Preface

The Adventure of Numbers... This title may surprise you. People tend to think of arithmetic as an emotionless activity, without the slightest tremor or the smallest risk. But I really am inviting you to go on an adventure.

The adventure is human above all. We will see not only discoveries but also controversies, meet engaging characters, witness hesitations and fruitful doubts that bring new ideas. The characters in our history are first of all mathematicians. Some of the earliest are anonymous—perhaps legendary— others are more familiar to us, still others are our contemporaries. They all contributed to refining and extending our understanding of numbers. They were men of the mind, but they were also flesh and blood. Faced daily with an eternal reality, they were nonetheless products of their time and their culture. Proud of knowing they were at the frontiers of knowledge, they had the humility of pioneers before the vastness of the continent they would figure in ... and figure out. We will follow the sometimes steep trails they cleared for us.

Other actors in our adventure are intangible: the numbers themselves. But isn't mathematics the discipline where order and logic reign, a perfectly orchestrated minuet with no wrong notes? The harmony of mathematics, real though it is, shouldn't conceal that it is a living science. The object of this science is concepts—which, even though they originate in our perception of the material world, are abstract. This does not mean that they bend to our will or respond to our wishes. The immense army of numbers is less a stately procession than an unruly mob, able to burst through the boundaries within which we might want to confine it. This confrontation won't lack surprises.

Now for a few words about this book and its author. Though I can introduce myself as a professional mathematician, I have to qualify that by saying that I am in no sense whatsoever a specialist in the history of mathematics. Any expert who looked in this book for a serendipitous find or an original bit of historical information would be disappointed. There is nothing encyclopedic about my book. I have simply tried to sketch a few striking moments that lead us to modern concepts. History gives me an opportunity to convince the reader that modern abstractions are natural and in some sense *necessary*. There may be some novelty in how certain crucial steps are put into perspective. I don't know, and I prefer to take shelter behind the popular saying that copying from one author is plagiarism

but copying from several is research. So I've done a researcher's work by consulting a number of publications. Some of the most important and most accessible are cited in the bibliography.

There is no attempt here to defend or refute any theory whatsoever. Certain themes are persistent. Among them is the deep connection between a mathematical advance and the prevailing scientific and philosophical view of the world at the time that advance is made. But it would be going too far to make this connection our guideline. The history before us is more swarming than linear, and despite my chronological presentation, I don't claim to chart the peaks of mathematical progress. But I did not refrain from speaking freely when I seemed to glimpse an idea. Ultimately, I tried to write an approachable text, particularly for readers who have little mathematical background—and who, in that regard, may have retained some bad memories from their adolescence. A certain amount of effort (not strictly mathematical) will be necessary, especially in the last chapters. How could it be otherwise when you try to understand recent progress? In any case, skipping the calculations won't keep you from understanding the book as a whole. Readers who may find it fun to practice on the little problems scattered throughout can consult the solutions at the end of each chapter. The most frequently used mathematical terms are defined in a glossary. Some more technical information is collected in the appendices at the end of the book, so that motivated readers can test and enhance their culture on some specific points.

I must, however, state clearly that my objective is not actually to teach the reader anything at all. My goal is much more modest: to share the joy of discovering and understanding this great and beautiful adventure of the mind. It will certainly be reached if you enjoy this journey to the fascinating land of numbers.

It is a pleasure to conclude this introduction by expressing my gratitude to the people and institutions who helped bring this book into being. My brother Yves Godefroy, who was not only my first mathematical mentor but also my first philosophy teacher, was the impetus for my work, and he devoted long hours to reading and commenting on the preliminary versions. Jacques Bienvenu got me to understand the history of complex numbers and awakened my interest in these questions; I would never have grasped their implications without him. On the advice of Nigel Kalton and Elias Saab, the University of Missouri at Columbia invited me to give lectures on this subject, enabling me to get my thoughts in order; then the ENSAE, on the initiative of Dominique Ladiray, accepted the French versions of these lectures. Jean-François Serval convinced me that it might be possible to publish my notes. Many colleagues and friends were kind enough to give their advice on the manuscript or inform me about topics of which I knew little, and their constructive criticism and encouragement were invaluable. This book owes a great deal to Benoît Bossard, Robert Cabane, Manuel Cepedello, Sylvie Delabrière, Philippe Esperet, Catherine Goldstein, Halvard Hervieu,

Daniel Li, Alain Louveau, Étienne Meyer, Bernard Randé, Marc Rogalski, Thomas Serval, Jean Saint-Raymond, Éric Van der Oord, and Michel Waldschmidt. The National Center for Scientific Research and the analysis group at the University of Paris VI let me devote myself freely to completing this work. The distinguished publishing house of Odile Jacob accepted my modest contribution, and Gérard Jorland granted me many discussions that were as fascinating as they were profitable. Finally, Michèle, Noémie, and Clémence patiently endured, throughout long months, the book's obsessive presence. I am deeply grateful to all of them.

Acknowledgments for the English edition

I am very grateful to the American Mathematical Society for their decision to publish an English version of the French text and for their role in this project. Leslie Kay completed the translation with extreme care, and perfect linguistic and mathematical competence.