Preface

This book grew out of two series of five two-hour lectures, given by Jean Berstel and Christophe Reutenauer in March 2007. The lectures were delivered during the school on "Combinatorics on Words" organized by Srečko Brlek, Christophe Reutenauer and Bruce Sagan that took part within the theme semester on *Recent Advances in Combinatorics on Words* at the Centre de recherches mathématiques (CRM), Montréal, Canada.

Notes for the lectures were written down by Aaron Lauve and Franco Saliola. They have augmented their notes with several topics and have added more than 100 exercises. There has been a lot of work in adding bibliographic references and a detailed index.

The text is divided into two parts. Part I, based on the lectures given by Christophe Reutenauer, is a comprehensive and self-contained presentation of the current state of the art in Christoffel words. These are finitary versions of Sturmian sequences. It presents relationships between Christoffel words and topics in discrete geometry, group theory, and number theory. Part I concludes with a new exposition of the theory of Markoff numbers.

Part II, based on the lectures by Jean Berstel, starts with a systematic exposition of the numerous properties, applications, and interpretations of the famous Thue – Morse word. It then presents work related to Thue's construction of a square-free word, followed by a detailed exposition of a linear-time algorithm for finding squares in words. This part concludes with a brief glimpse of several additional problems with origins in the work of Thue.

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