Introduction

The point-set topology of parametrized spaces is surprisingly subtle. Parametrized mapping spaces are especially delicate, and to have them one must leave the most commonly accepted convenient category of topological spaces. Such issues are dealt with in Chapter 1. Rather than give complete proofs, we shall collate results from the extensive literature on the subject to arrive at the framework that we find most convenient.

While in Chapter 1 we focus on single categories, in Chapter 2 we study "change" functors between categories, focusing on change of base space and change of groups. There are myriads of such functors, and sorting out all of the relationships among them is a thankless task. In fact, from a categorical point of view, a full theory of coherence relating them is well beyond current reach. Analogous compatibility relations in algebraic geometry are well-known to be as important to the applications as they are tedious to prove. We are interested here in the point-set level, preparing the way for our later study of these relations in derived homotopy categories.

Chapter 3 gives foundations for the generalization of parts of our theory from compact Lie groups to general Lie groups. It was already observed by Palais [136] that many results in equivariant homotopy theory can be generalized to Lie groups, or even to locally compact groups, provided that one restricts to proper actions. In the parametrized world, the homotopy theory is captured on fibers. When we restrict to proper actions on base spaces, the fibers have actions by the compact isotropy groups of the base space. So even though our primary interest is in compact Lie groups of equivariance, proper actions on the base space seem to provide the right natural level of generality. We set the stage for such a theory by generalizing various classical results about equivariant homotopy types and equivariant bundles and fibrations to the setting of proper actions by Lie groups. In Part II, we develop foundations for space level parametrized homotopy theory in that generality, but we will not go on to the spectrum level analogue in this book. Little of the material in Chapter 3 is needed in the nonequivariant specialization of our work.