Preface

This book is based on a series of fifteen advanced undergraduate lectures I gave at the Park City Mathematics Institute (PCMI) during the summer of 2001. A separate book based on the program of graduate lectures is being published in the Park City Mathematics Series.

My charge was to give students exposure to some of the important ideas related to modern enumerative geometry as it has evolved in recent years. My goals were to provide some background in and inspiration from classical enumerative geometry, to explain the rudiments of stable maps and Gromov-Witten theory, and to explain connections to string theory in physics.

The students were quite good, as I had been told to expect. Students were selected from a competitive application process. Furthermore, PCMI runs two series of undergraduate lectures: an "elementary" lecture series and an "advanced" lecture series. The students self-select which lecture series they will attend, so my lectures were populated by the strongest of this already talented group of undergraduates. The elementary lectures were given by Ruth Gornet on the subject of differential geometry.

While my experience with the undergraduate students was close to my expectations, there was however a big surprise: my lectures

were heavily attended by graduate students. Graduate students repeatedly told me that my lectures helped them solidify their knowledge of Gromov-Witten theory, as I presented a more leisurely introduction than they had seen before, made clearer than they had seen before how Gromov-Witten theory builds upon classical enumerative geometry, gave many examples, and connected to physics more than they had seen before. For this reason, while writing this book primarily for advanced undergraduates, I am keeping graduate students in mind as an important secondary audience.

Designing the course was a bit of a challenge. I could assume that the students were smart and willing to work hard and that they had "mathematical maturity", but I could not assume exposure to any specific area of mathematics beyond a standard undergraduate course in linear algebra. For that reason, the lectures contained introductory material on abstract algebra, geometry, analysis, and topology. Most of the participating students already knew some of these topics, but few knew all of them. Thus the lectures served both as a review and as an introduction to a range of areas in undergraduate mathematics.

The incorporation of physics presented another challenge, as I could not assume anything more than exposure to a first undergraduate physics course. Here, I did not even pretend to be pedagogical or complete. I cut corners by explaining a range of relevant ideas of physics via the simplest examples, emphasizing connections to enumerative geometry throughout. While the physics lectures were undoubtedly the most difficult part of the course for the students, I hoped that they would get a firm impression of the myriad connections between geometry and physics through this very brief introduction. I have similar hopes for the reader of this book.

It has certainly been gratifying to see a number of "my" PCMI undergraduate students currently pursuing graduate studies in this research area, near and dear to my own heart. But the PCMI program has a broader purpose—to give students a research experience that will benefit them in their chosen careers. I propose that the model adopted here for undergraduate training—shooting for some reasonably advanced ideas from graduate level mathematics while filling in

a broad range of topics en route—is a good way to help advanced undergraduates integrate what they have learned and prepare for their careers. Whether or not they pursue graduate studies in the particular focus areas seems secondary to me, even though I am quite fond of the subject area of this book.

Turning the lectures into a book introduced another set of challenges. Undergraduate students reading this book will probably not have the benefit of lectures at which they can ask questions as they go along. In addition, the PCMI students were helped by an expert teaching assistant, Artur Elezi. Since I assume that the reader will not have direct access to a faculty member, I have added much background material that was left out of the lectures, while trying to keep the informal feel throughout. I added material on differential geometry which was not needed in the original lectures since this topic was covered in Ruth Gornet's lectures. Even so, *the reader is warned that this book is not self-contained*. In particular, working through this book is not a substitute for a more thorough and more pedagogical treatment of any of the background topics reviewed here. I have certainly cut out much important foundational material to streamline the process of getting to the desired results. Even worse, sometimes for the sake of expediency I have given very nonstandard treatments of subjects. Some of these have a somewhat clumsy feel to them. This is especially true of the ad hoc treatment I have given of the beautiful subject of algebraic geometry. An undergraduate student interested in algebraic geometry reading this book is urged to consult the references provided. A graduate student interested in algebraic geometry reading this book is urged to provide the standard definitions of all of the concepts which have been introduced here in an ad hoc fashion.

Each of the chapters of the book corresponds reasonably closely to one of the lectures, the exception being that the material from a lecture entitled "More on bundles" has been divided up and distributed through other chapters, so that there are only fourteen chapters replacing fifteen lectures. Since material has been added beyond the content of the original lectures, each chapter contains substantially more than one lecture's worth of material.

As I have indicated above, this book will be quite challenging for an undergraduate student, and, to be honest, will possibly be too hard to understand completely in a few spots, especially the physics-related material near the end. But I encourage all undergraduate students to "go for it" and persist with the difficult parts. If you reach for the stars, you will at least get to the moon. That said, students would likely get more out of the book if they could consult a faculty member or graduate student when needed, as in a directed reading class.

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Undoubtedly a number of typos and other errors are present in the final version, for which I alone bear responsibility. I am maintaining a website for the book. It can be reached from the AMS webpage for the book, whose URL is given on the copyright page and on the back cover of the book. The website will contain lists of typographical errors and their corrections.

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November 2005 *Sheldon Katz*