CONTEMPORARY MATHEMATICS

691

Around Langlands Correspondences

International Conference Around Langlands Correspondences June 17–20, 2015 Université Paris Sud, Orsay, France

Farrell Brumley Maria Paula Gómez Aparicio Alberto Mínguez Editors



American Mathematical Society

Around Langlands Correspondences

CONTEMPORARY MATHEMATICS

691

Around Langlands Correspondences

International Conference Around Langlands Correspondences June 17–20, 2015 Université Paris Sud, Orsay, France

Farrell Brumley Maria Paula Gómez Aparicio Alberto Mínguez Editors



American Mathematical Society Providence, Rhode Island

EDITORIAL COMMITTEE

Dennis DeTurck, Managing Editor

Michael Loss Kailash Misra Catherine Yan

2010 Mathematics Subject Classification. Primary 20G25, 22E50, 11S37, 19L47, 22E45, 11F55, 11R39, 20C33, 11F85, 11F67.

Library of Congress Cataloging-in-Publication Data

- Names: Brumley, Farrell, 1976– editor. | Gomez Aparicio, Maria Paula, 1980– editor. | Minguez, Alberto, 1979– editor.
- Title: Around Langlands correspondences: International conference on around Langlands correspondences, June 17-20, 2015, Université Paris Sud, Orsay, France / Farrell Brumley, Maria Paula Gómez Aparicio, Alberto Mínguez, editors.

Description: Providence, Rhode Island: American Mathematical Society, [2017] | Series: Contemporary mathematics; volume 691 | Includes bibliographical references.

Identifiers: LCCN 2016049805 | ISBN 9781470435738 (alk. paper)

Subjects: LCSH: Algebraic number theory – Congresses. | Galois theory – Congresses. | AMS: Group theory and generalizations – Linear algebraic groups and related topics – Linear algebraic groups over local fields and their integers. msc | Topological groups, Lie groups – Lie groups – Representations of Lie and linear algebraic groups over local fields. msc | Number theory – Algebraic number theory: local and p-adic fields – Langlands-Weil conjectures, non-abelian class field theory. msc | K-theory – Topological K-theory – Equivariant K-theory. msc | Topological groups, Lie groups – Lie groups – Vertex (Number theory – Topological groups, Lie groups and automorphic forms – Other groups and their modular and automorphic forms (several variables). msc | Number theory – Algebraic number theory: global fields – Langlands-Weil conjectures, nonabelian class field theory. msc | Group theory and generalizations – Representation theory of groups – Representations of finite groups of Lie type. msc | Number theory – Discontinuous groups and automorphic forms – p-adic theory, local fields. msc | Number theory – Discontinuous groups and automorphic forms – special values of automorphic L-series, periods of modular forms, cohomology, modular symbols. msc

Classification: LCC QA247 . A627 2015 | DDC 512/.2–dc23

Contemporary Mathematics ISSN: 0271-4132 (print); ISSN: 1098-3627 (online)

DOI: http://dx.doi.org/10.1090/conm/691

Send requests for translation rights and licensed reprints to reprint-permission@ams.org.

Excluded from these provisions is material for which the author holds copyright. In such cases, requests for permission to reuse or reprint material should be addressed directly to the author(s). Copyright ownership is indicated on the copyright page, or on the lower right-hand corner of the first page of each article within proceedings volumes.

© 2017 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights

except those granted to the United States Government.

Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability. Visit the AMS home page at http://www.ams.org/

 $10 \ 9 \ 8 \ 7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1 \qquad 22 \ 21 \ 20 \ 19 \ 18 \ 17$

LC record available at https://lccn. loc.gov/2016049805

Color graphic policy. Any graphics created in color will be rendered in grayscale for the printed version unless color printing is authorized by the Publisher. In general, color graphics will appear in color in the online version.

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy select pages for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Permissions to reuse portions of AMS publication content are handled by Copyright Clearance Center's RightsLink[®] service. For more information, please visit: http://www.ams.org/rightslink.

In memory of François Courtès

Contents

Preface	ix
List of talks	xi
List of posters	xiii
Change of weight theorem for pro- <i>p</i> -Iwahori Hecke algebras NORIYUKI ABE	1
Conjectures about <i>p</i> -adic groups and their noncommutative geometry ANNE-MARIE AUBERT, PAUL BAUM, ROGER PLYMEN, and MAARTEN SOLLEVELD	15
Introduction to the Rapid Decay property INDIRA CHATTERJI	53
A second adjoint theorem for $SL(2, \mathbb{R})$ TYRONE CRISP and NIGEL HIGSON	73
A functoriality principle for blocks of <i>p</i> -adic linear groups JEAN-FRANÇOIS DAT	103
Poids de Serre dans la conjecture de Breuil–Mézard Agnès David	133
Affinoids in Lubin-Tate surfaces with exponential full level two NAOKI IMAI and TAKAHIRO TSUSHIMA	157
An automorphic variant of a conjecture of Deligne JIE LIN	181
Paquets d'Arthur des groupes classiques complexes COLETTE MOEGLIN and DAVID RENARD	203
Proof of the Aubert-Baum-Plymen-Solleveld conjecture for split classical groups	
Ahmed Moussaoui	257
From crystalline to unitary representations ENNO NAGEL	283
Representations of GL_N over finite local principal ideal rings: An overview ALEXANDER STASINSKI	337
The geometry and combinatorics of Springer fibers JULIANNA TYMOCZKO	359

Preface

Over the past 50 years, the Langlands program has served as an organizing principle in such disparate mathematical areas as arithmetic, physics, geometry, and representation theory, while steadily revealing a fascinating web of underlying laws that link them. At the heart of the program are the Langlands correspondences, relating the spectra of reductive algebraic groups over local and global fields and parametrizing their representations by Galois theoretic data.

In the classical version of the correspondence, the coefficients for the representations are taken in the field of complex numbers. In recent years, even more mysterious Langlands correspondences have been proposed with more general underling fields, injecting an already rich subject with substantially more arithmetic and geometric features. For instance, there is now a modular version, in both mod $\ell \neq p$ and mod p incarnations; there is a p-adic Langlands correspondence and a geometric Langlands correspondence. All of these are in various states of development. On the other hand, the existence of subtle links have progressively emerged between the classical local Langlands correspondence and noncommutative geometry via the Baum-Connes conjecture.

This volume contains the proceedings of the conference "Around Langlands Correspondences", the purpose of which was to examine recent developments in various versions of the Langlands correspondence and in related domains, and also to highlight the fruitful interactions among them. The event was held in Orsay, at Université Paris Sud, from June 17–20, 2015. The contributions cover a wide range of topics, including representations of pro-p-Iwahori Hecke algebras, enhanced Langlands parameters, critical values for automorphic L-functions, functoriality principles, representations of real and p-adic reductive groups and of groups over finite local rings, Arthur packets, Galois representations, Springer fibers and Schubert varieties, and the Rapid Decay property for finitely generated groups. It presents both expository and research articles.

The conference was part of a larger project, "La correspondance de Langlands", financed by a newly established program by the Centre Nationale de la Recherche Scientifique (CNRS) whose primary goal was to promote the role of women in mathematics. This is an exploratory grant project "Egalité", which was advanced by the Institut National des Sciences Mathématiques et de leurs Interactions (INSMI) of the CNRS, the Mission pour la Place des Femmes au CNRS, as well as the Institutional Transformation for Effecting Gender Equality in Research (INTEGER) European Project. It is our hope that the success of this conference will serve to bolster the continuation of such initiatives.

Prior to the main research conference, a one-day meeting for French high school students, "Correspondences", was held in April 2015 at Université Pierre et Marie

PREFACE

Curie. There were several interactive workshops with small groups of students and a general audience talk on Bhargava's composition law on cubes. The day ended with a speedcubing performance by the inimitable French mathematician François Courtès, who managed to hold the otherwise tired students' rapt attention. François passed away suddenly in September 2016, leaving behind a body of mathematical work notable for its technical mastery and creativity. We dedicate these proceedings to his memory.

> Farrell Brumley Maria Paula Gómez Aparicio Alberto Mínguez

x

List of talks

Ramla Abdellatif ENS Lyon Extensions between simple Hecke-Iwahori modules for SL(2, F)

Noriyuki Abe Hokkaido University and IMJ-PRG A classification of irreducible admissible modulo p representations of reductive p-adic groups

Anne-Marie Aubert IMJ-PRG, UPMC A bridge between local Langlands and Baum-Connes correspondences

Ana Caraiani Princeton University On vanishing of torsion in the cohomology of Shimura varieties

Indira Chatterji Université de Nice An introduction to the Rapid Decay Property

Joel Cohen 12M, Marseille Une identité spectrale pour une certaine intégrale orbitale tordue

Jean-Francois Dat IMJ-PRG, UPMC Principe de fonctorialité pour les blocs de groupes p-adiques

Maria Paula Gomez Aparicio Université Paris Sud 11 The Baum-Connes Conjecture and Bost's Oka's principle Nigel Higson Penn State University The Langlands classification and the Baum-Connes-Kasparov isomorphism

Vincent Lafforgue Université d'Orléans Chtoucas pour les groupes réductifs et paramétrisation de Langlands

Judith Ludwig Universität Bonn *p-adic Langlands functoriality and L-indistinguishability of overconvergent forms*

Colette Moeglin IMJ-PRG, UPMC Sur les paquets d'Arthur aux places réelles

Enno Nagel IMJ-PRG, UPMC The Fourier Basis of p-adic Differentiable Functions

Dipendra Prasad Tata Institute of Fundamental Research Branching laws and period integrals for non-tempered representations

Benjamin Schraen Université Versailles St-Quentin Caractère infinitésimal et représentations de Banach p-adiques

Maarten Solleveld Radboud Universiteit Nijmegen Reduction of the local Langlands correspondence to supercuspidal representations Alexander Stasinski Durham University Constructing the regular representations of GL(N) over local rings

Julianna Tymoczko Smith College Geometry and combinatorics of Springer fibers

xii

List of posters

Agnès David Université de Franche-Comté Genetics of Galois deformations

Naoki Imai Université de Tokyo Affinoids in the Lubin-Tate perfectoid space and simple epipelagic representations

Jie Lin IMJ-PRG, UPMC Critical values for automorphic L-functions

Sergio Mendes Université de Lisbonne On L-packets and depth for $SL_2(K)$

Manish Mishra University of Heidelberg Bernstein center of supercuspidal blocks

Ahmed Moussaoui IMJ-PRG, UPMC Centre de Bernstein stable et conjecture d'Aubert-Baum-Plymend-Solleveld

Sanaz Pooya Université de Neuchâtel Simple reduced L^p -operator crossed products with unique trace

Anna Szumowicz IMJ-PRG and Université Jagellonne de Cracovie Minimising volumes in number fields

Selected Published Titles in This Series

- 691 **Farrell Brumley, Maria Paula Gómez Aparicio, and Alberto Mínguez, Editors,** Around Langlands Correspondences, 2017
- 688 Jeffrey Bergen, Stefan Catoiu, and William Chin, Editors, Groups, Rings, Group Rings, and Hopf Algebras, 2017
- 687 Fernanda Botelho, Raena King, and T. S. S. R. K. Rao, Editors, Problems and Recent Methods in Operator Theory, 2017
- 686 Alp Bassa, Alain Couvreur, and David Kohel, Editors, Arithmetic, Geometry, Cryptography and Coding Theory, 2017
- 685 Heather A. Harrington, Mohamed Omar, and Matthew Wright, Editors, Algebraic and Geometric Methods in Discrete Mathematics, 2017
- 684 Anna Beliakova and Aaron D. Lauda, Editors, Categorification in Geometry, Topology, and Physics, 2017
- 683 Anna Beliakova and Aaron D. Lauda, Editors, Categorification and Higher Representation Theory, 2017
- 682 Gregory Arone, Brenda Johnson, Pascal Lambrechts, Brian A. Munson, and Ismar Volić, Editors, Manifolds and K-Theory, 2017
- 681 Shiferaw Berhanu, Nordine Mir, and Emil J. Straube, Editors, Analysis and Geometry in Several Complex Variables, 2017
- 680 Sergei Gukov, Mikhail Khovanov, and Johannes Walcher, Editors, Physics and Mathematics of Link Homology, 2016
- 679 Catherine Bénéteau, Alberto A. Condori, Constanze Liaw, William T. Ross, and Alan A. Sola, Editors, Recent Progress on Operator Theory and Approximation in Spaces of Analytic Functions, 2016
- 678 Joseph Auslander, Aimee Johnson, and Cesar E. Silva, Editors, Ergodic Theory, Dynamical Systems, and the Continuing Influence of John C. Oxtoby, 2016
- 677 Delaram Kahrobaei, Bren Cavallo, and David Garber, Editors, Algebra and Computer Science, 2016
- 676 **Pierre Martinetti and Jean-Christophe Wallet, Editors,** Noncommutative Geometry and Optimal Transport, 2016
- 675 Ana Claudia Nabarro, Juan J. Nuño-Ballesteros, Raúl Oset Sinha, and Maria Aparecida Soares Ruas, Editors, Real and Complex Singularities, 2016
- 674 Bogdan D. Suceavă, Alfonso Carriazo, Yun Myung Oh, and Joeri Van der Veken, Editors, Recent Advances in the Geometry of Submanifolds, 2016
- 673 Alex Martsinkovsky, Gordana Todorov, and Kiyoshi Igusa, Editors, Recent Developments in Representation Theory, 2016
- 672 Bernard Russo, Asuman Güven Aksoy, Ravshan Ashurov, and Shavkat Ayupov, Editors, Topics in Functional Analysis and Algebra, 2016
- 671 Robert S. Doran and Efton Park, Editors, Operator Algebras and Their Applications, 2016
- 670 Krishnendu Gongopadhyay and Rama Mishra, Editors, Knot Theory and Its Applications, 2016
- 669 Sergii Kolyada, Martin Möller, Pieter Moree, and Thomas Ward, Editors, Dynamics and Numbers, 2016
- 668 **Gregory Budzban, Harry Randolph Hughes, and Henri Schurz, Editors,** Probability on Algebraic and Geometric Structures, 2016
- 667 Mark L. Agranovsky, Matania Ben-Artzi, Greg Galloway, Lavi Karp, Dmitry Khavinson, Simeon Reich, Gilbert Weinstein, and Lawrence Zalcman, Editors, Complex Analysis and Dynamical Systems VI: Part 2: Complex Analysis, Quasiconformal Mappings, Complex Dynamics, 2016

For a complete list of titles in this series, visit the AMS Bookstore at www.ams.org/bookstore/comseries/.

This volume contains the proceedings of the international conference "Around Langlands Correspondences", held from June 17–20, 2015, at Université Paris Sud in Orsay, France.

The Langlands correspondence (nowadays called the usual Langlands correspondence), conjectured by Robert Langlands in the late 1960s and early 1970s, has recently seen some new mysterious generalizations: the modular Langlands correspondence, the *p*-adic Langlands correspondence, and the geometric Langlands correspondence, the last of which seems to share deep connections with the Baum-Connes conjecture.

The aim of this volume is to present, through a mix of research and expository articles, some of the fascinating new directions in number theory and representation theory arising from recent developments in the Langlands program. Special emphasis is placed on nonclassical versions of the conjectural Langlands correspondences, where the underlying field is no longer the complex numbers.



