CONTEMPORARY MATHEMATICS

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Centre de Recherches Mathématiques Proceedings

Invariant Subspaces of the Shift Operator

CRM Workshop
Invariant Subspaces of the Shift Operator
August 26–30, 2013
Centre de Recherches Mathématiques,
Université de Montréal, Montréal

Javad Mashreghi
Emmanuel Fricain
William Ross
Editors

American Mathematical Society
Providence, Rhode Island

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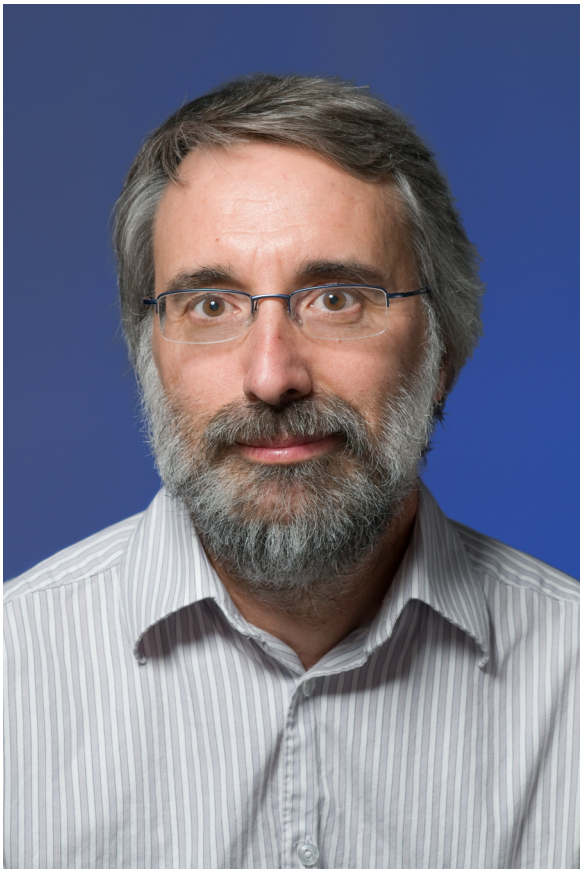

Professor Boivin (1955–2014) completed his Ph.D. at the Université de Montréal in 1984 under the direction of Paul Gauthier. He joined the University of Western Ontario as an Assistant Professor in 1986, after holding postdoctoral fellowships at UCLA and University College, London. He was promoted to Associate Professor in 1991, and then to Professor in 2004. He was appointed as Chair of Western's Department of Mathematics in 2011. His research specialties were complex analysis and approximation theory, and he was the author of multiple papers in these areas. He gave tireless service to granting agencies and selection committees in Québec and Ontario, and was a frequent conference organizer.

Professor Boivin participated in the conference on *Invariant Subspaces of the Shift Operator* and even contributed to this proceedings. His sudden death as a result of a heart failure was a great shock for all of us. With great affection for André and profound regret for his passing, we devote the present volume to his memory.

Contents

Preface

The main theme of this proceedings volume is the invariant subspaces of the shift operator S, or its adjoint S^*, on certain reproducing kernel Hilbert spaces of analytic functions on the open unit disk. Such spaces include the Hardy spaces H^2, the Dirichlet space $\mathcal{D}$, the de Branges-Rovnyak spaces $\mathcal{H}(b)$, and the model spaces K_Θ.

Model spaces have many fascinating aspects. For one, they represent, via Beurling's theorem, which characterizes the invariant subspaces of the shift operator $Sf = zf$ on H^2, the complete set of invariant subspaces of the backward shift operator $S^*f = (f - f(0))/z$. Using the theory of pseudo continuations developed by H. S. Shapiro, the description of these backward shift invariant subspaces was described in a seminal paper of Douglas, Shapiro, and Shields. The concept of pseudocontinuation continues to have an uncanny way of appearing in many unexpected areas of analysis.

These backward shift subspaces K_Θ of H^2 became to be known as model spaces via the Nagy-Foiaş theory when the compression S_Θ of the shift S to K_Θ was shown to represent a wide class of contraction operators on Hilbert space. Though these spaces make connections to operator theory and some areas of mathematical physics, they are a fascinating Hilbert space of analytic functions in the own right. They have interesting reproducing kernels and boundary behavior. The rank-one unitary perturbations of S_Θ (the Clark unitary operators) are an interesting class of operators with an even more interesting and useful set of spectral measures (Clark measures).

Parallel to the theory of Sz.-Nagy and Foias, de Branges and Rovnyak developed another model based on $\mathcal{H}(b)$ spaces, where b is an analytic function in the closed unit ball of H^∞ (bounded analytic functions on the open unit disk). When b is inner, $\mathcal{H}(b)$ coincides with the model space K_b. In the general case, $\mathcal{H}(b)$ spaces are not a closed subspace of H^2, but they are equipped with a norm to become a Hilbert space contractively embedded in H^2. Their structure is fascinating and depends on whether or not $\log(1 - |b|)$ is integrable on the unit circle, equivalently, non-extreme or extreme points in the closed unit ball of H^∞. Since the foundation developed by de Branges and Rovnyak, $\mathcal{H}(b)$ spaces continue to be a precious tool in various questions in analysis such as function theory (resolution of the Bieberbach conjecture by de Branges, rigid functions, Schwarz–Pick inequalities), operator theory (invariant subspace problem, composition operators, kernel of the Toeplitz operators), systems and control theory.

Related to the Hardy space, model spaces, and de Branges-Rovnyak spaces, is the Dirichlet space. This space connects to many areas and tools of analysis. For example, the definition of the Dirichlet space (analytic functions on the disk

whose image has finite area measure) certainly connects to geometry. As the name Dirichlet suggests, the Dirichlet space connects to Dirichlet's method for solving Laplace's equation. This naturally leads to another connection, logarithmic potential theory. Though much is known about this space, there remain several important open problems, most notably the characterization of its zero sets and of its shift-invariant subspaces.

This proceedings is the outcome of a conference that was held at CRM (Centre de Recherches Mathématiques) in Montreal from 26 to 30 August 2013. Numerous international experts in this area presented their ongoing research. Moreover, for graduate students and the newcomers to the field, there were several mini-courses on the basics of the H^2, K_Θ, $\mathcal{H}(b)$, and $\mathcal{D}$. The Editors would like to thank CRM for hosting this event and for its generous financial support for the invited speakers.

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Invariant Subspaces of the Shift Operator • Mashreghi et al., Editors

This volume contains the proceedings of the CRM Workshop on Invariant Subspaces of the Shift Operator, held August 26–30, 2013, at the Centre de Recherches Mathématiques, Université de Montréal, Montréal, Quebec, Canada.

The main theme of this volume is the invariant subspaces of the shift operator (or its adjoint) on certain function spaces, in particular, the Hardy space, Dirichlet space, and de Branges–Rovnyak spaces.

These spaces, and the action of the shift operator on them, have turned out to be a precious tool in various questions in analysis such as function theory (Bieberbach conjecture, rigid functions, Schwarz–Pick inequalities), operator theory (invariant subspace problem, composition operator), and systems and control theory.

Of particular interest is the Dirichlet space, which is one of the classical Hilbert spaces of holomorphic functions on the unit disk. From many points of view, the Dirichlet space is an interesting and challenging example of a function space. Though much is known about it, several important open problems remain, most notably the characterization of its zero sets and of its shift-invariant subspaces.

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