



# FIELDS INSTITUTE COMMUNICATIONS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

## Lectures on Global Optimization

Panos M. Pardalos  
Thomas F. Coleman  
Editors



American Mathematical Society  
The Fields Institute  
for Research in Mathematical Sciences





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Providence, Rhode Island

**The Fields Institute**  
**for Research in Mathematical Sciences**  
Toronto, Ontario



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## Preface

A large number of mathematical models in many diverse areas of science and engineering have lead to the formulation of optimization problems where the best solution (globally optimal) is needed. Due to the interdisciplinary nature of global optimization, there has been astonishing progress in this field during the last few decades. Many powerful computational algorithms and new theoretical developments have been introduced to solve a spectrum of hard problems in several disciplines.

In this book, we cover some of the recent important topics in global optimization with emphasis on recent theoretical developments and scientific applications.

During May 11-12, 2007, a workshop on “Global Optimization: Methods and Applications” was held at the Fields Institute. The organizers were Thomas F. Coleman (University of Waterloo), Panos Pardalos (University of Florida), and Stephen Vavasis (University of Waterloo). The purpose of the workshop was to explore recent practical progress made in the area of global optimization, especially in reference to industrial applications. Our goal was to develop connections among researchers and field engineers in this area and raise awareness of current methodologies and approaches among practitioners. This book is based on the presentations at the workshop and reflects the diverse spectrum of theoretical results and applications.

The target audience of the book includes graduate students in mathematics, engineering, and sciences, academic researchers, as well as practitioners who use global optimization for their specific needs and applications.

We would like to take this opportunity to express gratitude to the Fields Institute, the School of Mathematics (University of Waterloo), and the Center for Applied Optimization (University of Florida), for their help in organizing this workshop. We would also like to thank the authors of the papers, the anonymous referees, and Debbie Iscoe of the Fields Institute for helping us with the final typesetting of the book.

Panos M. Pardalos (University of Florida)  
Thomas F. Coleman (University of Waterloo)





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