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HOW MACHINES CAN MAKE MATHEMATICS MORE CONGRESSIVE

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Abstract. I argue that since technology is already changing the way we do mathematics, we can use this technology to allow mathematics to be more congressive without human mathematicians becoming redundant in the face of technological advances. In thinking about what it means to "do mathematics," I examine the following aspects of technology in mathematics: teaching and learning, asking questions, collaboration, dissemination, and the act of doing research. This is not intended to be a rigorous analysis but an informed reflection based on my experience as a mathematician.

Introduction

When I was a child our teachers relished telling us that we wouldn't always have calculators in our pockets. This dire warning was supposed to convince us how important it was to become proficient at mental arithmetic. That reasoning turned out to be fatally flawed: most of us now have a smartphone with us all the time, which not only gives us a calculator but also access to the internet and therefore almost the entirety of human knowledge.

People across history issued dire warnings about the effects of new technology. Even if we aren't historians (which I'm not) we can see traces of these dire warnings in fiction from the past. In *The Winslow Boy* by Terence Rattigan [R], Arthur Winslow is appalled that his son has brought a gramophone into the drawing room. Going further back, *Pride and Prejudice* by Jane Austen [A] depicts Mr Collins's distaste for circulating libraries, by which means young ladies could access—heaven forbid—works of fiction. And going further back still, Plato [P] decried the promotion of writing and reading, fearing that this would have deleterious effects on what he considered to be the important intellectual skill of memorization.

By contrast, in current times gramophones have passed into normality and already exited into outdatedness, libraries are a critical resource promoting equity and access to books for people of all backgrounds, and literacy is considered to be a basic need for participation in society, as well as a way of measuring the level of basic development of a society.

So where did these unnecessarily dire warnings come from? Were they merely a product of general skepticism and fear of new technologies, and can we separate out the genuinely harmful aspects of new technology from the potential benefits? There are many instances when we fear that gaining X will cause us to lose Y, when in reality the only reason we ever needed Y was because we didn't have X. I don't have much use for mental arithmetic now that I can just whip out my phone to do

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the calculation, but I still meet plenty of pushback when I say that I don't think memorizing multiplication tables is very important. A common wisdom is that we should not throw out the baby with the bathwater, but equally we should not keep the bathwater after the baby is gone.

The question is: how can technology help mathematics, and does this outweigh the perceived drawbacks? I am going to argue that technology is already changing the way we do mathematics, making certain old skills redundant. Rather than lamenting the loss of those skills, I celebrate that this allows more people into mathematics. I am going to argue that technology can make mathematics more inclusive. I will look at some case studies of how technology has already changed our mathematical practices, examine why there is resistance to this change, and explain why I am not afraid of human mathematicians becoming redundant in the face of technological advances.

Congressive mathematics. In [C1] I coined the terms "ingressive" and "congressive" to describe character traits and behaviors that I have seen to be pertinent to mathematics, education, and broader society. "Ingressive" behavior is individualistic and competitive, where "congressive" behavior takes the community into account and recognizes the interconnectedness of all people and things. Ingressive math is about being first and fastest, getting the right answers, and selecting the most brilliant people, whereas congressive math is about exploration, discovery, connections, and bringing more people in. Ingressive students are reassured by the idea of "right" and "wrong" answers, are convinced they are right, and enjoy believing they are more brilliant than others. By contrast, congressive mathematicians are drawn to open-ended problems which they think about for long periods of time, perhaps years. They worry that they are wrong, and suspect that they are stupid, all of which is likely to make them better mathematicians.

I believe that the world, education, and particularly math education has been rewarding ingressive behavior for far too long at the expense of congressive behavior which is arguably more beneficial to society and to mathematics. Using this terminology has helped me to focus on ways to improve my teaching, and to think about ways in which we could build a better and more inclusive math community. In this article I will focus on ways that technology has already helped to make math more congressive and could continue to do so, to the benefit of us all.

Redundant skills. Technology has already been changing mathematics by taking over some of the more routine parts so that humans don't have to do those anymore. We don't need human computers anymore. We don't need people to sit down and spend months of their lives calculating some more digits of π. When I was studying statistics in high school I became very adept at using the little book of log tables and normal distributions, but that skill is now completely redundant, and I'm glad.

We must be careful not to fall into a default stance that "it was all better in the old days" or to continue believing in skills that have become redundant. I hope we can all agree that it is no longer at all important to know how to use a slide rule, but there is much greater controversy around whether it's important to be good at mental arithmetic, or indeed to memorise facts—about math or anything else.

I have always been skeptical of the memorization of facts and deemed it not to be a very worthwhile intellectual activity. In fact, this is one of the reasons I always loved math, because it was the one subject in which I felt I didn't have to memorize

anything, just understand it. My skepticism of this form of intelligence was soon vindicated by the advent of the internet, which means we don't need to hold facts in our brain because we can look them up at any moment. Now a new skill has become much more relevant: the ability to find good information online, which is really the ability to assess what counts as good information and what doesn't. Perhaps it's expected that those who spent a lot of time developing old skills resist a world in which those skills become redundant, but I strongly believe we should resist people who are motivated by that sort of fear. Fortunately, there are plenty of people who have different values, and who are not just motivated by their own personal dominance.

Background on my view of technological advances. It is sometimes assumed that older people have an inherent resistance to new technology—that young people embrace whatever is new when they are young, before becoming resistant themselves when they are older and set in their ways. However, I grew up in a tech-forward family. My mother was among a fairly early wave of computer programmers, and her first job was to train bank workers in the first implementation of computer systems for banking. She taught me how to program when I was four, and a few years later I was, I was told, the first person at my school to submit an essay that was typed on a computer.

To this day, my parents—despite (obviously) being older than me—remain ahead of me in terms of technology, and so I know that technophilia is not just proportional to age. My parents had cellphones and then smartphones before me (my first was a hand-me-down from them). I say this to try and describe where I sit in between the extremes of technophobe and technophile. I know people at both extremes: those who immediately buy every iteration of an iPhone, and those who still don't have any kind of mobile phone, not even a flip phone. The latter say that they don't want to be contactable all the time, and I have sympathy with that. It took me a while to work out what the benefits were, and how to get them without suffering what I perceived as the disadvantages. This is also key to thinking about how machines and technology in general are changing mathematics, and what we might do about it.

Outline. In order to think about how technology can change how we do mathematics, we need to think about what it means to "do mathematics". I will look at a few case studies that fall broadly under the following headings as I examine different aspects of technology in mathematics:

(1) Teaching and learning
(2) Asking questions
(3) Collaboration
(4) Dissemination
(5) The act of doing research

These are, of course, not separated by clear boundaries, but I will use this as a guide as I examine different aspects of technology in mathematics, and then draw what conclusions I can. This essay is not trying to be a rigorous analysis but an informed reflection based on my experience of being a mathematician, and my observations of the community around me.

1. Teaching and learning

When I first started teaching I thought hard about lectures: what they're for, how they work (or don't), and how to deliver them most effectively. The main thing I realized was that I wanted to give the students something more than they would get from reading a book. The origin of lectures is from before the technology of printing presses, or at least before books became widely accessible. Therefore, the only way for a large group of people to access the information in a book was to turn up to a "lecture", literally a reading, where someone would stand at the front and read the book out.

This is an example of where human practice did not develop fast enough with the technology. Hundreds of years later books had become widespread, but plenty of my undergraduate lectures at the University of Cambridge consisted more or less of a lecturer just reading out notes and writing them on a chalkboard while we all furiously copied them down.

Technology has now developed further. Back when I started teaching, I had been dreaming of writing a textbook for category theory with a DVD at the back, a bit like language books that came with a video (or a cassette tape) so that we could learn from recordings of native speakers. I wanted videos for category theory because the way diagrams are used is very dynamic and it's hard to show that in a fixed printed text book. At the time I started thinking about this, it was prohibitively expensive to make a video, let alone turn it into a DVD and distribute it. But a few short years later YouTube came along, and it suddenly became possible to make videos at very little expense and distribute them to anyone in the world for free. I started making YouTube videos explaining math, and I was surprised by their popularity. I realized I needed to change my baseline for lecturing. It was no longer enough to be more than a book—it needed to be more than a video.

Some lecturers fear that if we make notes and videos for our students then we will all become redundant and have no job. However, I firmly believe that if technology can do my job for me then I shouldn't prevent its implementation just to keep my job. But I also believe that there always will be things I can do that technology can't. This simultaneously makes me not fear being made obsolete by technology, and also makes me actively relish the idea of letting technology do everything it can, saving my human skills for where they are really needed.

I think that, aside from the fear of being made redundant, there is a fear of change that is valid for teachers. Most teaching jobs come with no incentive to make changes to courses, and indeed disincentives to do so. That is, there are no material incentives (pay, promotion) as teaching is rarely valued in material ways compared with research papers and large grants. And it takes so much initial work to prepare a new course that it is only really worth it if we can keep teaching it the same way for some years. So the incentive has to be moral instead. One has to believe in investing time to make a better experience for students.

I moved away from standard "chalk-and-talk" teaching because it was not working for my students as they were not good at taking notes. We might lament the dying art of real-time note taking, but now that everything can be preserved on video it's not clear that real-time note taking is a skill worth clinging onto. Standard chalk-and-talk teaching is inherently ingressive as it consists of one person standing at the front and delivering a large quantity of information. My baseline

of bringing more value than a video has pushed me to teach in a much more interactive way, with active learning from the students, more group activities, and more discussion. It's true that I am now teaching liberal arts math to art students, rather than rigorous advanced math to math majors, but technology has both pushed and enabled me to teach math more congressively, to help those who have been put off ingressive math in earlier parts of their life.

2. Asking questions

One important aspect of learning mathematics is asking questions. I think about this both from the point of view of the asker and the "askee". In [C2] I wrote about devising a more congressive form of question time at the end of talks. I found that ordinary raised-hand question time was too ingressive, and, as a result, heavily skewed towards questions from white men, while congressive people (largely women) would privately ask me much more interesting questions afterwards. I tried to use technology to overcome those ingressive influences, but without much success—in fact I found that the technology even exacerbated them. That is, using online question submission technology encouraged ingressive people to ask even more obnoxious questions, and did not encourage congressive people to speak up. I found that it was better to use humanity to overcome those problems—that is, I now go round the room and chat to people face to face to hear their questions.

The possibility of technology perpetuating ingressive structures in the rest of society is something we need to be careful about, even as tech appears to be opening up new avenues for participation. When it comes to asking questions, anyone trying to learn math can now ask questions and get answers from anywhere around the world, on forums such as MathOverflow and Mathematics Stack Exchange. This has qualitatively changed the way we can learn and do research, but we need to be careful because it hasn't necessarily changed the hierarchical, competitive, and offputting social structures, the ones I call ingressive. I personally do read and benefit from answers to other people's questions on those sites, but after a few offputting incidents early on, I declined to participate actively.

I have recently launched what I hope is a more congressive forum for people to ask questions, in the form of a virtual book club for *The Joy of Abstraction* [C3], which is an introduction to category theory without requiring any formal math background. The book club is hosted by the Topos Institute, and the format is that anyone can submit questions anonymously, which I then answer in a video, chapter by chapter. The anonymity is to help people overcome the fear of being thought stupid, and also to remove the incentive to ask questions that sound impressive, rather than questions coming from a genuine desire to understand something. Unfortunately, anonymity seems to bring out the worst in people online, so we also have a crucial intermediate step where the questions are moderated by someone at the Topos Institute, so that I don't need to see anything that is rude or unconstructively attacking me. I also take steps in my responses to be encouraging about congressive questions, and temper the negative effects of questions that might be showing off.

I am very sympathetic about the difficulty of asking questions in ingressive environments, because it's something I struggle with myself. I have developed ways of using technology when I'm in the audience of a talk to engage more with the presentation, to get more out of it, and to be able to dare to ask more questions. I download related papers and, if possible, the slides for the talk so that I can go

back and forth even if the speaker has moved on. I look up as many things as possible when I need more explanation than the speaker has given. A different kind of person might just ask the speaker out loud but I have never been bold enough to do that, and tech saves me from having to be. Then, finally, if I have done a deep search online during the talk and I still have a question, I feel more confident that my question isn't "stupid". The tech helps me, but it is only needed if the environment is somewhat ingressive in the first place. If the community were just more welcoming of questions about math, perhaps we wouldn't need tech to intervene to help individuals overcome those barriers.

3. Collaboration

Technology has made research collaboration much easier, from a sheer logistical standpoint. Gone are the days of handwritten letters moving slowly across the world between collaborators. Email already made collaboration at a distance much faster, but collaborative blogs and wikis have changed it more qualitatively. Ideas can be shared in a format that is much less formal than a research paper or even a research seminar, perhaps more akin to a chat over a coffee in a common room. Indeed, the major collaborative blog in category theory is called the n-Category Café. People can share research ideas and get almost instant feedback from around the world.

Technology means that we can be in a research group without having to be geographically together. Having a local research group is one thing I miss about being part of a standard math department, since I quit my tenured job for a "portfolio career" in 2013. But then we had a pandemic and we all figured out how to talk to each other remotely whether we were geographically in the same city or not, and I suddenly realized that I could use the same technology to collaborate with people on research. I used to think it would only work if we were sitting in the same place and could see each other's writing, but webcams and shared whiteboards make up for that. It's a shame that it took a pandemic to get us there, just like it took a pandemic for some universities to realize that large lectures with more than 100 students might as well be a video.

Whether we're collaborating at a distance or not, tech can help with other practical aspects of research. We now have the possibility of essentially infinite pieces of paper, instant sharing of notes, and copying and pasting for large diagrams, for example. I remember when it felt very tech-savvy to be able to take photos of a blackboard we had been working on rather than copying it onto paper afterwards. But now we collaborate on virtual whiteboards and everything is instantly saved. Moreover, every time we go back to it, our notes from last time are still there, so we never even have to erase anything.

Mathematics research has the potential to become increasingly congressive as a result of tech improving the possibilities for both collaboration and dissemination.

4. Dissemination

Technology has also dramatically changed how we write—and indeed publish—papers, as the internet means that information has been democratized. There are still gatekeepers for academic journals, but the journals no longer have a monopoly on how mathematical writing can be disseminated around the world. This has resulted in what I consider to be a very productive grey area in the publication

process. Previously, it was very black-and-white, all-or-nothing, but now there are many levels of sharing on the way to formal publication. There have always been many levels of talk, from local departmental seminars to conferences of gradually increasing scopes. Now there are also many levels of publishing, from informal personal blogs to more carefully curated shared blogs, and we mostly all share our papers online in preprint form before they're formally published. As a result, I would argue that formal publication is not really part of the dissemination process anymore. We have a much more inclusive and equitable model with the arXiv, and we mathematicians all know that this is actually how people access papers. In reality the crucial moment for dissemination of our papers is when we put them on the arXiv, not when they are officially published.

The official publishing is there for filtering and status—that is, gatekeeping. The idea is that we need an official system of prestige recognition so that we know which papers are important and so that institutions can in turn decide who is prestigious and deserves jobs, promotions, and grants. The connotations of prestige came from the competition involved with getting published in various journals, and that came from the natural limits on what they could or would publish in the days before technology.

But in the current era this competition is fabricated. Technology means there are much better ways to disseminate our papers, and technology means that publishing doesn't have to be expensive, as has been shown by new online journals. So where is the competition coming from? I personally think the only reason we need it any more is because we haven't dared to let go of it. Math, and certainly individual fields of pure math, is not large enough or prolific enough that we really need extreme filters on what papers are made available. So we just need some system of "prestige labels" for the purposes of job applications, promotions, and grants. But that doesn't have to be journals. We could have exactly the same system of peer review but without the journals.

In my experience, mathematicians who are wedded to the old system are either too successful in it to admit there's anything wrong with it, or too afraid of how we would transition and what we might lose along the way. For example, if experimental scientists continue with the old method of journal prestige, will mathematicians lose out on grant money by giving up on it? As a result of such fears we have become stuck in an outdated system that has not made any sense for some years, not to mention that we (universities) are paying ludicrous amounts of money to profit-making academic publishers who take all our work for free and then charge us money to access it.

Not only do mathematicians do the research and write the papers, we now also typeset the papers, as well as referee and edit them, all without being paid by the publishers. Typesetting is a skill that mathematicians now need, and in some fields they need particular advanced skills. I don't think it's a coincidence that the development of diagrammatic reasoning in category theory has gone in parallel with the development of mathematical typsetting so that we can actually publish those diagrams. In the early stages of my career I developed advanced skills in writing talks on transparencies, with colour coding and a complex system of overlay. Those skills are now completely redundant, but I've replaced them with, I think, quite profound skills in LaTeX and PSTricks. As category theory is so dependent on diagrams, being able to typeset complex ones is much more useful to me than

having a good memory, especially as it means I can dream up complex diagrammatic calculations without being daunted by the fear of having to typeset them later.

This does all mean that mathematicians do even more free work than they used to, but it is perhaps an improvement on the recent past, even quite late into the twentieth century, when mathematicians (who were mostly male) wrote papers by hand that were typed up by their secretaries (who were typically female, and were sometimes the mathematicians' wives).

I think we could use technology much more radically in how we write papers. Terseness used to be paramount for mathematical writing, but now space doesn't matter any more, so we could provide much more explanation in papers. Some people object to exposition (I have had plenty of referee's reports complaining that I explain too much) but with electronic papers I dream of a "telescoping" version where the reader can expand exposition if they need it, and hide it where they don't.

Technology could help us make the refereeing process more congressive, to make it more interactive rather than a stark one-off exchange of a report and then revisions. That was the most practical approach when communication was slow (by snail mail) but now that communication is instant we could have a much more progressive exchange, so that instead of a referee bashing their head against a paper that is hard to understand, they could ask for clarification from the author as they go along, and the author could make gradual revisions instead of making them all at once.

Technology has facilitated new journals that are entirely online and free to access. The *New York Journal of Mathematics* launched in 1994 and *Theory and Applications of Categories* in 1995, both free and online from the start, and many other new journals have since followed. I hope we can use technology to change the entire structure of the publishing system before too long.

Another important aspect of dissemination is conferences. The pandemic finally forced us to work out how to have conferences remotely, but there are pros and cons to this. It is better for the environment if we don't fly, and it is helpful for those who lack the funds to travel, or who can't get away from caring duties, or who can't travel for health reasons. However, the talks themselves are only a small part of a conference, and the rest is about the informal interactions in between talks. Moreover, part of the value of a conference is exactly the act of going away from home, and being isolated for a week or so with nothing to do except think about math and go to talks. All that is lost with a virtual conference.

I was grateful to be able to give public talks virtually during covid, including giving talks all round the world without ever leaving my kitchen. I could reach more people that way, because potentially anyone in the world with an internet connection could attend the talk. But I still believe I can help people more in person. I can reach them more powerfully.

5. The act of doing research

Technology has dramatically changed how I do research. At a prosaic but very crucial level, having a smartphone means I can do research anywhere. I have always loved being able to do my very abstract research with just a pen and paper. I love doing it in cafes, bars, trains, on the beach. In the old days the only trouble was that if I needed to look something up I'd be stuck. I used to sit in a cafe where

I knew there was a pay phone, and if I needed to look something up I'd phone a friend who was at a computer, and ask them to look it up. Now I can look things up wherever I am, using my phone. This is my way of making this technology work for me: I have a smartphone but keep it permanently on silent with no notifications. In this way I can look things up, and contact people if I need to, and be "at work" wherever I like, but not have to be disturbed.

Of course, a crucial part of this is that technology has changed what it means to "look things up". Now that all our journals are online, including historical ones, I have not consulted a physical math journal for at least fifteen years. Knowledge used to be power, but it isn't any more; the ability to search the internet and distinguish between the hits is much more important. I am in awe of mathematicians with encylopædic knowledge of the literature, but I mostly make up for it with good skills at internet searching. More critical for me is an awareness of which sorts of category theorists work on which sorts of things. I don't have much memory of facts, but I have a good memory of the general sense of conference talks (without any of the details) and this means I can home in on what I need without having to remember exactly what it is. I appreciate the fact that search engines allow us into mathematical research without needing encyclopædic brains. This allows more people in. I was never very good at remembering definitions and references, and now it doesn't matter.

However, the myth of memory being important for math endures. We need other skills instead, and these are the more profound intellectual skills of math research. Thus far I have really only addressed ways in which tech has changed the *business* of math, not the math itself. Has technology changed the intellectual parts of math? Can it do so in the future, and should we embrace that?

While there are now some computer-aided proof checkers and even proof generators, I don't think technology has really yet encroached on the deepest, most creative, and therefore most human aspects of math research. The deep creative parts involve coming up with ideas in the first place—ideas for definitions, ideas for proofs, ideas for making connections between different parts of math, ideas for new ways to express things, ideas for notation and terminology (which is surprisingly critical sometimes), ideas for diagrammatic reasoning, and ideas for visual representation. To get machines to do math research we have to figure out ways to tell them to do it, and if we don't know how to do it ourselves yet then it's hard for us to tell them how to do it. Machines can do a certain amount of proof checking, but secretly mathematicians all know that we don't write completely rigorous proofs—we make arguments based in logic and backed up by logical steps of a size that we think our peers will be able to fill in. We don't have definitions of what those sizes of step are, so it would be hard to tell a machine to do it. And generating proofs is a whole different skill from just checking them, as any math student knows. Being able to follow someone else's proof is a great deal easier than coming up with a new one yourself.

None of this is to say it will never be possible for computers to surpass us in mathematical research abilities. Perhaps machines can learn how to do it, just as humans have to, or perhaps do it differently. After all, machines have been learning how to play chess, and it is key that they don't do it in the way humans do: they have a much bigger capacity for searching through all possible moves and their consequences. Human mathematicians would probably not get very far if they

simply searched through all the possible logical consequences of what is currently known now in order to try and come up with new math. Coming up with new math requires leaps of imagination, guesswork, gut feeling. Understanding what that means enough to get computers to do it is a very interesting idea but I can't imagine it happening very soon. On the other hand, perhaps machines will learn to do differently. Humans went wrong trying to make flying machines because they started out trying to make them flap their wings like birds. They also went wrong trying to make mechanical hearts by starting out trying to make them beat like an organic heart. In both cases the key was to realise that machines have different strengths and weaknesses, so emulating nature might not be the best way to get a machine to do something.

6. Conclusions

There is a certain amount of fear that artificial intelligence is going to start taking creative jobs away from people, now that there are AI bots that are reasonably accomplished at writing and really quite impressive at digital art. But technology has always been taking people's jobs. I was once told off by a train conductor for buying my ticket online, as I was "putting ticket agents out of work". Travel agents have basically almost disappeared now that most of us book our plane tickets online. Development of camera film has almost gone as well. But I believe that if machines can take over human work we should think about a universal basic income, rather than prevent technology doing things just to keep people in work.

Technology can be a democratizing force and that is good and bad, but I firmly believe that the good outweighs the bad. Technology lets people in without so many gatekeepers in the way, just like actual democracy does—or should. And just like actual democracy, it's highly imperfect, and other side effects arise from not having gatekeepers. But the principle of inclusion is so important that we must find ways to minimize the bad side effects.

One possible ingressive response to problems with technology—other than charging ahead disregarding those problems—is to legislate against it. This might happen at the level of actual law or just at the level of rules in education, such as making rules and punishments to stop people using AI to generate essays, or to stop students looking things up online during a test. Perhaps there is a more congressive approach that does not attempt to prevent the advance of technology, but still takes into account its human, environmental, economic, or political implications. We could accept the technology and acknowledge that if everyone always has access to it then it is now a valid part of every human endeavour. Perhaps we don't need to test students' abilities to write without the help of AI, because they'll always have access to AI. I'll be happy if AI can do more and more things and take over the more boring tasks we used to have to do, like knowing our way around a city or memorizing phone numbers. I would rather save my brain for much more interesting things and not hold onto skills that technology has made redundant.

We should not hold onto the bathwater just to keep the baby, but rather, we should work out how to extract the baby and get rid of the bathwater. We might even find that the baby has grown up and walked away.

What if one day machines can do actual research in category theory better than I can? I believe I'll be delighted to let the machines resolve the research problems I've been grappling with for years. What if machines can explain mathematics

better than I can, so that my skills as a teacher are also no longer needed? What if machines can teach math so well that nobody is excluded from mathematics and nobody has math phobia?

In that case I think I'll retire from math and become a pianist, with great joy.

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