

Introduction

This is the second volume in the Mathematical Circles Library devoted to the Moscow Mathematical Olympiad (MMO). Its contents and those of the first volume, *Moscow Mathematical Olympiads 1993–1999*, were originally published in Russian in one volume by the Moscow Center for Continuous Mathematical Education, or MCCME. More volumes are in preparation.

The introduction to the first volume contains a few paragraphs about the purpose of the Olympiad, its distinguishing features, and the way it is organized. Here we repeat only enough information to make the book self-contained.

Difficulty level. The numbering of the problems was changed from the original, the Russian class level (8, 9, 10, 11) being replaced by a letter (A, B, C, D), to avoid the implication that level-11 problems, say, should necessarily be accessible to 11th grade students in the United States. In fact a better fit for level-D problems would be 12th graders, but the equivalence is not exact, especially since the curriculum in American high schools is far from standardized.

Within a given level, the MMO organizers try to order the problems by order of difficulty. Ideally, the first problem is the easiest and ends up being solved by a quarter to half of all the participants, while the last problem is solved by only a few. Problems that we feel are too difficult *for their positions* are given a star in this book. (The star is not an indication of absolute difficulty: that Problem 02.B.6 has no star does not imply that it is easier than 00.B.2.)¹ Whenever statistics from the Olympiads were available, we used them as a guide in assigning stars.

Solutions. We always tried to give a solution formulated from within the context of the (Russian) high-school curriculum; in most cases we managed to do this. For some problems we give a second solution, one that requires facts or ideas beyond the curriculum. In some cases, the second solution rests on the first, in the sense that we assume that the reader who has studied the first solution does not mind learning a more elegant one.

¹The notation 02.B.6 indicates Problem 6 in Level B for the year 2002. Problems starting with 9 can be found in *Moscow Mathematical Olympiads 1993–1999*.

Reference facts. This part contains the concepts and theorems used most frequently in the solutions. We tried not to include those generally covered by high-school textbooks, except in cases where we deemed the curriculum inadequate. For every fact, we give a cross-referenced list of problems. Should you encounter a fact you are not familiar with while reading a solution, try to solve other problems where this fact is also used.

Hints. We've tried to offer hints that don't give away the answer, so don't expect a problem to become trivial after reading a hint. If you still cannot solve the problem, read the answer; if even this does not help, read the beginning of the solution.

Acknowledgements

Every Olympiad is the result of the work of many problem authors; a list of authors for this volume is given at the end. We also want to mention the enormous work of those who selected the problems, compiled the tests, proctored the tests, and graded the papers. Finally, Olympiads have no *raison d'être* without their participants—motivated high-school students. We note with great pleasure that many of the students who solve Olympiad problems as contestants go on to become professional mathematicians.

After each Olympiad, a booklet with solutions is printed and handed out to participants at the closing ceremony. We have used these booklets as the basis for our book. We have corrected mistakes (and added new ones, no doubt), improved solutions, provided greater detail and illustrations, added second solutions, and so on. We are thankful to all those who participated in writing these booklets.

Sometimes it is difficult for a writer to know whether a given solution will be intelligible to a student. We are grateful to A. D. Blinkov, A. M. Fedorova, T. I. Golenishcheva-Kutuzova, A. V. Manucharova, D. D. Mukhina, and the students of Form v-2008 of the Moscow State 57th School for pointing out potentially confusing passages.

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