Preface

The study of representations of p-adic groups was born over fifty years ago when Friedrich Mautner published a paper on spherical functions of p-adic PGL(2). In this 1958 paper he introduced principal series representations "as induced representations following the by now classical procedure of Frobenius, Schur, Bargmann, Gelfand, Naimark, Mackey and others" [Mau58, §3]. The companion paper to [Mau58], which was not published until 1964, exhibited the first examples of supercuspidal representations [Mau64]. Taken together, these two papers represent the first steps taken toward the solution of a problem which is deceptively simple to state: classify and construct all irreducible admissible representations of reductive p-adic groups.

By 1970, Harish-Chandra's work on the Plancherel formula for reductive p-adic groups had appeared in the Séminaire Bourbaki [HC70a], for which he developed the idea of the character of an admissible representation as a distribution on the Hecke algebra of the p-adic group. On the other hand, in the intervening years, Robert Langlands introduced L-parameters of admissible representations of p-adic groups as part of the celebrated Local Langlands Correspondence [Lan68].

The Leitfaden for this volume is based directly on the remarkable work done in this early period and follows three threads: the study of smooth representations as begun by Mautner (Part 1); character theory beginning with the work of Harish-Chandra (Part 2); and the Local Langlands Correspondence, framed by Robert Langlands (Part 3).

The first part begins with Alan Roche's chapter, in which he develops the Bernstein decomposition, thus setting up the theory of types. Part 1 continues with Jiu-Kang Yu's introduction to Bruhat-Tits theory — a theory which is indispensable in p-adic representation theory. Ju-Lee Kim's chapter provides an elementary introduction to supercuspidal representations, and goes on to describe her work proving that Jiu-Kang Yu's construction of supercuspidal representations [Yu01] is complete in many cases. The material in Part 1 beautifully frames Mautner's two original papers in the field.

The chapter by Paul Sally and Loren Spice gives an introduction to character theory beginning with Harish-Chandra and focuses on the history of calculations of character values. On the same theme of tools for computing character values, Part 2 continues with Julia Gordon and Yoav Yaffe's chapter on arithmetic motivic integration as an alternative to classical p-adic integration. This chapter also serves as a segue to some current research on character value computations.

The final part of this volume picks up the thread introduced in 1968 by Robert Langlands. Paul Mezo's chapter reviews local class field theory and sketches the Local Langlands Correspondence, which is now a theorem in many cases. Jiu-Kang

Yu's chapter in Part 3 then describes the Local Langlands Correspondence as it applies to algebraic tori.

The tapestry woven with the ideas found in the early papers by Mautner, Harish-Chandra and Langlands is remarkably rich and represents a testament to the inventiveness of the many mathematicians who have worked in this area during the last half-century. Nevertheless, much work remains to be done. For example, the Local Langlands Correspondence, when taken together with the structure of local L-packets, offers a parametrization of admissible representations which is, *a priori*, very different from that described in Part 1. The reconciliation of these two perspectives on admissible representations is an area of active research.

This volume evolved from two Fields Institute Workshops held at the University of Ottawa: the *Workshop on the Representation Theory of p-adic Groups* held in May 2004, organised by Jason Levy and Monica Nevins; and the *Workshop on the Representation Theory of Reductive Algebraic Groups*, held in January 2007, organised by Clifton Cunningham and Monica Nevins. Each workshop was constructed around three mini-courses, as well as a two-day conference. The goal was to present some of the main themes and results of the representation theory of *p*-adic groups to a broad audience.

The first workshop began with three six-hour mini-courses:

- Alan Roche (University of Oklahoma), *The Bernstein Centre and Types*;
- Jiu-Kang Yu (Purdue University), *Bruhat-Tits Theory and Buildings*;
- Paul Mezo (Carleton University), *The Local Langlands Program.*

It also featured a colloquium lecture by Paul Sally, Jr., University of Chicago, entitled *Characters for Reductive p-adic Groups.* Each mini-course lecturer graciously provided meticulous notes. The value of these notes as introductions to some of the most beautiful aspects of the subject was soon apparent and many participants encouraged their publication.

Nevertheless, it was not until some two and a half years later that steps were taken to ensure the longevity of these excellent resources. The second workshop included three four-hour mini-courses:

- Julia Gordon (University of British Columbia), *Motivic Integration and its Applications to p-adic Groups*;
- Ju-Lee Kim (University of Illinois at Chicago, now Massachusetts Institute of Technology), *Recent Progress in the Classification of Supercuspidal Representations*;
- Phil Kutzko (University of Iowa), *Plancherel Measure and Reducibility of Parabolic Induction via Types and Covers*;

as well as a colloquium talk by A. Raghuram (Oklahoma State University) entitled *Arithmetic of L-functions.* It was in fact Raghuram who finally convinced the organisers to undertake the creation of this volume even though, regretfully, his own talk did not fall under its scope.

We offer our profound thanks to all the mini-course lecturers for their diligence and generosity. They were instrumental in making these workshops a great success and this volume would not have been possible without their dedication to this project.