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Masayoshi Miyanishi

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Masayoshi Miyanishi

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Preface

“Open algebraic surface” is a synonym for “algebraic surface which is not necessarily complete”. An open algebraic surface is understood to be a Zariski open set of a complete algebraic surface. An affine algebraic surface is a typical example of an open algebraic surface, and it is embedded into a projective algebraic surface as the complement of a hypersurface section of the projective surface.

There is a long history of research on projective algebraic surfaces and we have the beautiful Enriques–Kodaira classification of such surfaces. Thanks to the work of many predecessors, we have powerful methods to look into the geometry and topology of projective algebraic surfaces. Compared to the status of the theory of projective (or complete) algebraic surfaces there was limited knowledge to study the geometric or topological structure of open algebraic surfaces until the early 1970's, when those people like C. P. Ramanujam, S. S. Abhyankar, T. T. Moh, and M. Nagata began to consider the following problems:

(1) Embedding of the affine line into the affine plane.
(2) The cancellation problem, which asks whether or not an affine variety X is isomorphic to the affine space $\mathbb{A}^n$ of dimension n provided $X \times \mathbb{A}^m$ is isomorphic to the affine space $\mathbb{A}^{n+m}$.
(3) The structure of the automorphism group $\mathrm{Aut}(\mathbb{A}^n)$ of the affine space $\mathbb{A}^n$ of dimension n.

The difficulty lies in the lack of methods to examine the geometric structure of affine varieties and the behavior of a curve, say in the affine plane $\mathbb{A}^2$, when it approaches the points at infinity. It was gradually revealed through the works, [**1, 70, 74**], for example, how to overcome the difficulty.

A turning point in research into non-complete algebraic varieties probably occurred when Iitaka [**33**] introduced the notion of logarithmic Kodaira dimension of an algebraic variety and indicated the possibility of classifying non-complete algebraic varieties via logarithmic Kodaira dimension in a way parallel to the classification of projective algebraic varieties via Kodaira dimension (or canonical dimension). Indeed, when a smooth algebraic variety X is embedded into a smooth projective algebraic variety V in such a way that $D := V - X$ is a divisor with simple normal crossings, then the logarithmic Kodaira dimension of X is defined as an invariant to measure the growth of $\dim H^0(V, n(D + K_V))$ when n tends to the positive infinity. It is shown to be proper to the variety X, that is to say, independent of the embedding X into V of the above kind. In the surface case, this suggested considering the Enriques–Kodaira classification in the framework of open algebraic surfaces.

This book is organized to follow this suggestion. Namely, we recall the Enriques–Kodaira classification in Chapter 1 and develop the classification theory of open algebraic surfaces in Chapter 2. To compare the development of both theories, we explain the complete case by going rather too much into the details. But we believe that the theory of open algebraic surfaces can be better understood by the comparison of both theories. To write Chapter 1, we basically followed the references [**6, 67**]. The author believes that the theory of open algebraic surfaces should be extended also to the case where the ground field has positive characteristic. Hence Chapter 1 is written to cover the case of positive characteristic as well. Chapter three is mostly for the accounts how the theory of open algebraic surfaces is applied to affine algebraic surfaces.

The author was invited to write a book on open algebraic surfaces by Peter Russell of McGill University and Martin Goldstein of the Centre de recherches mathématiques, Université de Montréal when the author visited Montreal in 1994 on the occasion of the Canadian Mathematical Society Winter Meeting. He is very grateful to them for constant encouragement during the writing of the manuscript.

M. Miyanishi
July, 1999

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Open Algebraic Surfaces

Masayoshi Miyanishi

Open algebraic surfaces are a synonym for algebraic surfaces that are not necessarily complete. An open algebraic surface is understood as a Zariski open set of a projective algebraic surface. There is a long history of research on projective algebraic surfaces, and there exists a beautiful Enriques-Kodaira classification of such surfaces. The research accumulated by Ramanujan, Abhyankar, Moh, and Nagata and others has established a classification theory of open algebraic surfaces comparable to the Enriques-Kodaira theory. This research provides powerful methods to study the geometry and topology of open algebraic surfaces.

The theory of open algebraic surfaces is applicable not only to algebraic geometry, but also to other fields, such as commutative algebra, invariant theory, and singularities. This book contains a comprehensive account of the theory of open algebraic surfaces, as well as several applications, in particular to the study of affine surfaces. Prerequisite to understanding the text is a basic background in algebraic geometry. This volume is a continuation of the work presented in the author's previous publication, *Algebraic Geometry*, volume 136 in the AMS series, Translations of Mathematical Monographs.

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