

PREFACE

*Mathematics is too serious and,
therefore, no opportunity should be
missed to make it amusing.*

Blaise Pascal

Mathematical puzzles and games have been in evidence ever since man first began posing mathematical problems. The history of mathematics is replete with examples of puzzles, games, and entertaining problems that have fostered the development of new disciplines and sparked further research. Important connections exist between problems originally meant to amuse and mathematical concepts critical to graph theory, geometry, optimization theory, combinatorics, and number theory, to name only a few.

As a motivating force, then, the inclination to seek diversion and entertainment has resulted in the unintended revelation of mathematical truths while also tempering mathematical logic. In fact, Bertrand Russell (1872–1970) once noted: “*A logical theory may be tested by its capacity for dealing with puzzles, and it is a wholesome plan, in thinking about logic, to stock the mind with as many puzzles as possible, since these serve much the same purpose as is served by experiments in physical science.*”

Perhaps the popularity of mathematical puzzles and games endures because they fulfill the need for diversion, the desire to achieve mastery over challenging subject matter or simply to test our intellectual capacities. Of equal importance, mathematical amusements also offer an ample playing field to both the amateur and the professional mathematician. That mathematicians from antiquity to the present have always taken interest and delighted in puzzles and diversions might lend credence to the notion that creative stimulus and aesthetic considerations are closely interwoven. Edward Kasner and James Newman in their essay *Pastimes of past and present times* (in *The World of Mathematics*, Vol. 4 (ed. James Newman), Dover, Mineola 2000) declare: “... *No branch of intellectual activity is a more appropriate subject for discussion than puzzles and paradoxes Puzzles*

in one sense, better than any other single branch of mathematics, reflect its always youthful, unspoiled, and inquiring spirit Puzzles are made of the things that the mathematician, no less than the child, plays with, and dreams and wonders about, for they are made of the things and circumstances of the world he lives in."

In attempting to bring the reader closer to the distinguished mathematicians, I have selected 127 problems from their works. Another 50 related problems have been added to this collection. The majority of these mathematical diversions find their basis in number theory, graph theory and probability. Others find their basis in combinatorial and chess problems, and still others in geometrical and arithmetical puzzles. Noteworthy mathematicians ranging from Archimedes, Cardano, Kepler, Pascal, Huygens, Newton, Euler, Gauss, Hamilton, Cayley, Sylvester, to von Neumann, Banach, Erdős and others, have all communicated brilliant ideas, methodological approaches leavened with humor, and absolute genius in mathematical thought by using recreational mathematics as a framework.

This book also explores the brain-teasing and puzzling contributions of contemporary scientists and mathematicians such as John E. Littlewood, John von Neumann, Stephen Banach, Paul Erdős, (H. S. M.) Donald Coxeter, the Nobel-Prize winning physicist Paul Dirac, the famous mathematical physicist Roger Penrose, the eminent mathematician and puzzle composer John Horton Conway and the great computer scientist and mathematician Donald Knuth.

I have purposely selected problems that do not require advanced mathematics in order to make them accessible to a variety of readers. The tools are simple: nothing but pencil and paper. What's required is patience and persistence, the same qualities that make for good careful mathematical research. Restricting problems to only those requiring the use of elementary mathematics consequently forces the omission of other equally celebrated problems requiring higher mathematical knowledge or familiarity with other mathematical disciplines not usually covered at the high school level. Even so, I have made several exceptions in the application of certain nonstandard yet elementary techniques in order to solve some problems. To help readers, I have provided outlines in the book's four appendices because I believe that the time and effort needed to master any additional material are negligible when compared to the reader's enjoyment in solving those problems.

At some point when writing a book of this kind, most authors must limit their choices. The dilemma I most frequently confronted as I selected problems was this: *What determines whether a task is recreational or not?* As already mentioned, in centuries past almost all mathematical problems (ex-

cluding, of course, real-life problems of measurement and counting) existed chiefly for intellectual pleasure and stimulation. Ultimately, however, deciding the recreational merits of a given problem involves imposing arbitrary distinctions and artificial boundaries. Over time, a significant number of recreational mathematics problems have become integral to the development of entirely new branches in the field. Furthermore, scientific journals often take as their subject of study problems having the same features as those that characterize recreational mathematics problems. If the reader takes pleasure in squaring off with the problems included here, then the author may regard his selections as satisfactory.

Although several tasks may appear trivial to today's amateur mathematician, we must recall that several centuries ago, most of these problems were not easy to solve. While including such problems provides historical insight into mathematical studies, we must also remain alert to their historical context.

As this book is intended principally to amuse and entertain (and only incidentally to introduce the general reader to other intriguing mathematical topics), without violating mathematical exactitude, it does not always strictly observe the customary rigorous treatment of mathematical details, definitions, and proofs. About 65 intriguing problems, marked by *, are given as exercises to the readers.

I note that, in some instances, difficulties arose with respect to reproducing exact quotes from various sources. However, I trust that these minor inconveniences will not detract from the book's overall worth.

Last, a few comments regarding the arrangement of materials. The table of contents lists the tasks by their title, followed by the author's name in parentheses. Mathematicians whose tasks are included appear in the book's index in boldface. Brief biographies of these contributors appear in chronological order on pages 299–310. The page location indicating a particular biography is given in the text behind the name of the contributor and his puzzle (for example, → p. 299). Furthermore, when introducing the tasks themselves, I have included sometimes amusing anecdotal material since I wanted to underscore the informal and recreational character of the book. Given that the majority of terms, mathematical or otherwise, are familiar to readers, there is no subject index.

Acknowledgments. In the course of writing this book, I received great support from my family, friends and editors.

An especially warm thank you goes to my wife and colleague Ljiljana Petković, and my elder son Ivan, for their comments during the preparation of the manuscript, and their never-failing support and love.

Most of all, I thank my younger son Vladimir Petković for his eye-catching illustrations. A very talented computer artist and designer, Vladimir not only created computer artworks that are both aesthetically attractive and mathematically flavored, but he also solved several intricate problems discussed in this book.

I have made use of personal materials collected over a twenty year period from university libraries in Freiburg, Oldenburg, and Kiel, Germany; London, England; Strasbourg (Université Louis Pasteur), France; Tsukuba, Japan; Minneapolis, Minnesota, Columbia University; Vienna, Austria; the Department of Mathematics, Novi Sad, Serbia and the Institute of Mathematics, Belgrade, Serbia. I wish to thank the staff of these libraries for their assistance.

I would also like to say thanks to dear friends Dr. Martyn Durrant and Professor Biljana Mišić-Ilić, who read a great deal of the manuscript and suggested some improvements in language and style.

Many thanks go to Sergei Gelfand, the Publisher of the American Mathematical Society, for his helpful comments, suggestions and encouragement, and to Christine M. Thivierge, Associate Editor for Proceedings of the American Mathematical Society, for her great effort and persistence in providing the photos of great mathematicians and several very valuable illustrations, and Luann Cole, a production editor at the American Mathematical Society, who read the manuscript from beginning to end and made a number of improvements. I am also thankful to the anonymous referees; their helpful suggestions, remarks and criticism greatly improved the exposition.

I especially wish to acknowledge the assistance and exceptional efforts of Margaret Alice Hanson who helped me rewrite and edit sections of the manuscript to prepare it for publication.

Miodrag S. Petković
University of Niš, Serbia
February 2009