Preface

This book grew out of the notes for ten lectures given by the author at the CBMS Conference at Texas A&M University, College Station, during the week of May 20-24, 2002. Paulo Lima Filho, J. Maurice Rojas and Hal Schenck did a fantastic job of organizing this conference and taking care of more than 80 participants, many of them graduate students working in a wide range of mathematical fields. We were fortunate to be able to listen to the excellent invited lectures delivered by the following twelve leading experts: Saugata Basu, Eduardo Cattani, Karin Gatermann, Craig Huneke, Tien-Yien Li, Gregorio Malajovich, Pablo Parrilo*, Maurice Rojas, Frank Sottile, Mike Stillman*, Thorsten Theobald, and Jan Verschelde*.

Systems of polynomial equations are for everyone: from graduate students in computer science, engineering, or economics to experts in algebraic geometry. This book aims to provide a bridge between mathematical levels and to expose as many facets of the subject as possible. It covers a wide spectrum of mathematical techniques and algorithms, both symbolic and numerical. There are two chapters on applications. The one about statistics is motivated by the author's current research interests, and the one about economics (Nash equilibria) recognizes Dave Bayer's role in the making of the movie *A Beautiful Mind*. (Many thanks, Dave, for introducing me to the stars at their kick-off party in NYC on March 16, 2001.)

At the end of each chapter there are about ten exercises. These exercises vary greatly in their difficulty. Some are straightforward applications of material presented in the text while other "exercises" are quite hard and ought to be renamed "suggested research directions". The reader may decide which is which.

We had an inspiring software session at the CBMS conference, and the joy of computing is reflected in this book as well. Sprinkled throughout the text, the reader finds short computer sessions involving polynomial equations. These involve the commercial packages `Maple`® and `MATLAB`® as well as the freely available packages `Singular`[1], `Macaulay 2`[2], `PHCpack`[3], and `SOSTools`[4]. Developers of the last three programs spoke at the CBMS conference. Their names are marked with a star above.

There are many fine computer programs for solving polynomial systems other than the ones listed above. Sadly, I did not have time to discuss them all. One

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[1] Singular is a free software distributed under the GNU license. © Department of Mathematics and Centre for Computer Algebra, University of Kaiserslautern, Germany

[2] Macaulay 2: © Daniel R. Grayson and Michael E. Stillman (1993-2001) and is distributed free under the GNU license

[3] PHCpack: © 1998, Katholieke Universiteit Leuven, Department of Computer Science, Heverlee, Belgium

[4] SOSTools is a MATLAB® toolbox and freely available under the GNU license at: http://www.cds.caltech.edu/sostools or http://www.aut.ee.ethz.ch/~parrilo/sostools

such program is `CoCoA`[5] which is comparable to `Singular` and `Macaulay 2`. The textbook by Kreuzer and Robbiano [**KR00**] does a wonderful job introducing the basics of Computational Commutative Algebra together with examples in `CoCoA`.

Software is necessarily ephemeral. While the mathematics of solving polynomial systems continues to live for centuries, the computer code presented in this book will become obsolete much sooner. I tested it all in May 2002, and it worked well at that time, on our departmental computer system at UC Berkeley. And if you would like to find out more, each of these programs has excellent documentation.

I am grateful to the students in my graduate course, *Math 275: Topics in Applied Mathematics*, for listening to my ten lectures at home in Berkeley while I first assembled them in the spring of 2002. Their spontaneous comments proved to be extremely valuable for improving my performance later on in Texas. After the CBMS conference, the following people provided very helpful comments on my manuscript: John Dalbec, Jesus De Loera, Mike Develin, Alicia Dickenstein, Ian Dinwoodie, Bahman Engheta, Stephen Fulling, Karin Gatermann, Raymond Hemmecke, Serkan Hoşten, Robert Lewis, Gregorio Malajovich, Pablo Parrilo, Francisco Santos, Frank Sottile, Seth Sullivant, Caleb Walther, and Dongsheng Wu.

Special thanks go to Amit Khetan and Ruchira Datta for helping me while in Texas and for contributing to Sections 4.5 and 6.2 respectively. Ruchira also assisted me in the hard work of preparing the final version of this book. It was her help that made the rapid completion of this project possible.

Last but not least, I wish to dedicate this book to the best team of all: my daughter Nina, my son Pascal, and my wife Hyungsook. A million thanks for being patient with your papa and putting up with his crazy early-morning work hours.

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Bernd Sturmfels*
Berkeley, June 2002

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