ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

194. Warren Ambrose: *Remark on measures on locally compact topological groups.*

Let G be a locally compact topological group with Haar measure m. Let m' be another left invariant measure which satisfies André Weil's condition: $(x, y) \to (x, xy)$ is measure preserving. The measure m' is a *sub-measure* of the Haar measure m if and only if every m-measurable set of positive m-measure contains an m'-measurable set of positive m'-measure. It is shown that every sub-measure m' gives rise in a natural way to a locally compact topology on G which is a refinement of the given locally compact topology, and in which G is a topological group. Strong use is made of André Weil's fundamental theorem about measure giving rise to a topology, but it is necessary to modify his topology, in general, to obtain local compactness. (Received August 8, 1945.)

195. Reinhold Baer: *Null systems in projective space.*

Extending a result by O. Veblen-Young, R. Brauer has characterized the null-systems over an n-dimensional projective space P, proving in particular that n must be odd, provided that P is the n-dimensional projective space over a commutative field of coordinates. It is the object of the present note to remove this last hypothesis. More precisely it is proved that the existence of a null-system in an n-dimensional projective space P with $1 < n$ is equivalent to the fact that n is odd and that P is the n-dimensional projective space over a commutative field of coordinates. (Received August 22, 1945.)

196. Reinhold Baer: *Representations of groups as quotient groups.* I.

If N is a normal subgroup of the group H, and if the groups G and H/N are isomorphic, consider H/N a representation of the group G. In this first part of a systematic theory of such representations the author is concerned with three problems: the classification of representations; the derivation of invariants of classes of representations; the construction of "interesting" classes of representations. (Received August 22, 1945.)

197. Reinhold Baer: *Representations of groups as quotient groups.* II. *Minimal central chains of a group.*

If N is a normal subgroup of the group G, define the subgroups ${}_iN$, ${}^iG$ inductively by ${}^iG = (G, {}^{i-1}G)$, ${}_iN = (G, {}_{i-1}N)$ where (X, Y) is the subgroup generated by all the commutators $x^{-1}y^{-1}xy$ for x in X and y in Y. It is shown that ${}^iG/{}_iN$ is finite, if G/N is finite. For G a free group, a survey is given of the manifold of the groups ${}^iG/W$ where ${}_{i+1}N \leqq W$ and ${}_iN/{}_{i+1}N$ is the direct product of $W/{}_{i+1}N$ and of $({}_iN \cap {}^{i+1}G)/{}_{i+1}N$. These groups ${}^iG/W$ may be considered to be generalizations of I. Schur's "Darstellungsgruppen." (Received August 22, 1945.)

198. Reinhold Baer: *Representations of groups as quotient groups. III. Invariants of classes of related representations.*

In the first part of this investigation the representations of a group as quotient groups have been divided into classes of related representations; and it has been shown that these classes of related representations possess a vast system of structural invariants. The present note is devoted to the investigation of the interrelatedness of these invariants and to applications of these invariants, in particular for deriving criteria for a homomorphism to be an isomorphism. (Received August 22, 1945.)

199. Reinhold Baer: *The homomorphism theorems for loops.*

This note concerns itself with the isomorphism laws, including Zassenhaus' formula, which play a fundamental role in the proof of the Jordan-Hölder-Schreier-Zassenhaus Theorem. Proofs of these theorems, of (new) converses and extensions of them are given which apply not only to classical group theory, but also to loop theory. Preference is given to working with homomorphisms, instead of doing computations with elements. (Received August 22, 1945.)

200. Richard Brauer: *A note on systems of homogeneous algebraic equations.*

Assume that the field K has the following property (A): For every positive integer r there exists an integer $\psi(r)$ such that for $n \geqq \psi(r)$ every equation $a_1x_1^r + \cdots + a_nx_n^r = 0$ with coefficients a_i in K has a nontrivial solution $(x_1, \cdots, x_n)$ in K. It is shown that for every system of positive integers $r_1, r_2, \cdots, r_h$ and for every integer $m \geqq 0$ there exists an expression $\Omega(r_1, r_2, \cdots, r_h; m)$ such that for $n \geqq \Omega(r_1, \cdots, r_h; m)$ every system of h homogeneous algebraic equations of degrees $r_1, r_2, \cdots, r_h$ respectively with coefficients in K is satisfied by all points of an m-dimensional linear manifold M, defined in K. If K does not have the property (A), a similar statement holds, if one allows that M is not defined in K but in a soluble extension field of K. The p-adic fields have the property (A). As a further application, it is shown that if l_n denotes the smallest number such that the general algebraic equation $F(x) = 0$ of degree n can be solved by means of algebraic functions of at most l_n arguments (Hilbert's resolvent problem), then $\lim_{n\to\infty} (n - l_n) = \infty$. (Received August 11, 1945.)

201. Richard Brauer: *On the representation of a group of order g in the field of the gth roots of unity.*

It has long been surmised that every representation L of a group G of order g can be written in the field Ω of the gth roots of unity, that is, that L is similar to a representation with coefficients in Ω. In the present paper, this conjecture is proved for the first time. The proof is obtained by combining the methods of I. Schur and of H. Hasse with results concerning the modular representations of groups. (Received August 21, 1945.)

202. I. S. Cohen and Irving Kaplansky: *Rings with a finite number of prime elements.*

Let R be an integral domain in which every element is a product of prime elements, of which it is assumed there are but a finite number n. Every ideal in R has a finite basis (this is proved more generally for any integral domain having a finite number of prime ideals, each of which has a finite basis), and its prime ideals $M_1, \cdots, M_h$ are all maximal. Each prime element of R is contained in just one M_i, and if a_i and b_i are products of prime elements in M_i and $\prod_{i=1}^h a_i = \prod_{i=1}^h b_i$, then $(a_i) = (b_i)$, $i = 1, \cdots, h$. The factorization problem can then be reduced to the case of a local ring. If R is a local ring and factorization is not unique, then $n \geqq 3$ and the residue class field contains at most $n-1$ elements; the completion of R is free from zero-divisors and has exactly the same primes as R. The multiplicative structure of R is explicitly determined for $n=3, 4, 5$. (Received August 10, 1945.)

203. I. S. Cohen and Abraham Seidenberg: *Prime ideals and integral dependence.*

Let $\mathfrak{R}$ and $\mathfrak{S}$ be commutative rings, $\mathfrak{R} \subseteq \mathfrak{S}$, with the same identity element, and let $\mathfrak{S}$ be integrally dependent on $\mathfrak{R}$. The following theorems are proved: (1) If $\mathfrak{p}$ is a prime ideal in $\mathfrak{R}$, then there exists a prime ideal $\mathfrak{P}$ in $\mathfrak{S}$ such that $\mathfrak{P} \cap \mathfrak{R} = \mathfrak{p}$. (2) If $\mathfrak{q}$ and $\mathfrak{p}$ are prime ideals in $\mathfrak{R}$, $\mathfrak{q} \subset \mathfrak{p}$, and if $\mathfrak{Q}$ is a prime ideal in $\mathfrak{S}$ such that $\mathfrak{Q} \cap \mathfrak{R} = \mathfrak{q}$, then there exists a prime ideal $\mathfrak{P}$ in $\mathfrak{S}$ containing $\mathfrak{Q}$ such that $\mathfrak{P} \cap \mathfrak{R} = \mathfrak{p}$. (3) Assume now that no element of $\mathfrak{R}$ is a zero-divisor in $\mathfrak{S}$ and that $\mathfrak{R}$ is integrally closed. If $\mathfrak{q}$ and $\mathfrak{p}$ are as in (2) and if $\mathfrak{P}$ is a prime ideal in $\mathfrak{S}$ such that $\mathfrak{P} \cap \mathfrak{R} = \mathfrak{p}$, then there exists a prime ideal $\mathfrak{Q}$ in $\mathfrak{S}$ contained in $\mathfrak{P}$ such that $\mathfrak{Q} \cap \mathfrak{R} = \mathfrak{q}$. The present proofs are simpler than those of Krull (Math. Zeit. vol. 42 (1937) pp. 745–766), which in addition assume that $\mathfrak{R}$ and $\mathfrak{S}$ are integral domains. (Received August 10, 1945.)

204. Jakob Levitzki: *Chains of multipa and semi-primarity.*

A descending chain of right ideals $R_1 \supset R_2 \supset R_3 \supset \cdots$ of an arbitrary ring S is called a chain of multipla, if for each n there exists a right ideal R_n' for which $R_{n+1} = R_n' R_n$. As in the classical theory of ideals, each descending chain of right ideals in a semi-simple ring is a chain of multipla. A slight modification of the usual argument leads to the following statement: A ring S is semi-simple if and only if (1) S is without nilpotent ideals and (2) each chain of multipla in S is finite. The corresponding theorem of E. Noether assumes instead of (2) the finiteness of each descending chain of right ideals. It is further proved that from condition (2) alone follows that S is semi-primary (that is, the radical N of S is nilpotent, and S/N is semi-simple). A necessary and sufficient condition for semi-primarity is obtained by assuming condition (2) for the right ideals which either contain or are contained in the radical of S. For the truth of this last theorem it is irrelevant which of the various existing definitions of the radical is chosen. (Received September 1, 1945.)

205. Ivan Niven: *A remark on Gaussian integers.*

The author has shown (Trans. Amer. Math. Soc. vol. 48 (1940) p. 410) that every Gaussian integer of the form $a+2bi$ is expressible as a sum of three squares, and has given necessary and sufficient conditions for a sum of two squares. The question is now raised, how many representations are possible? It is shown that any $a+2bi$ has an infinite number of three square, but a finite number (with formula given) of two square, representations. (Received August 20, 1945.)

206. W. V. Parker: *The characteristic roots of matrices.*

If λ is a characteristic root of a matrix A, and if $\rho_1^2 \geqq \rho_2^2 \geqq \cdots \geqq \rho_n^2$ $(\rho_i \geqq 0)$ are the characteristic roots of $A\bar{A}'$, Browne has shown that $\rho_1 \geqq |\lambda| \geqq \rho_n$. In this paper it is shown that for any complex number μ such that $\rho_1 \geqq |\mu| \geqq \rho_n$ there exists a matrix B which has the characteristic root μ and is such that $B\bar{B}' = A\bar{A}'$. Consideration is also given to the distribution of the roots of B within the annular region. (Received August 10, 1945.)

207. Edward Rosenthall: *On the sums of two squares and the sum of cubes.*

In this paper the complete rational integer solution of each of the diophantine systems $x_1^2+y_1^2=x_2^2+y_2^2= \cdots =x_n^2+y_n^2$ and $(x_1^2+y_1^2)(x_2^2+y_2^2) \cdots (x_n^2+y_n^2)=(u_1^2+v_1^2) \cdots (u_m^2+v_m^2)$ is obtained in terms of integral parameters. Also, a method is given for finding all the sets of rational integers satisfying the diophantine equation $x_1^3+x_2^3 + \cdots +x_{2n}^3=0$. This method reduces the resolution of this equation to a system of linear homogeneous equations in which the number of unknowns always exceeds the number of equations. The above results are deduced from the complete integer solution in the quadratic fields $Ra(i)$, $Ra((-3)^{1/2})$ of certain connected simple and extended multiplicative equations. (Received September 4, 1945.)

Analysis

208. R. P. Agnew: *A simple sufficient condition that a method of summability be stronger than convergence.*

Let $\sigma_n = \sum_{k=1}^{\infty} a_{nk}s_k$ be a regular Silverman-Toeplitz transformation by which a sequence $s_1, s_2, \cdots$ is summable to σ if $\sigma_n \to \sigma$ as $n \to \infty$. It is shown that if $a_{nk} \to 0$ as $n, k \to \infty$, then some divergent sequences of zeros and ones are summable. A more general theorem applies to transformations which are not necessarily regular. If (i) $\sum |a_{nk}| < \infty$ for each fixed $n=1, 2, \cdots$ and if (ii) as $n \to \infty$, the maximum for $k=1, 2, \cdots$ of $|a_{nk}|$ converges to 0, then some divergent series of zeros and ones are summable. The theorems furnish criteria for determination of relations among methods of summability. (Received August 20, 1945.)

209. R. P. Agnew: *Characterization of methods of summability effective for power series inside circles of convergence.*

A matrix b_{nk} of real or complex constants determines a series-to-sequence transformation $\sigma_n = \sum_{k=0}^{\infty} b_{nk}u_k$ by means of which a series $\sum u_n$ is summable B to σ if $\sigma_0, \sigma_1, \cdots$ exist and $\lim \sigma_n = \sigma$. In order that the matrix b_{nk} be such that $\sum u_n$ is summable B whenever $\sum u_n z^n$ has radius of convergence greater than 1, it is necessary and sufficient that (1) constants $\beta_0, \beta_1, \cdots$ exist such that $\lim_{n\to\infty} b_{nk} = \beta_k$ when $k=0, 1, 2, \cdots$ and (2) to each number θ in the interval $0<\theta<1$ corresponds a constand $M(\theta)$ such that $|b_{nk}\theta^k| < M(\theta)$ when $n, k=0, 1, 2, \cdots$. If (1) and (2) hold and $\sum u_n z^n$ has radius of convergence greater than 1, then $\sum \beta_n u_n$ converges absolutely and the number $B\ \{\sum u_n\}$ to which $\sum u_n$ is summable B is $\sum \beta_n u_n$. In order that $B\{\sum u_n\} = \sum u_n$ whenever $\sum u_n$ has radius of convergence greater than 1, it is necessary and sufficient that (1) and (2) hold with $\beta_k = 1$ for each k. (Received August 3, 1945.)

210. Joshua Barlaz: *On some triangular summability methods.*

A class of triangular sequence-to-sequence summability methods is given by the transform $t_n = e^{-x_n} \sum_{\nu=0}^{\infty} s_\nu x_n^\nu / \nu!$, $n=0, 1, 2, \cdots$, where $\{x_n\}$ is a sequence of numbers