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Ramsey Theory on the Integers

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10 9 8 7 6 5 4 3 2 1 09 08 07 06 05 04

To

Eleanor

Emma and Sarah

–Bruce

To

Elisa

Quinn and Ava

–aaron

Contents

List of Tables

Preface

Ramsey Theory on the Integers covers a variety of topics from the field of Ramsey theory, limiting its focus to the set of integers – an area that has seen a remarkable burst of research activity during the past twenty years.

The book has two primary purposes: (1) to provide students with a gentle, but meaningful, introduction to mathematical research – to give them an appreciation for the essence of mathematical research and its inescapable allure and also to get them started on their own research work; (2) to be a resource for all mathematicians who are interested in combinatorial or number theoretical problems, particularly "Erdős-type" problems.

Many results in Ramsey theory sound rather complicated and can be hard to follow; they tend to have a lot of quantifiers and may well involve objects whose elements are sets whose elements are sets (that is not a misprint). However, when the objects under consideration are sets of integers, the situation is much simpler. The student need not be intimidated by the words "Ramsey theory," thinking that the subject matter is too deep or complex – it is not! The material in this book is, in fact, quite accessible. This accessibility, together with the fact that scores of questions in the subject are still to be answered, makes Ramsey theory on the integers an ideal subject for a student's first research experience. To help students find suitable

projects for their own research, every chapter includes a section of "Research Problems," where we present a variety of unsolved problems, along with a list of suggested readings for each problem.

Ramsey Theory on the Integers has several unique features. No other book currently available on Ramsey theory offers a cohesive study of Ramsey theory on the integers. Among several excellent books on Ramsey theory, probably the most well-known, and what may be considered *the* Ramsey theory book, is by Graham, Rothschild, and Spencer (*Ramsey Theory,* 2nd *Edition* [**127**]). Other important books are by Graham (*Rudiments of Ramsey Theory* [**122**]), McCutcheon (*Elemental Methods in Ergodic Ramsey Theory* [**184**]), Nešetřil and Rödl (*Mathematics of Ramsey Theory* [**199**]), Prömel and Voigt (*Aspects of Ramsey Theory* [**207**]), Furstenberg (*Dynamical Methods in Ramsey Theory* [**111**]), and Winn (*Asymptotic Bounds for Classical Ramsey Numbers* [**274**]). These books, however, generally cover a broad range of subject matter of which Ramsey theory on the integers is a relatively small part. Furthermore, the vast majority of the material in the present book is not found in any other book. In addition, to the best of our knowledge, ours is the only Ramsey theory book that is accessible to the typical undergraduate mathematics major. It is structured as a textbook, with numerous (over 150) exercises, and the background needed to read the book is rather minimal: a course in elementary linear algebra and a 1-semester junior-level course in abstract algebra would be sufficient; an undergraduate course in elementary number theory or combinatorics would be helpful, but not necessary. Finally, *Ramsey Theory on the Integers* offers something new in terms of its potential appeal to the research community in general. Books offering a survey of solved and unsolved problems in combinatorics or number theory have been quite popular among researchers; they have also proven beneficial by serving as catalysts for new research in these fields. Examples include *Old and New Problems and Results in Combinatorial Number Theory* [**92**] by Erdős and Graham, *Unsolved Problems in Number Theory* [**135**] by Guy, and *The New Book of Prime Number Records* [**220**] by Ribenboim. With our text we hope to offer mathematicians an additional resource for intriguing unsolved problems. Although not

nearly exhaustive, the present book contains perhaps the most substantial account of solved and unsolved problems in Ramsey theory on the integers.

This text may be used in a variety of ways:

- as an undergraduate or graduate textbook for a second course in combinatorics or number theory;
- in an undergraduate or graduate seminar, a capstone course for undergraduates, or an independent study course;
- by students working under an REU program, or who are engaged in some other type of research experience;
- by graduate students looking for potential thesis topics;
- by the established researcher seeking a worthwhile resource in its material, its list of open research problems, and its somewhat enormous (often a fitting word when discussing Ramsey theory) bibliography.

Chapter 1 provides preliminary material (for example, the pigeonhole principle) and a brief introduction to the subject, including statements of three classical theorems of Ramsey theory: van der Waerden's theorem, Schur's theorem, and Rado's theorem. Chapter 2 covers van der Waerden's theorem; Chapters 3–7 deal with various topics related to van der Waerden's theorem; Chapter 8 is devoted to Schur's theorem and a generalization; Chapter 9 explores Rado's theorem; and Chapter 10 presents several other topics involving Ramsey theory on the integers.

The text provides significant latitude for those designing a syllabus for a course. The only material in the book on which other chapters depend is that through Section 2.2. Thus, other chapters or sections may be included or omitted as desired, since they are essentially independent of one another (except for an occasional reference to a previous definition or theorem). We do, however, recommend that all sections included in a course be studied in the same order in which they appear in the book.

Each chapter concludes with a section of exercises, a section of research problems, and a reference section. Since the questions contained in the Research Problem sections are still open, we cannot say

with certainty how difficult a particular one will be to solve; some may actually be quite simple and inconsequential. The problems that we deem most difficult, however, are labeled with the symbol $*$. The reference section of each chapter is organized by section numbers (including the exercise section). The specifics of each reference are provided in the bibliography at the end of the book.

The material covered in this book represents only a portion of the subject area indicated by the book's title. Many additional topics have been investigated, and we have attempted to include at least references for these in the reference sections. Yet, for every problem that has been thought of in Ramsey theory, there are many more which that problem will generate and, given the great variety of combinatorial structures and patterns that lie in the set of integers, countless new problems wait to be explored.

We would like to thank Dr. Edward Dunne and the members of the AMS production staff for their assistance in producing this book. We also thank Tom Brown, Scott Gordon, Jane Hill, Dan Saracino, Dan Schaal, Ralph Sizer, and the AMS reviewers for their helpful comments and advice, which greatly improved the manuscript. We also express our gratitude to Ron Graham and Doron Zeilberger for their support of this project. We owe a big debt to the pioneers and masters of the field, especially Ron Graham, Jarik Nešetřil, Joel Spencer, Neil Hindman, Tom Brown, Timothy Gowers, Hillel Furstenberg, Vitaly Bergelson, Vojtěch Rödl, Endre Szemerédi, László Lovász (we had to stop somewhere), and of course Bartel van der Waerden, Issai Schur, Richard Rado, and Frank Ramsey. To all of the others who have contributed to the field of Ramsey theory on the integers, we extend our sincere appreciation. Finally, we want to acknowledge that this book would not exist without the essential contributions of the late Paul Erdős. But beyond the content of his achievements, he has personally inspired the authors as mathematicians. Our professional lives would have had far less meaning and fulfillment without his work and his presence in our field. For that pervasive, though perhaps indirect, contribution to this text, we are in his debt.

Notation

Notation	Description	Page
$\lceil \cdot \rceil$	Ceiling function	10
$\lfloor \cdot \rfloor$	Floor function	10
$\oplus$	Modular addition	107
$[a, b]$	$\{a, a+1, \ldots, b\}$	9
$A - B$	$\{x \in A : x \notin B\}$	9
A_D	Family of arithmetic progressions with gaps in D	104
AP	Family of arithmetic progressions	14
$AP_{a(m)}$	Family of arithmetic progressions with gaps congruent to $a \pmod m$	163
$AP^*_{a(m)}$	$AP_{a(m)} \cup A_{\{m\}}$	168
$AP_{(m)}$	Set of arithmetic progressions (mod m)	164
AUG_b	Family of augmented progressions with tail b	152
$B(f; r)$	Brown number	278
cul_j	Culprit of color j	32
$\Delta_{\{a,b,c\}}$	Triangle on vertices a, b, c	205
$\Delta(D, k; r)$	Diffsequence Ramsey-type number	266
doa	Degree of accessibility	270
dor	Degree of regularity	138
dor_k	Degree of regularity for $T_{k-1}(a)$	145
$DW(k)$	Descending wave 2-color Ramsey-type number	70
$F(k; r)$	Folkman number	262
$\Gamma_m(k)$	Least s guaranteeing k-term arithmetic progressions in all s-term $[1, m]$-gap sequences	73
$GQ_\delta(k)$	2-color Ramsey-type function for generalized quasi-progressions	67

Notation	Description	Page
$g(r,k)$	Least integer such that for all S with $\|S\| = g$ and all $\chi : S \to \{0,\ldots,r-1\}$, whenever $\|\chi(S)\| = k$ there exist $t_1, t_2, \ldots, t_r \in S$ with $(t_1, t_2, \ldots, t_r)$ zero-sum	286
gs	Gap size	278
(k,n,d)-progression	k-term quasi-progression with diameter n and low-difference d	57
$H(s_1,\ldots,s_k)$	2-color Ramsey-type function for homothetic copies of $\{1, 1+s_1, \ldots, 1+s_1+\cdots+s_k\}$	147
$\lambda(c,k;r)$	Special r-coloring of $[1, cr(k-1)^2]$ that avoids monochromatic k-term c-a.p.'s	112
$\mathcal{L}(t)$	Equation $x_1 + \cdots + x_{t-1} = x_t$	212
$\mu(k)$	$\min\{\|\mathcal{E}\| : \Gamma = (V,\mathcal{E})$ is a hypergraph not satisfying Property B and $\|E\| = k$ for all $E \in \mathcal{E}\}$	36
$M_\chi(n)$	Number of monochromatic Schur triples in $[1,n]$ under χ	205
$\nu(k)$	Erdős and Turán function	41
$\Omega_m(k)$	Least n so that every $\{x_1,\ldots,x_n\}$ with $x_i \in [(i-1)m, im-1]$ contains a k-term arithmetic progression	182
P_n	Family of sequences generated by iteration of a polynomial of degree n	81
$P_{n,k}$	Family of k-term members of P_n	81
$Q_n(k)$	2-color Ramsey-type function for quasi-progressions	57
$R(AP^*_{a(m)}, k, l; r)$	Generalization of $R(AP^*_{a(m)}, k; r)$	168
$r(\mathcal{E}; s)$	Rado number for equation $\mathcal{E}$	230
$R(\mathcal{F}, k; r)$	Ramsey-type number for family $\mathcal{F}$	14
$R(k_1,\ldots,k_r)$	r-color (classical) Ramsey number	8
$R_r(k)$	$R(\underbrace{k,\ldots,k}_{r})$	8
$RR(S;r)$	Reverse r-regular number	185
$S(k_1,\ldots,k_r)$	Generalized Schur number	212
$\hat{S}(k_1,\ldots,k_r)$	Strict generalized Schur number	218
S_n	Family of sequences generated by iteration of a polynomial of degree at most n	82
$S_{n,k}$	Family of k-term members of S_n	82
$SP_m(k)$	Ramsey-type number for semi-progressions	72
$s(r)$	Schur number	201
$\hat{s}(r)$	Strict Schur number	217
$T_{a,b}$	Set of (a,b)-triples	136
$T(a,b;r)$	Ramsey-type number for (a,b)-triples	136

Notation	**Description**	**Page**
$T(a_1, \ldots, a_{k-1})$	Ramsey-type function for generalization of (a, b)-triples	145
$\Theta(n)$	Set of permutations of $[1, n]$ with no 3-term arithmetic progression	194
$\theta(n)$	$\lvert\Theta(n)\rvert$	193
V_m	$\{x \in \mathbb{Z}^+ : m \nmid x\}$	273
$V_{m,n}$	$\{x \in \mathbb{Z}^+ : m \nmid x, n \nmid x\}$	289
$w(k)$	$w(k; 2)$	25
$w(k; r)$	van der Waerden number	11
$w(k_1, \ldots, k_r; r)$	Mixed van der Waerden number	33
$w'(c, k; r)$	Ramsey-type number for arithmetic progressions with gaps at least c	111
$w'(f(x), k; r)$	Ramsey-type number for f-a.p.'s	116
$w^*(k, j)$	Ramsey-type number for arithmetic progressions with color discrepancy at least j	187
$\mathbb{Z}^+$	Positive integers	9

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