Preface

In February of 2007, I converted my "What's new" web page of research updates into a blog at `terrytao.wordpress.com`. This blog has since grown and evolved to cover a wide variety of mathematical topics, ranging from my own research updates, to lectures and guest posts by other mathematicians, to open problems, to class lecture notes, to expository articles at both basic and advanced levels.

With the encouragement of my blog readers, and also of the American Mathematical Society, I published many of the mathematical articles from the first two years of the blog as [**Ta2008**] and [**Ta2009**], which will henceforth be referred to as *Structure and Randomness* and *Poincaré's Legacies Vols. I, II* throughout this book. This gave me the opportunity to improve and update these articles to a publishable (and citeable) standard, and also to record some of the substantive feedback I had received on these articles by the readers of the blog.

The current text contains many (though not all) of the posts for the third year (2009) of the blog, focusing primarily on those posts of a mathematical nature which were not contributed primarily by other authors, and which are not published elsewhere. It has been split into two volumes.

The first volume (referred to henceforth as *Volume 1*) consisted primarily of lecture notes from my graduate courses on real analysis that I taught at UCLA. The current volume consists instead of sundry articles on a variety of mathematical topics, which I have divided (somewhat arbitrarily) into expository articles (Chapter 1) which are introductory articles on topics of relatively broad interest, and more technical articles (Chapter 2) which are narrower in scope and often related to one of my current research interests.

These can be read in any order, although they often reference each other as well as articles from previous volumes in this series.

A remark on notation

For reasons of space, we will not be able to define every single mathematical term that we use in this book. If a term is italicised for reasons other than emphasis or for definition, then it denotes a standard mathematical object, result, or concept, which can be easily looked up in any number of references. (In the blog version of the book, many of these terms were linked to their Wikipedia pages, or other on-line reference pages.)

I will however mention a few notational conventions that I will use throughout. The cardinality of a finite set E will be denoted $|E|$. We will use the asymptotic notation $X = O(Y)$, $X \ll Y$, or $Y \gg X$ to denote the estimate $|X| \leq CY$ for some absolute constant $C > 0$. In some cases we will need this constant C to depend on a parameter (e.g., d), in which case we shall indicate this dependence by subscripts, e.g., $X = O_d(Y)$ or $X \ll_d Y$. We also sometimes use $X \sim Y$ as a synonym for $X \ll Y \ll X$.

In many situations there will be a large parameter n that goes off to infinity. When that occurs, we also use the notation $o_{n \to \infty}(X)$ or simply $o(X)$ to denote any quantity bounded in magnitude by $c(n)X$, where $c(n)$ is a function depending only on n that goes to zero as n goes to infinity. If we need $c(n)$ to depend on another parameter, e.g., d, we indicate this by further subscripts, e.g., $o_{n \to \infty; d}(X)$.

We will occasionally use the averaging notation $\mathbf{E}_{x \in X} f(x) := \frac{1}{|X|} \sum_{x \in X} f(x)$ to denote the average value of a function $f : X \to \mathbf{C}$ on a nonempty finite set X.

Acknowledgments