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Classifying Spaces of Sporadic Groups

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ABSTRACT. For each of the 26 sporadic finite simple groups, we construct a 2-completed classifying space, via a homotopy decomposition in terms of classifying spaces of suitable 2-local subgroups; this leads to an additive decomposition of the mod 2 group cohomology. We also summarize the current status of knowledge in the literature about the ring structure of the mod 2 cohomology of those groups.

Our decompositions arise via recent homotopy colimit theorems of various authors: in those results, the colimit is indexed by a collection of 2-subgroups, which is "ample" in the sense of affording the desired cohomology decomposition. Furthermore our decompositions are "sharp" in the sense of being simplified by the collapse of an underlying spectral sequence.

Among the various standard ample collections available in the topological literature, we make a suitably minimal choice for each group—and we further interpret that collection in terms of an equivalent "2-local geometry" from the group theory literature: namely as a simplicial complex determined by certain 2-local subgroups. In particular, we complete the verification that for each sporadic group, an appropriate 2-local geometry affords a small ample collection. One feature which emerges in each case is that the geometry has "flag-transitive" action by the group, so that the orbit complex (the quotient space modulo that action) is a simplex: and then the diagram of classifying spaces of subgroups which indexes the homotopy decomposition is a pushout n-cube of the relevant dimension n (though sometimes that orbit simplex diagram is further simplified via cancellations).

The work begins with a fairly extensive initial exposition, intended for non-experts, of background material on the relevant constructions from algebraic topology, and on local geometries from group theory. The subsequent chapters then use those structures to develop the main results on individual sporadic groups.

Contents

Introduction

The 26 sporadic simple groups have played an exceptional role in the theory and classification[1] of the finite simple groups, ever since the discovery of the first few such groups in the late 1800s. In the last decade or so, various authors have also observed exceptional behavior in the mod p group cohomology of some of the sporadic groups, especially at the prime $p = 2$; see for example Benson [**24, 25, 26**], Benson and Wilkerson [**29**], Milgram [**92, 93, 94**]. Determining (or at least describing) the cohomology of sporadic groups represents an area of considerable current activity; and such study combines methods of algebraic topology with information and techniques from finite group theory.

In this work, we provide for each sporadic group a description of a 2-completed classifying space, in terms of classifying spaces of suitable 2-local subgroups. In particular, this leads to an additive decomposition of the mod 2 group cohomology. Similar decompositions can be obtained for odd primes p; but the indexing set of p-local subgroups is usually smaller and often less interesting.

Although our focus here is on sporadic groups, we emphasize that this work should be also be viewed in relation to work on cohomology of the other simple groups. For example, our decompositions for sporadic groups are largely inspired by a standard decomposition (see Example 5.1.18) of the mod p cohomology for a group of Lie type of characteristic p. However, even in that model case for Lie type groups, it remains a major open problem to give an explicit description of the individual component terms and their cohomology. In contrast, the cohomology and completed classifying spaces of groups of Lie type in coprime characteristic are rather well understood in the light of the extraordinary work of Quillen [**103**] for the general linear groups, and subsequent work along the same lines by Fiedorowicz and Priddy [**60**] for the remaining classical groups and Kleinerman [**81**] for the groups of exceptional Lie type. Cohomology and classifying spaces of symmetric and alternating groups are understood to some extent (Nakaoka [**99**], Mui [**97**], Mann [**89, 90**], Gunawardena, Lannes and Zarati [**70**], Ha and Lesh [**71**], etc).

The context of this work. Naturally the structure of the p-local subgroups[2] in a finite group G should provide an important ingredient in the determination of its mod-p group cohomology. One p-local approach to cohomology, often used as a tool in modern work on individual sporadic groups, is provided by the topological literature on *cohomology decompositions.* In particular, we mention Dwyer's analysis [**51, 53**] of *ample* collections $\mathcal{C}$ of p-subgroups of G, namely collections which

[1] The reader who is uneasy with the classification of the finite simple groups may insert "known" before "sporadic simple groups" here. The classification is not used in our analysis in this work, which studies only the properties of those 26 individual sporadic groups.

[2] A *p-local subgroup* is the normalizer of a non-identity p-subgroup.

afford an additive decomposition of the mod p group cohomology of G, where the components are given by the cohomology of certain p-local subgroups of G (for example, the normalizers of the p-groups in $\mathcal{C}$). The origins of these decompositions go back to work of Brown, Quillen, and Webb in the 1970s and 1980s. More recently, treatments of certain ample collections (notably by Jackowski, McClure, Oliver, Dwyer, and Grodal) consider *homotopy decompositions*, namely corresponding decompositions not at the level of cohomology, but instead at the deeper topological level of the classifying space BG. Here the decomposition is in fact afforded by a homotopy colimit, in which the component terms are classifying spaces of suitable subgroups of G, and the terms are indexed by a category determined by the collection $\mathcal{C}$. For these purposes, there are various standard ample collections in the literature, such as $\mathcal{S}_p(G)$ defined by all nontrivial p-subgroups; but often the standard collections are too large to be useful for practical computation, so that it is also of interest to determine ample subcollections which are as *small* as possible.

Several years ago, the first author initiated the project that became this work: namely the construction, via a homotopy decomposition over subgroups, of a 2-completed classifying space $BG_2^\wedge$ for each sporadic group G. One motivation was the observation that for a number of the sporadic groups, the group theory literature (especially that on *2-local geometries*) already contained information sufficient to determine a small ample collection, and hence a homotopy decomposition—for example, collections corresponding to the 2-local geometries treated in Smith and Yoshiara [**116**], and the geometry used in Benson [**24**]. For the remaining sporadic groups, progress was initially slower, since the larger groups require substantially more complicated analysis. However, recent work of various authors on the 2-local structure of large sporadic groups has led for example to the completion in Yoshiara [**131**] of the determination of the 2-radical subgroups of the remaining sporadic groups. So, combining such results in group theory with the homotopy colimit theorems indicated above, in the present work we exhibit for each of the 26 sporadic groups a small ample collection related to a 2-local geometry, and the corresponding homotopy decomposition. In each case, since the group acts flag-transitively on the geometry, the relevant homotopy colimit is over an indexing category given by a *simplex*; so that the diagram of classifying spaces is that of a pushout n-cube in the relevant dimension n. (Occasionally cancellations simplify the calculation to an even smaller subdiagram.) We show furthermore that each decomposition is *sharp*, in Dwyer's sense of affording an alternating sum decomposition of group cohomology via the formula of Webb—this simplification arises from the collapse of an underlying spectral sequence. In particular, these results confirm a conjecture (see [**116**, Conj. 1, p. 376]) concerning such a connection between group theory and algebraic topology. That conjecture was necessarily somewhat vague, since the 2-local geometries in the literature were defined in a number of different ways; however, in this work we do indicate for each group a 2-local geometry suitable for our purposes.

Outline of the work. Our treatment is fairly lengthy, so we indicate some of its main features in overview.

Chapter 1 gives a brief summary of our main results.

Since we wish to make our account accessible both to group theorists and to homotopy theorists, we then present in Part 1 an extensive exposition for non-experts of some of the background material involved in each area.

Most of Part 1 is devoted to a review of selected topological material leading up to, and into, the modern literature[3] on decompositions of group cohomology: Chapter 2 recalls some basics of the group cohomology of a finite group G; including aspects of the topological approach via the classifying space BG, and of "approximating" BG—via the Borel construction on a suitable G-space. In the classical literature, these spaces of interest are viewed as topological spaces; but in Chapter 3, we will indicate the more modern viewpoint on spaces as simplicial sets—emphasizing the standard equivalence (due to Quillen) of those two viewpoints, in terms of the homotopy categories they determine. In Chapter 4, we then adopt the viewpoint of simplicial sets—since that is the appropriate general context for our description there of two important constructions due to Bousfield and Kan, namely completions and homotopy colimits. Chapter 5 then reviews some of the more specific literature on homotopy decompositions of the classifying space, given in Dwyer's viewpoint via a homotopy colimit indexed by a collection of p-subgroups; the chapter then examines some of the standard particular collections which are ample in the sense of affording such a decomposition.

Part 1 concludes with Chapter 6, which reviews various notions from the group theory literature, primarily relevant to 2-local geometries for sporadic groups; especially from the viewpoint of their observed connections with the ample collections of 2-subgroups and decompositions described in Chapter 5.

Part 2 then presents the main results of our work. Chapter 7 contains a section on each of the individual sporadic groups G: We indicate a homotopy decomposition for the 2-completed classifying space $BG_2^\wedge$, in terms of a small ample collection determined by a suitable 2-local geometry—with a corresponding additive decomposition of the mod 2 group cohomology of G. We also add, where available, further remarks on the status of knowledge in the literature about the full ring structure of the mod 2 cohomology. In some cases, our results are fairly immediate deductions from results already in the literature; but in the remaining cases where we require a more substantial argument, we sometimes postpone the detailed proofs to sections corresponding to the relevant groups in our final Chapter 8.

We close this Introduction with a brief mention of a topic which we will touch on repeatedly later, but for which we have *not* tried to develop our own exposition in this work:

Foreword on Lie type groups and buildings. Our main results in Chapter 7 are concerned with the 26 sporadic simple groups, viewed via their 2-local geometries; but at various points in our expository Chapters 4–6, it will also be natural to consider examples given instead by the simple groups of Lie type, together with their natural geometries given by Tits buildings. There are a number of reasons for this, among them:

• Buildings are simplicial complexes providing convenient and very natural spaces for a number of the topological constructions we will be examining.

[3] In particular we will frequently quote from a very useful recent exposition on decompositions and their background by Dwyer [**53**]—which we strongly urge readers to consult alongside our exposition.

• Various standard results on groups of Lie type and buildings provide a "model case" for homology decompositions and related research areas—often providing motivation for generalizations to other groups, or even to all finite groups.

• The 2-local geometries for sporadic groups were inspired by certain analogies with buildings.

In particular, our main results for sporadic groups involve a homotopy decomposition indexed by a simplex—and these can be regarded as analogues of the corresponding standard result for Lie type groups, see Example 5.1.18.

When we discuss examples from the theory of Lie type groups, our approach will be to quote the relevant properties from the literature, wherever they arise in our development, for example in Example 4.6.3—rather than attempting to provide a single exposition of that material beforehand; for nowadays a number of fuller introductions to the Lie theory are readily available. (But we do review a fair number of relevant properties at Example 5.1.1.)

In particular, an expository treatment of groups of Lie type, tailored to the general context of our work here, can be found in Section 6.8 of the book [**22**] of the first author. We also mention the expository discussion of buildings given in Chapter 2 of the book [**115**] now in preparation[4] by the second author. These two treatments also list various additional references: on groups of Lie type, we mention especially Carter [**43**] as well as Ch. 2–4 in the third volume [**64**] of the series of Gorenstein, Lyons, and Solomon; and on buildings, Brown [**36**] and Ronan [**105**].

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[4]As of this writing (June 2007), revised Chapters 1–3 are visible on the Web at the URL http://www.math.uic.edu/~smiths/book/book.ps; further revisions will continue at least through later 2007.

Some general notation and conventions

We indicate here at the outset some fairly standard general notation, which we normally follow throughout this work. (Of course we will introduce more specific notation as needed later on in the course of the text.)

We write $A := B$ or $B =: A$ to indicate that A is *defined* to be equal to B; whereas we write $A = B$ to denote an equality between already defined objects.

Conventions for groups. We use G to denote some *finite* group.We emphasize that at least initially, G can be an arbitrary finite group; we will not restrict our focus to simple groups (and in particular, sporadic groups) until Chapter 6. We indicate below some common conventions for general finite groups; for further details, see standard treatments such as Gorenstein, Lyons, and Solomon [**64**].

For a prime p, we write $O_p(G)$ for the largest normal p-subgroup of G, and $O^p(G)$ for the smallest normal subgroup of G for which $G/O^p(G)$ is a p-group. Thus $O^p(G)$ is generated by the p'-elements of of G.

If H and K are subgroups of a group G, we write $[H, K]$ for the subgroup of G generated by the commutators $h^{-1}k^{-1}hk$ where $h \in H$ and $k \in K$. We write $L_n(G)$ for the nth term in the *lower central series* of G, defined by $L_0(G) := G$ and $L_{n+1}(G) := [L_n(G), G]$. We write $L_\infty(G)$ for the intersection of the $L_n(G)$, namely the final term in the lower central series. This is the smallest normal subgroup of G for which the quotient is nilpotent. Since a finite nilpotent group is the direct product of its Sylow p-subgroups, we have

$$L_\infty(G) = \bigcap_p O^p(G); \qquad G/L_\infty(G) \cong \prod_p G/O^p(G). \tag{0.0.1}$$

Suppose that $H \le K \le G$ are finite groups. We say that H is *weakly closed* in K with respect to G if the only G-conjugate of H in K is H itself. In particular, this implies that H is normal in K.

Atlas conventions, with some variations. For the names of simple groups, and for various notation related to their subgroup structure, we use the conventions of the Atlas [**47**]; see the Atlas for fuller details. We summarize some of these usages below, indicating several small modifications that we use in this work. Many of these conventions can be seen in subgroups of the sporadic groups given in the table in Chapter 1.

Group extensions: The notation $N{:}H$ indicates a split extension with normal subgroup N and quotient H; while $N{}^{\cdot}H$ indicates a nonsplit extension. The notation $N\,H$ makes no statement about whether the extension splits. For example, all three of these notations for extensions arise for subgroups in the entry for the Fischer group Fi_{24}' in the table in Chapter 1.

Structure of certain p-groups: If m and n are positive integers, we write m^n for an abelian group $(\mathbb{Z}/m)^n$, and in particular we use a number m to denote $\mathbb{Z}/m$, except where confusion might result from these abbreviations. The notation $p^{a+b+\cdots}$ denotes a p-group having an ascending central series with quotients of rank $a, b, \cdots$; square brackets $[a]$ indicate order p^a with no assertion about a series. In particular, p^{1+2n} denotes an extraspecial p-group of width n; with exponent p when p is odd. When $p = 2$, the two types of extraspecial 2-groups of width n are denoted by 2_+^{1+2n} and 2_-^{1+2n}, where the sign indicates the type of the orthogonal form on the central quotient. In width $n = 1$, we have $2_+^{1+2} = D_8$, the dihedral group of order 8; and $2_-^{1+2} = Q_8$, the quaternion group of order 8. More generally D_{2m} denotes the dihedral group of order $2m$.

Almost-simple groups: We write Σ_n for the symmetric group of degree n, and Alt_n for the alternating group of degree n (the Atlas uses S_n and A_n).[5] We write $\Omega_{2n}^+(2)$ and $\Omega_{2n}^-(2)$ for the simple subgroup (usually the commutator subgroup, of index two) in the orthogonal group over the field $\mathbb{F}_2$ of two elements (the Atlas uses $O_{2n}^+(2)$ and $O_{2n}^-(2)$, which elsewhere is often used instead for the full orthogonal group). Finally, we write $Sp_{2n}(2)$ for the symplectic group over $\mathbb{F}_2$ (the Atlas uses $S_{2n}(2)$). We follow the Atlas in writing $L_n(q)$ and $U_n(q)$ for the simple linear and unitary groups; and $G_2(2)$ for the group of exceptional Lie type G_2 over $\mathbb{F}_2$.

We write $G \wr H$ for the wreath product (copies of G permuted by H); there is really only one instance where we use this, namely in one of the 2-local subgroups of the Harada–Norton group HN in Section 7.19, where

$$\mathrm{Alt}_5 \wr 2 = (\mathrm{Alt}_5 \times \mathrm{Alt}_5){:}2.$$

Conventions for coefficient rings and homology. We let R denote a commutative ring with unit; typically R appears as the coefficient ring for homological algebra.

Often we will specialize R to rings of particular interest: we write $\mathbb{Z}$ for the ring of rational integers, and $\mathbb{F}_p$ for the Galois field of p elements. We write $\mathbb{Z}_{(p)}$ for the ring of p-local integers; namely the local subring of the rationals $\mathbb{Q}$ obtained from $\mathbb{Z}$ by inverting all primes other than p. The quotient ring $\mathbb{Z}_{(p)}/p\mathbb{Z}_{(p)}$ is isomorphic to the field $\mathbb{F}_p$. In Chapter 7 on the sporadic groups, we will specialize to $p = 2$ (though similar results can be obtained for odd p).

For homology and cohomology, we follow the convention that a comma used as a delimiter indicates *group* (co)homology, in expressions such as $H^*(G, R)$; while a semicolon as a delimiter, in expressions such as $H^*(X; R)$, indicates the (co)homology of a *space* X. See for example Proposition 2.2.7 (1) where both notations appear.

NOTATION 0.0.2 (Spectral sequences). Beginning at Section 2.6, we will be considering certain spectral sequences for homology and cohomology. For more detail on general spectral sequences, see for example Chapter 3 of [**22**].

We index homology spectral sequences $E_{s,t}^2$ and cohomology spectral sequences $E_2^{s,t}$ in such a way that s is the horizontal coordinate and t is the vertical coordinate.[6] Consequently the vertical axis is given by the terms with $s = 0$.

[5] We have chosen to use Alt_n, to avoid confusion with our later practice in Chapters 6–8 of using A_n to denote an elementary abelian 2-group of rank n.

[6] Warning for the non-expert: notice this is the convention for Cartesian coordinates; it is the opposite of the linear algebra convention for indexing the rows and columns of a matrix.

We say that the spectral sequence is *first quadrant* if the terms are zero for $s < 0$ and for $t < 0$. The spectral sequences we encounter will be first quadrant.

We recall, for example from [**22**, p. 99]: A (homology) spectral sequence is said to be *strongly convergent* if given s and t, there exists some finite value $n \geq 2$ such that $E^n_{s,t} = E^\infty_{s,t}$; and further for fixed m, the terms $E^\infty_{s,t}$ on the diagonal $s + t = m$ form a set of filtration quotients for the m-th homology group that the spectral sequence is designed to compute. For cohomology spectral sequences, one uses the corresponding notation $E_n^{s,t}$ and $E_\infty^{s,t}$ in the definition. *It is standard that first quadrant spectral sequences are strongly convergent, so this convergence holds for the sequences we will encounter.* $\diamondsuit$

NOTATION 0.0.3 (Limits and colimits). We shall use the terms *limit* and *colimit* (written $\varprojlim$ and $\varinjlim$) to refer to what are sometimes known as *inverse limit* and *direct limit* (see for example Spanier [**117**, p. 18]), or as *projective limit* and *inductive limit* respectively (depending on when and where you were educated).

We recall some features of these functors related to the standard setting in homological algebra of exactness, derived functors, and adjointness; for further reference, see for example Benson [**21**, p. 22] [**22**, Sec. 7.2] or Weibel [**125**, Sec. 2.6]. For modules, the functor $\varprojlim$ is left exact; and the right derived functors of $\varprojlim$ are written $\varprojlim^i$. The functor $\underrightarrow{\mathrm{colim}}$ is right exact; and its left derived functors are written $\underrightarrow{\mathrm{colim}}_i$. (See for example [**21**, Exer., p. 244]; and 2.6.9 in [**125**], as well as 2.6.4 and 2.6.1 there.) These left derived functors will be described in more detail when we need them later, in Section 4.8. $\diamondsuit$

Conventions for spaces. Through Chapter 2, the letter X will usually denote a topological space. For reference on classical algebraic topology, see standard texts such as Spanier [**117**].

In the chapters thereafter, much of the topological material will be developed instead in the language of simplicial sets (see for example May [**91**] for further reference); so starting just before Section 3.7, we will use the word "space" to mean simplicial set. Then X will usually denote such a space, and we will apply various topological notions to spaces in this new sense.

However, we will recall in Section 3.6 that the viewpoints of topological spaces and of simplicial sets are in fact equivalent for our present purposes. Hence even in our later chapters, the reader can typically still think of a space X informally just as a classical topological space.

We denote a space (in either of the above senses) which consists of a single point by a star $\star$; sometimes $\star$ will be contained as a basepoint in a larger space. We use a 5-pointed star, to avoid conflict with the use of an asterisk $*$ (6-pointed star) for several other purposes: namely the customary sequence of dimensions in homological expressions such as $H^*(G, R)$, and the join $X * Y$ of simplicial complexes or topological spaces.

If f and g are maps of spaces from X to Y, we write $f \simeq g$ if f and g are homotopic. We write $X \simeq Y$ if spaces X and Y are homotopy equivalent. (We use the symbol $\cong$ to denote an isomorphism, and we usually then also specify the category in which that map is a morphism.)

We follow the notational convention from topology that if X is a set or a space admitting action of the group G, then X^g denotes the fixed points of $g \in G$, and

X^G the fixed points of G. If X is a subset of G, we use $g^{-1}Xg$ to denote conjugation by g; this avoids the ambiguity of using the group theory convention of X^g for this conjugation.

We shall write $X_p^\wedge$ for the Bousfield–Kan $\mathbb{F}_p$-completion $(\mathbb{F}_p)_\infty X$ of X; see Notation 4.3.5.

Finally, we mention that our Index also serves as a reference for further notation and definitions.

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