Introduction

Complex Made Simple is intended as a text on complex analysis at the beginning graduate level — students who have already taken a course on this topic may nonetheless be interested in the results in the second half of the book, beginning somewhere around Chapter 16, and experts in the field may be amused by the proof of the Big Picard Theorem in Chapter 20.

The main prerequisite is a course typically called "Advanced Calculus" or "Analysis", including topics such as uniform convergence, continuity and compactness in Euclidean spaces. A hypothetical student who has never heard of complex numbers should begin with Appendices 1 and 2 (students who are familiar with basic manipulations with complex numbers on an informal level can skip Appendix 2, although many such students should probably read Appendix 1). Definitions and results concerning metric spaces are summarized in Appendix 4, with most proofs left as exercises. We decided not to include a similar summary of elementary point-set topology: General topological spaces occur in only a few sections, dealing with Riemann surfaces (students unfamiliar with general topology can skip those sections or pretend that a topological space is just a metric space). The only abstract algebra required is a rudimentary bit of group theory (normal subgroups and homomorphisms), while the deepest fact from linear algebra used in the text is that similar matrices have the same eigenvalues.

Of course the analysis here is really no simpler than that in any other text on the topic at the same level (although we hope we have made it simple to understand). A more accurate title might be *Complex Explained in Excruciating Detail*: Since our main intent is pedagogical, we place great emphasis on motivation, attempting to distinguish clearly between clever ideas and routine calculations, to explain what various results "really mean", to show

how one might have *found* a certain argument, etc. In several places we give two versions of a proof, one more "abstract" than the other; the reader who wishes to attain a clear understanding of the difference between the forest and the trees is encouraged to contemplate both proofs until he or she sees how they are really just different expositions of the same underlying idea.

Many results in elementary complex analysis (pointwise differentiability implies smoothness, a uniform limit of holomorphic functions is holomorphic, etc.) are really quite surprising. Or at least they *should* be surprising; we include examples from real analysis for the benefit of readers who might not otherwise see what the big deal is.

There are a few ways in which the content differs from that of the typical text. First, the reader will notice an emphasis on (holomorphic) automorphism groups and an explicit mention of the notion of covering spaces. These concepts are used in incidental ways in the first half of the book; for example linear-fractional transformations arise naturally as the automorphisms of the Riemann sphere instead of being introduced as an *ad hoc* class of conformal maps in which it just happens that various calculations are easy, covering maps serve to unify various results on analytic continuation, etc.; then it turns out that some not-quite-trivial results on automorphisms and covering maps are crucial to the proof of the Big Picard Theorem.

Probably the most unusual aspect of the content is the inclusion of a section on the relation between Brownian motion and the Dirichlet problem. In most of the text we have tried to achieve a fairly high standard of rigor, but in this section the notion of rigor simply flies out the window: We do not even include precise definitions of the things we're talking about! We decided to include a discussion of this topic even though we could not possibly do so rigorously (considering the prerequisites we assume) because Brownian motion gives the clearest possible intuition concerning the Dirichlet problem. Readers who are offended by the informal nature of the exposition in this section are encouraged to think of it not so much as a lecture but rather a conversation in the departmental lounge or over a few beers on a Friday afternoon.

Finally, the proof of the Big Picard Theorem will probably be new to most readers, possibly including many experts. The proof is certainly not simpler or shorter than the proofs found in typical texts, but it seems very interesting, at least to me: It proceeds by essentially a direct generalization of the standard "one-line" proof of the Little Picard Theorem. (See the discussion of Theorem A and Theorem B in Chapter 20.)

It will be clear to many readers that I first learned much of this material from [**R**]. Very few references are given; all of the results are quite standard, and I doubt that any of the proofs are new. Indeed, for some time I thought

that the proof of the Big Picard Theorem was original with me — Anthony Kable discovered that it is essentially the same as the original proof [**J**]. (This raises the question of why the original proof is not so well known; I conjecture that it fell out of favor because various concepts and techniques were much newer and fuzzier in Picard's time than they are at present.) The list of references at the end of the book should not be construed as a guide to the literature or even as a list of suggestions for further reading; it is simply a list of the references that happened to come up in the text. (I decided that including a "Further Reading" section would border on arrogance — further reading here could include topics in almost any area of mathematics.)

It is a pleasure to thank various students and past and present colleagues for mathematical and moral support through the years, including Benny Evans, Alan Noell, Wade Ramey, David Wright, and in particular Robert Myers, who gave a very careful reading of the sections on topology, and especially Anthony Kable, who made various valuable comments at every stage of the project. We enjoyed working with the people at the AMS: Barbara Beeton provided staggeringly competent and often witty TeXnical advice, and Edward Dunne was a very enthusiastic and helpful senior editor.

Any errors or omissions are the responsibility of the author. However, readers who feel that the whole book is just one big mistake need to discuss the matter with Walter Rudin: Before reading *Real and Complex Analysis* I had no idea I was interested in the subject. It seems presumptuous to publish another book in a field where there already exists a text so beautiful it makes your eyes hurt, but several people kept bugging me to write up my lecture notes — this seemed like the only way to shut them up.