a solution $u(x)$ of the differential equation $\sum_{i=0}^{n} a_i(x)u^{(i)}+g(x)=0$ such that $f(x)=u(x)$ a.e. on (a, b) if and only if for each such subinterval (α, β), $\int_\alpha^\beta \left|\sum_{i=1}^{n} a_i(x)h^{n-i}\Delta_h^i f(x)+h^n[a_0(x)f(x)+g(x)]\right| dx=o(h^n)$, where $\Delta_h^i f(x)$ denotes the ith forward difference of $f(x)$ with interval h. There is also given a corresponding integral criterion for harmonic functions. (Received August 10, 1945.)

224. C. E. Rickart: *Banach algebras with an adjoint operation.*

A "B^*-algebra" $\mathcal{A}$ is a normed ring (Gelfand, Rec. Math. (Mat. Sbornik) N.S. vol. 9 (1941)), not necessarily commutative, in which to each element $x \in \mathcal{A}$ corresponds a unique element $x^* \in \mathcal{A}$ with the properties: (1) $x^{**}=x$, (2) $(xy)^*=y^*x^*$, (3) $(\lambda x+\mu y)^*=\bar{\lambda}x^*+\bar{\mu}y^*$, (4) $\|xx^*\|=\|x\|^2$ (Gelfand and Neumark, ibid. vol. 12 (1943)). An element $u \in \mathcal{A}$ is called a "projection" if $u^2=u$ and $u^*=u$. A B^*-algebra is called a "B_p^*-algebra" if for every $x \in \mathcal{A}$ a projection $u \in \mathcal{A}$ exists such that $xu=x$ and $xy=0$ implies $yu=0$. Special examples of B_p^*-algebras are the factors of Murray and von Neumann (Ann. of Math. vol. 37 (1936)). The structure of a B_p^*-algebra $\mathcal{A}$ is studied here in terms of the projections in the algebra. $\mathcal{A}$ is generated by its projections. Notions of "equivalence" and "finiteness" of projections are used closely related to similar notions of Murray and von Neumann for factors. A B_p^*-algebra is simple if, and only if, all of its projections are finite. $\mathcal{A}$ is said to satisfy the "denumerability condition" if every family of orthogonal projections is denumerable. $\mathcal{A}$ is said to be "quasi-transitive" when $xAy=(0)$ implies either $x=0$ or $y=0$. If $\mathcal{A}$ is quasi-transitive, regular in the sense of von Neumann and satisfies the denumerability condition, then it is necessarily simple. Every quasi-transitive B_p^*-algebra is central. If the denumerability condition is satisfied, a central B_p^*-algebra is quasi-transitive. (Received August 11, 1945.)

225. H. E. Robbins: *Cesàro convergence of certain random sequences.*

Let $\{a_n\}$ be a sequence of numbers with $\sum a_n n^{-2} < \infty$ and which converges $(C, 1)$ to a limit α. It is shown that almost every sequence derived from $\{a_n\}$ by a random process of a certain general type, involving the dropping or repeating of terms of

$\{a_n\}$, will also converge $(C, 1)$ to α. This generalizes a theorem of Buck and Pollard (Bull. Amer. Math. Soc. vol. 49 (1943) p. 928, Theorem 2). It follows that with probability 1 certain random sequences of transformations exhibit ergodic behavior. (Received August 10, 1945.)

226. M. S. Robertson: *The coefficients of univalent functions.*

Let $f(z)=\sum_1^\infty c_n z^n$, $c_1=1$, be regular and convex in the direction of the imaginary axis for $|z|<1$ and be real on the real axis. Then the coefficients c_n satisfy the inequalities $c_{n-1}-c_{n+1}\leqq 4n(n^2-1)^{-1}\cdot(1-|c_n|)$, n even, $|c_{n-1}-c_{n+1}|\leqq 4n(n^2-1)^{-1}\cdot(1-c_n)$, n odd and greater than 1. The factor $4n\cdot(n^2-1)^{-1}$ cannot be replaced by a smaller one and the equality signs are attained by the function $z(1-z)^{-1}$. Let the function $F(z)=\sum_1^\infty a_n z^n$, $a_1=1$, be regular and univalent for $|z|<1$ and real on the real axis. Then $(n+1)a_{n-1}-(n-1)a_{n+1}\leqq 4(n-|a_n|)$, n even, $|(n+1)a_{n-1}-(n-1)a_{n+1}|\leqq 4(n-a_n)$, n odd and greater than 1, $4(1+a_2+a_3+\cdots+a_{n-1})-(n-2)a_n\leqq n^2$, $n>1$. The proofs depend upon the trigonometric inequalities $(n+1)\sin(n-1)\theta/\sin\theta-(n-1)\sin(n+1)\theta/\sin\theta\leqq 4(n-\sin n\theta/\sin\theta)$, $n^2+n\sin 2n\theta/\sin 2\theta-2(\sin n\theta/\sin\theta)^2\geqq 0$. (Received August 9, 1945.)

227. M. S. Robertson: *Univalent power series with multiply monotonic sequences of coefficients.*

The Cesàro partial sums of the first order $S_n^{(1)}(z)=\sum_{k=0}^n(n-k+1)z^k$ of the geometric series are univalent for $|z|\leqq 1$ (E. Egerváry, Math. Zeit. vol. 42 (1937) pp. 221–230). The author obtains a new proof which permits him to generalize this result to linear sums $\sum_{k=1}^m c_k S_{n+k-1}^{(1)}(z)$ with non-negative coefficients not all zero. These sums are univalent for $|z|\leqq 1$ provided that for all $\alpha, \sum_{k=2}^{m-1} B_k^m\{(\sin k\alpha/\sin\alpha)^2-k\sin 2k\alpha/\sin 2\alpha\}\geqq 0$ where $B_k^m=\sum_{s=1}^{m-k} c_s c_{s+k}$, and in particular if the sequence $\{B_2^m, B_3^m, \cdots, B_{m-1}^m, 0, 0, \cdots\}$ is monotonic of order 2. If the sequence $\{a_n\}$ is monotonic of order 2, $a_n\rightarrow 0$, and if for each $n>n_0$ the sequence $\{A_2^n, A_3^n, \cdots, A_{n-1}^n, 0, 0, \cdots\}$ is monotonic of order 2 where $A_k^n=\Delta_n^{(2)}a_{n-k}\cdot\Delta^{(1)}a_n+\sum_{s=1}^{n-k-1}\Delta^{(2)}a_s\cdot\Delta^{(2)}a_{s+k}$ then $f(z)=\sum_0^\infty a_n z^n$ is regular and univalent for $|z|<1$. This result is an extension of a theorem of L. Fejér (Trans. Amer. Math. Soc. vol. 39 (1936) pp. 18–59) and the improvement on his result by G. Szegö (Duke Math. J. vol. 8 (1941) pp. 559–564). The method of proof yields a positive lower bound for $|f(z_1)-f(z_2)|$, $z_1\neq z_2$. (Received August 7, 1945.)

228. P. C. Rosenbloom: *Convex bodies in Banach spaces.*

The notions of Minkowski functional, supporting functional, and polar body of a convex body can be generalized from finite-dimensional spaces. If E is a separable Banach space, then a necessary and sufficient condition that a convex body K^* in its conjugate space E^* be regularly convex—which is equivalent to its being the polar body of a convex body K in E—is that its Minkowski functional $K^*(f)$ be lower semicontinuous in the weak topology of E^*. We say that $K^*(f)$ is uniformly differentiable at f_0 if $\lim_{h\to 0}\{K^*(f_0+hf)-K^*(f_0)\}/h$ exists uniformly in $\|f\|\leqq 1$. If K is a convex body in E, and $f_0(x)=1$ is a supporting hyperplane of K, and if $K^*(f)$ is uniformly differentiable at f_0, where K^* is the polar body of K, then $f_0(x)=1$ has a unique point in common with K and this point is an extreme point of K. We say, in this case, that f_0 is tangent to K. If E^* is separable, then the set of points on the boundary of K^* which are tangent to K is dense on the boundary of K. This work is related to that of Mazur, Yosida, Fukamiya, Krein, Šmulian, Gantmacher, Milman, and Mackey. (Received August 17, 1945.)

229. C. A. Truesdell: *On the functional equation* $\partial F(z, \alpha)/\partial z = F(z, \alpha+1)$.

It is attempted to provide a theory which motivates and verifies many seemingly special relations among various familiar special functions. The recurrence relation $\partial F(z, \alpha)/\partial z = A(z, \alpha)F(z, \alpha)+B(z, \alpha)F(z, \alpha+1)$ satisfied by many familiar functions furnishes common ground for study. In all cases of interest this equation is reducible to the form $\partial F(z, \alpha)/\partial z = F(z, \alpha+1)$. If $\phi(\alpha)$ is bounded in a right half-plane a unique solution $F(z, \alpha)$, an integral function of z, exists such that $F(z_0, \alpha)=\phi(\alpha)$. Two solutions which agree in a right half-plane of α when $z=z_0$ agree for all values of z whether or not they are bounded functions of α. On the basis of these theorems it is possible to establish many relationships satisfied by certain classes of solutions of the F-equation: (1) power series solutions, (2) factorial and Newton series solutions, (3) contour integral solutions, (4) generating expansions, (5) definite integrals, (6) relations among various different solutions of the F-equation. Methods of discovery are stressed because the discovery of a relationship satisfied by some special function or functions is almost always more difficult and more interesting than the construction of an ad hoc rigorous proof. A number of these special relations are shown to be obtainable by substitution in general formulas. (Received August 7, 1945.)

Applied Mathematics

230. R. J. Duffin: *Nonlinear networks.* I.

A system of n nonlinear algebraic equations in n real variables is studied and shown to have a unique solution. A special case of this system is the equations which govern the distribution of current in a direct current electrical network when the conductors are *quasi-linear*. A quasi-linear conductor is one in which the potential drop across the conductor and the current through the conductor are nondecreasing functions of one another. It follows that the distribution of current among the conductors of a quasi-linear network is unique. (Received September 6, 1945.)

231. F. J. Murray: *Linear equation solvers.*

The theory and actual construction of certain devices for the solution of a system of linear equations $\sum_{j=1}^{n} a_{i,j}x_j = b_i$ is described. In these machines, the variables x_j are subject to the control of the operator and the machine indicates the value of $\mu = \sum_{i=1}^{n}(\sum_{j=1}^{n} a_{ij}x_j - b_i)^2$. The operator varies each x_j in turn to minimize this expression. This is equivalent to the Gauss-Seidel method applied to the symmetric system obtained by multiplying the given set of equations by the adjoint matrix. The expressions $\sum_{j=1}^{n} a_{i,j}x_j - b_i$ are realized (except possibly in sign) as the amplitudes of alternating current voltages by means of a combination of bell transformers and variable resistances. This can be done in a number of ways. These alternating currents are then rectified by means of diode vacuum tubes and the combined currents measured by a microammeter. The result is essentially μ. Emphasis is placed upon the possibility of amateur construction and standard radio parts are used. The cost of the part of the device associated with the coefficients is proportional to n^2. The sensitivity of modern vacuum tubes is utilized to minimize the constant of proportionality. Received August 3, 1945.)

232. H. E. Salzer: *Note on coefficients for numerical integration with differences.*