Preface

Dedekind and Weber's 1882 paper on algebraic functions of one variable is one of the most important papers in the history of algebraic geometry. It changed the direction of the subject, and established its foundations, by introducing methods from algebraic number theory. Specifically, they used rings and ideals to give rigorous proofs of results previously obtained, in nonrigorous fashion, with the help of analysis and topology. Also, by importing ideas from number theory, the paper revealed the deep analogy between number fields and function fields—an analogy that continues to benefit both number theory and geometry today.

The influence of the paper is obvious in 20th-century algebraic geometry, where the role of arithmetic/algebraic methods has increased enormously in both scope and sophistication. But, as the sophistication of algebraic geometry has increased, so has its detachment from its origins. While the Dedekind-Weber paper continues to be cited, I venture to guess that few modern algebraic geometers are familiar with its contents. There are a few useful commentaries on the paper, but those that I know seem to focus on a few of the concepts used by Dedekind and Weber, while ignoring others. And, of course, fewer mathematicians today are able to read the language in which the paper was written (and I don't mean only the German language, but also the mathematical language of the 1880s).

I therefore believe that it is time for an English edition of the paper, with commentary to assist the modern reader. My commentary takes the form of a Translator's Introduction, which lays out the historical background to Dedekind and Weber's work, plus section-by-section comments and footnotes inserted in the translation itself. The comments attempt to guide the reader through the original text, which is somewhat terse and unmotivated, and the footnotes address specific details such as nonstandard terminology. The historical background is far richer than could be guessed from the Dedekind-Weber paper itself, including such things as Abel's results in integral calculus, Riemann's revolutionary approach to complex analysis and his discoveries in surface topology, and developments in number theory from Euler to Dedekind. The background is indeed richer than some readers may care to digest, but it is a background against which the clarity and simplicity of the Dedekind-Weber theory looks all the more impressive.

I hope that this edition will be of interest to several classes of readers: historians of mathematics who seek an annotated edition of one of the classics, mathematicians interested in history who would like to know where modern algebraic geometry came from, students of algebraic geometry who seek motivation for the concepts they are studying, and perhaps even algebraic geometers who have not had time to catch up with the origins of their discipline. (It seems to an outsider that just the modern literature on algebraic geometry would take more than a lifetime to absorb.)

This translation was originally written in the 1990s, but in 2011 I was motivated to revise it and write an introduction in order to prepare for a summer school presentation on ideal elements in mathematics. I have also compiled a bibliography and index. The bibliography is mainly for the Translator's Introduction, but it is occasionally referred to in the commentary on the translation, so I have placed it after the translation.

The summer school, PhilMath Intersem, was organized by Mic Detlefsen, and held in Paris and Nancy in June 2011. I thank Mic for inviting me and for support during the summer school. I also thank Monash University and the University of San Francisco for their support while I was researching this topic and writing it up. Anonymous reviewers from the AMS have been very helpful with some technical details of the translation, and I also thank Natalya Pluzhnikov for copyediting. Finally, I thank my colleague Tristan Needham, my wife Elaine, and son Robert for reading the manuscript and saving me from some embarrassing errors.

John Stillwell

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