

# CONTEMPORARY MATHEMATICS

675

## Real and Complex Singularities

XIII International Workshop  
Real and Complex Singularities  
July 27–August 8, 2014  
Universidade de São Paulo, Campus São Carlos,  
São Carlos, Brazil

Ana Claudia Nabarro  
Juan J. Nuño-Ballesteros  
Raúl Oset Sinha  
Maria Aparecida Soares Ruas  
Editors



American Mathematical Society  
Real Sociedad Matemática Española



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American Mathematical Society  
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*This volume is dedicated to Carmen Romero Fuster  
on the occasion of her 60th birthday*



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## Preface

This volume contains the proceedings of the XIII International Workshop on Real and Complex Singularities, held in honour of María del Carmen Romero Fuster in celebration of her sixtieth birthday. The meeting was part of a highly successful series of biennial conferences organized by the Singularity Theory group at São Carlos, Universidade de São Paulo, Brazil. Its main theme was the singularity theory of spaces and maps, and the applications of singularity theory to differential geometry.

The 2014 workshop was held in two weeks. In the first week, 17 plenary and 47 parallel talks were presented by specialists in their fields and 27 posters were presented by young researchers. In addition, one mini-course on a current topic of research was also delivered by Jean-Paul Brasselet. The program of the second week consisted of 4 mini-courses, 2 public lectures, 10 short talks and 5 posters. The mini-courses were taught by Marcelo Hernandes, Shyuichi Izumiya, David Massey, and José Luis Cisneros and José Seade, the public lectures were delivered by Stanislaw Janeczko and Marcio Gomes Soares.

The meeting was attended by 183 participants from 25 countries (Algeria, Brazil, Bulgaria, China, Colombia, Denmark, England, EUA, France, Germany, Greece, Hungary, Iceland, Iran, Iraq, Japan, Mexico, Nigeria, Peru, Poland, Russia, Spain, Sweden, Turkey, Vietnam).

The high level of the book reflects the atmosphere of the conference, where the most recent and most important results in singularity theory and its applications were presented. The volume contains the notes of the mini-courses taught by J.-P. Brasselet on intersection homology and by D. Massey on non-isolated hypersurface singularities and Lê cycles. The remaining contributions are research articles which cover topics from the foundations of singularity theory (including classification theory and invariants) to topology of singular spaces (links of singularities and semi-algebraic sets), as well as applications to topology (cobordism and Lefschetz fibrations), dynamical systems (Morse-Bott functions) and differential geometry (affine geometry, Gauss-maps, caustics, frontals and non-Euclidean geometries). All the manuscripts have been carefully peer-reviewed. We thank the authors for their valuable contributions, and the referees for their careful and diligent work. Our thanks also go to the editor of the RSME-AMS series in Contemporary Mathematics Pedro J. Paúl Escolano as well as to the editorial staff of the American Mathematical Society for their patience and efficient help with the publication process of the volume.

We could not have organized the workshop without the help of many people and institutions. We start by thanking the members of the Organizing Committee: João Carlos Ferreira Costa, Thaís Maria Dalbelo, Nivaldo Grulha, Ton Marar, Luciana Martins, Aurélio Menegon Neto, Bruna Oréfice Okamoto, Miriam da Silva Pereira, Raimundo Nonato Araújo dos Santos and João Nivaldo Tomazella. We also thank the members of the Scientific Committee: Carolina Araújo, James Damon, Alexandre Gurgel Fernandes, Javier Fernández de Bobadilla, Shyuichi Izumiya, Stanislaw Janeczko, Isabel Labouriau, Anne Pichon, Federico Sánchez Bringas, Márcio Soares for their support. We are also very grateful to the administrative staff of the ICMC-USP. The workshop was funded by the Brazilian agencies and institutions FAPESP, CNPq, CAPES, PROEX-USP, INCTMAT, ICMC-USP, SBM, the Spanish DGICYT and the Japanese funding agency JSPS, whose support we gratefully acknowledge.

Ana Claudia Nabarro  
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Maria Aparecida Soares Ruas

## Carmen Romero Fuster

The meeting in 2014 was organized by the São Carlos and the València groups of singularities, and had the great pleasure to celebrate the 60<sup>th</sup> birthday of Carmen Romero Fuster, professor at the Universitat de València, Spain. Carmen is one of the leading experts in applications of singularity theory to geometry, widely known by her pioneer work on generic geometry of submanifolds in Euclidean and hyperbolic spaces. Carmen and the São Carlos group started their scientific collaboration in 1985, when she was an assistant professor at the Universidade Federal de Viçosa, Brazil. Her fruitful interaction with the Brazilian singularists includes about thirty joint papers and six former students spread across four Brazilian states, São Paulo, Minas Gerais, Rio Grande do Sul and Bahia. Furthermore, some of her Spanish students have spent long periods of time in São Carlos as postdocs or visiting professors, it was therefore natural to celebrate her 60<sup>th</sup> birthday at the ICMC-USP, São Carlos.

Carmen completed her undergraduate degree in the Universitat de València in 1976. Soon after that, in 1977, she started her Master's degree at the University of Warwick with E. C. Zeeman. Followingly, she went to Southampton to begin her PhD under the supervision of S. A. Robertson. Her PhD thesis, finished in 1981, was entitled *Sphere Stratifications and the Gauss Map* and was about global convexity properties of hypersurfaces in the Euclidean space. The results of the thesis were cited in the books of Arnold, Gussein-Zade and Varchenko, "Singularités des applications différentiables" (Ed. Mir, 1986) and Banchoff, Gaffney and MacCrory, "Cusps of Gauss Mappings" (Res. Not. Math. Pitman, 1982), two of the main references in Singularity Theory at that moment.

Soon enough, Carmen started applying singularity theory techniques to differential geometry also in higher codimension submanifolds. Her generalization of the four vertex theorem for generic space curves lead to fruitful collaborations with V. D. Sedykh or to the thesis of her first student J. J. Nuño Ballesteros. In 1985 Carmen met M. A. S. 'Cidinha' Ruas and after some joint publications they co-supervised the thesis of D. K. H. Mochida, which layed the foundations of generic geometry of surfaces in  $\mathbb{R}^4$  in a series of papers which are still amongst Carmen's most cited papers up to date. Other PhD students in the context of generic geometry of submanifolds in Euclidean spaces were E. Sanabria Codesal and R. A. Gonçalves in Spain and S. M. Moraes and R. R. Binotto in Brazil. In the 2000's, Carmen started a collaboration with S. Izumiya on geometry of submanifolds in the Minkowski space. Her interaction with the Japanese singularity group has been one of the most prolific and counts up to 12 joint publications.

In the same period, Carmen started to be interested in Vassiliev type invariants of spaces of mappings. This lead to the thesis of C. Mendes de Jesus on Arnold's

invariants, which was continued by defining certain weighted graphs dual to the singular set of a map from a surface to the plane as a global topological invariant which complements the Vassiliev invariants. The study of topological invariants produced two more students, R. Oset Sinha, who studied Vassiliev type invariants of stable maps from 3-manifolds to  $\mathbb{R}^3$  in his thesis, and C. Casonatto, who studied the case of 3-manifolds to  $\mathbb{R}^4$ .

The conference was a great success and most of Carmen's students attended the meeting. There was a plenary lecture prepared by J. J. Nuño Ballesteros, R. Oset Sinha and M. A. S. Ruas on Carmen's work, and at the conference dinner a video produced by E. Sanabria Codesal showed the non-research aspects of Carmen's academic life, a life in which she has made many good and lasting friendships around the world and in which she has shared her passion for mathematics with anyone who has been lucky enough to have their paths cross with hers.

Ana Claudia Nabarro  
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The volume contains the notes from two mini-courses taught during the workshop: on intersection homology by J.-P. Brasselet, and on non-isolated hypersurface singularities and Lê cycles by D. Massey. The remaining contributions are research articles which cover topics from the foundations of singularity theory (including classification theory and invariants) to topology of singular spaces (links of singularities and semi-algebraic sets), as well as applications to topology (cobordism and Lefschetz fibrations), dynamical systems (Morse-Bott functions) and differential geometry (affine geometry, Gauss-maps, caustics, frontals and non-Euclidean geometries).

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