CONTEMPORARY MATHEMATICS

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Advances in Non-Archimedean Analysis

13th International Conference p-adic Functional Analysis August 12–16, 2014 University of Paderborn, Paderborn, Germany

> Helge Glöckner Alain Escassut Khodr Shamseddine Editors



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Preface

This volume contains papers based on lectures given at the 13th International Conference on *p*-adic Functional Analysis, which was held at the University of Paderborn, Germany, from August 12 to August 16, 2014. The conference followed the tradition of 12 earlier meetings which took place in Laredo (Spain, 1990), Santiago (Chile, 1992), Clermont-Ferrand (France, 1994, 2004, 2010), Nijmegen (The Netherlands, 1996, 2002), Poznan (Poland, 1998), Ioannina (Greece, 2000), Concepcion (Chile, 2006), East Lansing (USA, 2008), and Winnipeg (Canada, 2012).

The 19 articles included in this book feature recent developments in various areas of non-archimedean analysis, ranging from non-archimedean functional analysis (sequence spaces, quantitative compactness theorems, operator theory) to representation theory, number theory, non-archimedean dynamical systems and a range of topics in non-archimedean analysis (like non-archimedean function theory, differential equations over the Amice ring, implicit function theorems over infinite rank valued fields and integration over the Levi-Civita field). The book also contains a survey article on the p-adic Hayman conjecture, and a survey of applications.

The preparations for the conference were overshadowed by the death of Wim H. Schikhof, who played an important part in the organization of the conference series throughout the years, was an influential researcher in the area, and a member of the current scientific board. Besides the personal loss, we feel the gap left in the scientific community. The conference was held in the memory of Wim Schikhof and this proceedings volume contains both the last research article he devised (jointly with Elena Olivos) and a text on his life and work, written by his long term collaborators Herminia Ochsenius, Elena Olivos, and Cristina Perez-Garcia. We also include an obituary note by Arnoud C. M. van Rooij and two photographs of Wim taken during the first conference of this series (1990 in Laredo) and the last meeting he attended (2010 in Clermont-Ferrand).

For their guidance and help, we are grateful to the further members of the scientific board of the conference (which comprised J. Aguayo Garrido, A. Escassut, A. K. Katsaras, A. N. Kochubei, H. Ochsenius, M. C. Perez Garcia, K. Shamseddine, and W. H. Schikhof[†]). The University of Paderborn provided some funding for the conference. Finally, we thank all authors and referees of these proceedings, as well as the AMS (notably Christine M. Thivierge) for their efforts.

Helge Glöckner, Alain Escassut, and Khodr Shamseddine



FIGURE 1. Photographs commemorating Wim Schikhof, as presented by Cristina Perez-Garcia and Alain Escassut during the Paderborn conference

Obituary note

In Memoriam Wim Schikhof.

He accepted his fate, including his approaching death.

He loved life, but always when something desirable was out of reach he took time to consider it and he consented, wistfully, maybe, but without bitterness. "This is not for me", he told me once, as if it was a shoe that was not his size.

We have known each other since he was a freshman and I was a sophomore at the University of Utrecht, sixty years ago. There was no special bond between us, then, but math students lived in a small world and Wim and I came from the same part of the country. In 1965 I got an appointment in Nijmegen, where he had gone a year earlier. He was writing a thesis on *p*-adic analysis. As it happened, I had recently become interested in the same subject, and we began to cooperate.

Together with Jan van Tiel and Marius van der Put we formed an unofficial research group, meeting regularly in Utrecht or in Nijmegen. It was on the train that Wim and I developed the integration theory along the lines set out by Monna and Springer.

Jan and Marius left Utrecht and *p*-adic analysis. Wim and I stayed in Nijmegen. We kept working together, both mathematically and as members of the faculty. Together with Ronald Kortram (whose mathematical interests were completely disjoint from ours), for about thirty years we took care of the analysis curriculum, sharing the same ideas about how mathematics should be taught. Meanwhile, Wim and I wrote a book on real functions (that almost got the title "A Second Curse on Real Functions"), and Wim was the student adviser in the days when student advisers had not been invented yet. It was precisely the work he was made for.

Wim and I each led our own lives – his was quite different from mine. But during all the years we were at the university we relied on each other. If it is true that a friend is someone who needs no explanations, we were friends.

When his time came, he consented. I hope I will do the same.

Arnoud van Rooij

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This volume contains the Proceedings of the 13th International Conference on p-adic Functional Analysis, held from August 12-16, 2014, at the University of Paderborn, Paderborn, Germany. The articles included in this book feature recent developments in various areas of non-Archimedean analysis, non-Archimedean functional analysis, representation theory, number theory, non-Archimedean dynamical systems and applications. Through a combination of new research articles and survey papers, this book provides the reader with an overview of current developments and techniques in non-Archimedean analysis as well as a broad knowledge of some of the sub-areas of this exciting and fast-developing research area.



