Advice to the Reader

The original idea of this book was to convey to the reader what we know about algebraic geometry codes, starting from the very beginning and up to the most recent results. By no means is this goal achieved. The twenty-five-year domain of several hundred published papers can hardly be fully explored in a single book. That is why we have restricted ourselves to a textbook for a reader already working in or planning to plough this or adjacent fields. We could not expect the reader to be acquainted with either algebraic geometry or coding theory. In the first two chapters we briefly explain necessary results. These chapters are not supposed to be textbooks in those fields, but we hope that a diligent reader seeing one or both of these theories for the first time may still get an adequate feel of what they look like and will continue his way in the chosen direction.

We should stress the role of exercises dispersed everywhere in the book. They are numerous and constitute an organic part of the exposition. Their formulations are to be read and studied together with other statements of the book. If the reader has no desire to solve an exercise, he should just consider it to be one of the propositions given without proof. It is, however, highly recommended that you solve a certain number of exercises, this being important to acquire firm knowledge of the material. A large number of them are quite accessible even for an inexperienced reader.

The authors cherish the hope that they are able to solve all the exercises. Those we cannot solve are also presented but called problems (see, however, the footnote on p. x of the Preface).

The book is designed for several categories of readers.

If you are a specialist in codes interested mostly in fast knowledge of what algebraic geometry codes are, then we advise you glance through Secs. 1.1 and 1.2, just to rub shoulders with our terminology, which often differs from the standard one. Sec. 1.1.2 should be studied in more detail. Read Secs. 2.1.1–2.1.3, 2.2.1, 2.2.2, and Sec. 2.5. Then try to read at least a part of Secs. 2.4 and 3.3, then Secs. 3.4.1 and 3.4.2. After that you are ready to pass to the fourth chapter, to algebraic geometry codes themselves. You will surely be able to read Sec. 4.1 (except maybe for what concerns self-dual codes in Sec. 4.1.2) and Sec. 4.5 (except for Sec. 4.5.2). To understand what is written about self-dual codes in Sec. 4.1.2, you need Secs. 2.1.4; to understand Sec. 4.4.2, you need Sec. 2.4.1 and 2.4.2 and a part of Sec. 3.3; before you read Sec. 4.5.2, you need to study Secs. 2.1.4 and 3.2.4. This way gives you a rather thorough acquaintance with algebraic geometry codes at little expense. Then you can briefly glance through the rest of the book.

If you are a specialist in codes interested first of all in asymptotic problems of coding theory, then it is reasonable to start by looking through Secs. 1.1 and 1.2 and a more intense reading of Sec. 1.3, containing what is somewhat hidden in most books on coding theory. Then read Secs. 2.1.1–2.1.3 and 2.2.1, then Secs. 2.2.2 and 3.2. Now you are ready to read Sec. 4.5 (to understand Sec. 4.5.2, you also need Secs. 2.1.4 and 3.2.4). All this gives some insight into the asymptotic possibilities of algebraic geometry codes. Then we advise you to take a look at the rest of the book.

For an algebraic geometer interested in algebraic geometry codes as a new area to which to apply the algebraic geometry, we advise reading thoroughly Chapter 1 and then glancing at Secs. 2.1.1–2.1.3, 2.2.1, and 2.2.2 and studying those where you feel yourself less confident. Then read Chapter 4 starting from Sec. 4.1.1; after that, just read this chapter choosing one of the scenarios described above according to your interests.

If you are an algebraic geometer looking for new problems in your domain generated by algebraic geometry codes, then after Chapter 1 it is advisable to look through Chapter 2, concentrating on the material you are less familiar with. Then study closely Chapter 3. Then read Secs. 4.1 and 4.3 and pass to Secs. 4.5 and 4.6.

If codes are of no interest to you at all, after glancing through Chapter 2 you can then concentrate on Chapter 3. In this way the book can serve you as a textbook on algebraic geometry over a finite field.

Now we address the largest audience. If you are not a specialist in either codes or algebraic geometry and you wish to go to their meeting-point as soon as possible, we recommend the following course: Secs. 1.1.1, 1.1.2, and 1.2.1; then Secs. 2.1.1–2.1.3, 2.2.1, 2.2.2, Secs. 3.1 and 3.4. In the fourth chapter first read Secs. 4.1.1, 4.1.2, 4.3.1, 4.4.2, and 4.4.3. Now you are acquainted with algebraic geometry codes. Then read the rest of Chapter 4, turning when necessary to those parts of Chapters 1 and 2 you need.

And last, if you are already a specialist in algebraic geometry codes, start with the contents and then follow your own choice. Do not forget about the historical and bibliographic notes at the end of each chapter.

We heartily advise all readers to try to solve at least some of the exercises.

We also advise you not to forget about the tables and diagrams in the Appendix and about the list of names and index.

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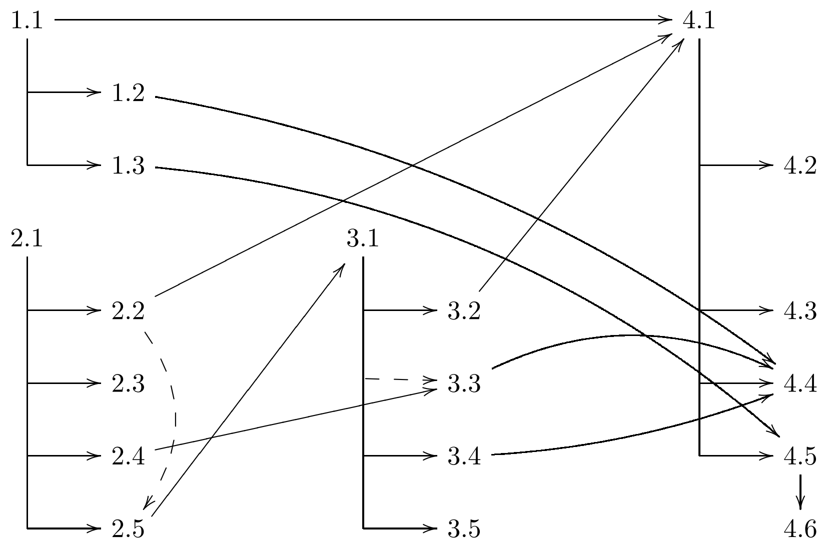
Chapter relation diagram.

$$
\begin{tikzcd}
\boxed{\text{1. Codes}} \arrow[dd] & & \boxed{\text{2. Curves}} \arrow[d] \\
& & \boxed{\text{3. Curves over Finite Fields}} \arrow[dll] \\
\boxed{\text{4. Algebraic Geometry Codes}} & &
\end{tikzcd}
$$

Section relation diagram.



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To avoid misunderstanding, we would like to point out that some terms and notation we use here differ somewhat from common use. In particular, in what follows:

- The notation $A \subset B$ means that A is a *proper* subset of B, i.e., $A \neq B$; when we do not exclude the possibility $A = B$, we write $A \subseteq B$. The set difference is denoted $A \setminus B$.
- An $[n,k,d]_q$ code may be linear or nonlinear. In the last case, k is not necessarily an integer; see Sec. 1.1.1.
- For *algebraic geometry codes* we use the notation $(X,\mathcal{P},D)_L$, $(X,\mathcal{P},D)_\Omega$, etc., while in many papers they are denoted, respectively, $C(G,D)$ and $C^*(G,D)$; see Sec. 4.1.1.
- By $\Omega(D)$ we denote the space

$$\Omega(D) = \{\omega \in \Omega(X)^* \mid (\omega) + D \geq 0\} \cup \{0\},$$

  which in many papers may be denoted by $\Omega(-D)$; see Sec. 2.1.1.
- When describing code families and constructions, we mostly keep the usual meaning of the term, sometimes differing from it in details; see Secs. 1.2.2 and 1.2.3.
- $\lfloor x \rfloor$ means the integer part of $x \in \mathbb{R}$, i.e., $\lfloor x \rfloor \in \mathbb{Z}$, $\lfloor x \rfloor \leq x < \lfloor x \rfloor + 1$; and $\lceil x \rceil$ is the least integer such that $x \leq \lceil x \rceil < x + 1$.