Preface

This book contains the material that was essentially covered in a course "de troisième cycle"[1] taught during the second semester of the 1996-1997 academic year at the University d'Orléans. The goal of this course was to give an exposition of an example of the use of logarithmic Sobolev inequalities coming primarily from two papers by B. Zegarlinski [**Zeg90, Zeg96**]. The example is concerned with real spin models with weak interactions on a lattice where one can apply a classic method due to Dobrushin; see notably [**Dob70**]. For these models, we give a proof of the uniqueness of the Gibbs measure by showing the exponential stabilization of the stochastic evolution of an infinite dimensional diffusion process which generalizes the case of the Glauber dynamics for the Ising model. Although these models are technically more complicated than the Ising model, one still uses familiar techniques, e.g., using Ito's stochastic integral calculus to construct and study diffusion processes, as well as utilizing the well-known properties of self-adjoint differential operators on $\mathbb{R}^n$ and Sobolev and Poincaré inequalities in their original setting. These models also utilize in a natural way some elegant results on logarithmic Sobolev inequalities such as the Bakry-Émery and Herbst inequalities. Interestingly, these models are simplifications of the Nelson models of Euclidean fields where Gross first introduced logarithmic Sobolev inequalities.[2]

In this book we introduce in a self-contained manner the basic notions of self-adjoint operators, diffusion processes, and Gibbs measures. The chapter on logarithmic Sobolev inequalities is enriched by adding applications to Markov chains so as not to remain in too special a setting. The reader will find indications of some recent applications of logarithmic Sobolev inequalities to statistical mechanics at the end of Chapter 5.

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Note added to the original Preface. The translation presented here differs from the French original by a small number of corrections. Since the original course was given, logarithmic Sobolev inequalities have been the subject of many articles. We recommend that interested readers consult

[1]Translator's note: "Un cours de troisième cycle" is equivalent to an advanced graduate course in an American university.

[2]Translator's note: These are now also called Gross inequalities.

[**Cor02, OR07**], and their bibliographies if they are interested in further study of the subjects treated here.