CONTEMPORARY MATHEMATICS

603

Commutative and Noncommutative Harmonic Analysis and Applications

AMS Special Session in Memory of Daryl Geller on Wavelet and Frame Theoretic Methods in Harmonic Analysis and Partial Differential Equations
September 22-23, 2012
Rochester Institute of Technology, Rochester, NY

Azita Mayeli
Alex Iosevich
Palle E. T. Jorgensen
Gestur Ólafsson
Editors

American Mathematical Society

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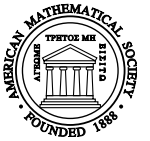

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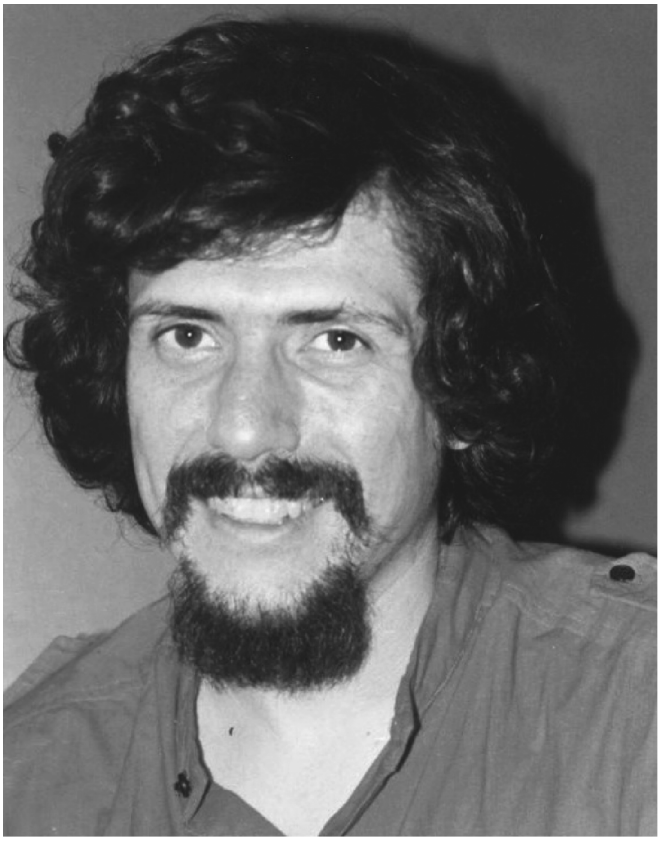

Photo courtesy of the Department of Mathematics, Stony Brook University

To the memory of Daryl Geller;
man of the Heisenberg group.

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Preface

In recent years, wavelet and frame theory advances in harmonic analysis have spawned new and exciting developments in various areas and have revealed some novel and unexpected connections between different parts of mathematics, pure and applied. For example, the use of wavelet and frame theory methods, multiscale analysis methods, and Calderón decomposition techniques have played a crucial role in a diverse range of problems in pure and applied mathematics and statistics.

This volume consists of invited survey and expositions stressing the interaction and connection between Darly Geller's area of research: commutative and abstract harmonic analysis, in particular harmonic analysis on the Heisenberg group. The invited potential authors of this volume have already made substantial contributions to mathematics and its applications.

The content of the volume is based on underlying fundamental ideas of relevance to the formal presentations at the conference for future research. Each article contains new research advances, and starts with a user friendly tutorial to make this volume accessible to young mathematicians.

For the purpose of organization, we have divided the book into three parts, noncommutative, commutative, and applications: The first group of papers is devoted to problems in noncommutative harmonic analysis, the second to topics in commutative harmonic analysis, and the papers in the third grouping are devoted to such applications as wavelet and frame theory, and to some real-world applications. Although some papers in the book in fact cover more than one of the three areas, we feel that a subdivision into three separate parts will still prove useful to readers.

The Editors:
Azita Mayeli
Alex Iosevich
Palle Jorgensen
Gestur Ólafsson
April 23, 2013

Remembering Daryl Geller
(1950 - 2011)

It is with great sadness that we report the death of Daryl Geller, Professor of Mathematics at the State University of New York at Stony Brook and former University of Toronto mathematics specialist. Daryl grew up in Toronto and attended the University of Toronto as a mathematics specialist, winning many awards including high placements on the Putnam Competition. Upon his graduation in 1972 he attended Princeton University, obtaining his Ph.D. in 1976 under the supervision of Elias Stein. He arrived at Stony Brook in 1979.

Daryl Geller worked on classical harmonic analysis and its neighboring areas. His research covered partial differential equations, harmonic analysis, several complex variables, and Lie groups. His early pioneering work on harmonic analysis on the Heisenberg group, and more generally on stratified Lie groups, paved the way to a number of trends in noncommutative analysis and geometry, entailing the theory of decompositions of unitary representations of Lie groups. Some of his papers are a bit technical and do not seem to have received the attention they deserve. We feel that a collection of research papers gathered in a single volume will help to remedy this.

The name of Geller will always be historically significant because of the importance of his contributions to the many areas of analysis, in particular to the harmonic analysis of the Heisenberg group.

Publications

The following is a list of some of Daryl Geller's publications.

(1) Pesenson, Isaac Z.; Geller, Daryl, *Cubature formulas and discrete Fourier transform on compact manifolds. From Fourier analysis and number theory to radon transforms and geometry*, 431–453, Dev. Math., 28, Springer, New York, 2013.

(2) Durastanti, Claudio; Geller, Daryl; Marinucci, Domenico, *Adaptive nonparametric regression on spin fiber bundles.* J. Multivariate Anal. 104 (2012), 16–38.

(3) Geller, Daryl; Mayeli, Azita, *Wavelets on manifolds and statistical applications to cosmology.* Wavelets and multiscale analysis, 259–277, Appl. Numer. Harmon. Anal., Birkhauser/Springer, New York, 2011.

(4) Geller, Daryl; Pesenson, Isaac Z., *Band-limited localized Parseval frames and Besov spaces on compact homogeneous manifolds.* J. Geom. Anal. 21 (2011), no. 2, 334–371.

(5) Geller, Daryl; Marinucci, Domenico, *Mixed needlets.* J. Math. Anal. Appl. 375 (2011), no. 2, 610–630.

(6) Geller, Daryl; Mayeli, Azita, *Nearly tight frames of spin wavelets on the sphere.* Sampl. Theory Signal Image Process. 9 (2010), no. 1-3, 25–57.

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(17) Geller, Daryl, *Approximate projections and analytic pseudolocality of the Szegö projection on domains in* $\mathbf{C^2}$. Several complex variables and complex geometry, Part 3 (Santa Cruz, CA, 1989), 165–172, Proc. Sympos. Pure Math., 52, Part 3, Amer. Math. Soc., Providence, RI, 1991.

(18) Geller, Daryl, *Analytic pseudodifferential operators for the Heisenberg group and local solvability.* Mathematical Notes, 37. Princeton University Press, Princeton, NJ, 1990. viii+495 pp. ISBN: 0-691-08564-1.

(19) Christ, Michael; Geller, Daryl, *Singular integral characterizations of Hardy spaces on homogeneous groups.* Duke Math. J. 51 (1984), no. 3, 547–598.

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(29) Geller, Daryl, *Necessary and sufficient conditions for local solvability on the Heisenberg group.* Harmonic analysis in Euclidean spaces (Proc. Sympos. Pure Math., Williams Coll., Williamstown, Mass., 1978), Part 2, pp. 219–226, Proc. Sympos. Pure Math., XXXV, Part, Amer. Math. Soc., Providence, R.I., 1979.

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(31) Geller, Daryl, *Fourier analysis on the Heisenberg group.* Proc. Nat. Acad. Sci. U.S.A. 74 (1977), no. 4, 1328–1331.

List of speakers

The following is the list of the names of the organizers and their speakers.

AMS special session organizers:
Alex Iosevich, Rochester University
Azita Mayeli, City University of New York, Queensborough C. College

Speakers at **Wavelet and Frame Theoretic Methods in Harmonic Analysis and Partial Differential Equations in Memory of Daryl Geller:**

Dynamical sampling: time-space trade-off
Akram Aldroubi, Vanderbilt University

Near-extremizers of affine-invariant inequalities, and arithmetic progressions
Michael Christ, University of California, Berkeley

Orbit spaces for linear actions of solvable groups
Bradley Currey, Saint Louis University

Trends in abelian and non-abelian harmonic analysis
Palle E. T. Jorgensen, University of Iowa

Continuous frames and operator valued-measures
David R Larson, Texas A&M University

Besov spaces and frames on stratified Lie groups
Gestur Òlafsson, Louisiana State University

Variational bounds for a dyadic model of the bilinear Hilbert transform
Eyvindur Ari Palsson, University of Rochester

Kolmogorov and linear widths of balls in Sobolev spaces on compact manifolds
Isaac Z Pesenson, Temple University and CCP

Weighted L^p estimates for the k-plane transforms
Boris Rubin, Louisiana State University

The indicator/interval testing characterization of the two weight inequality for the Hilbert transform
Eric T Sawyer, McMaster University

Square functions, trees, and the exponential integral
Leonid Slavin, University of Cincinnati

A 4D wavelet transform
Keith F. Taylor, Dalhousie University

Bilinear restriction estimates
Faruk Temur, University of Illinois at Urbana-Champaign

List of participants

Akram Aldroubi, Vanderbilt University

Michael Christ, The University of California, Berkeley

Bradley Currey, Saint Louis University

Alex Iosevich, University of Rochester

Palle E. T. Jorgensen, The University of Iowa

David R Larson, Texas A&M University

Bochen Liu, The University of Rochester

Azita Mayeli, City University of New York, Queensborough C. College

Brendan Murphy, The University of Rochester

Gestur Òlafsson, Louisiana State University

Vignon Oussa, Bridgewater State University

Eyvindur Ari Palsson, The University of Rochester

Isaac Z. Pesenson, Temple University and CCP

Boris Rubin, Louisiana State University

Eric T. Sawyer, McMaster University

Leonid Slavin, University of Cincinnati

Keith F. Taylor, Dalhousie University

Faruk Temur, University of Illinois at Urbana-Champaign

CONM 603

Topics in Harmonic Analysis • Mayeli et al., Editors

This volume contains the proceedings of the AMS Special Session on Wavelet and Frame Theoretic Methods in Harmonic Analysis and Partial Differential Equations, held September 22–23, 2012, at the Rochester Institute of Technology, Rochester, NY.

The book features new directions, results and ideas in commutative and noncommutative abstract harmonic analysis, operator theory and applications. The commutative part includes shift invariant spaces, abelian group action on Euclidean space and frame theory; the noncommutative part includes representation theory, continuous and discrete wavelets related to four dimensional Euclidean space, frames on symmetric spaces, C^*-algebras, projective multiresolutions, and free probability algebras.

The scope of the book goes beyond traditional harmonic analysis, dealing with Fourier tools, transforms, Fourier bases, and associated function spaces. A number of papers take the step toward wavelet analysis, and even more general tools for analysis/synthesis problems, including papers on frames (over-complete bases) and their practical applications to engineering, cosmology and astrophysics.

Other applications in this book include explicit families of wavelets and frames, as they are used in signal processing, multiplexing, and study of Cosmic Microwave Background (CMB) radiation.

For the purpose of organization, we have divided the book into three parts, noncommutative, commutative, and applications. The first group of papers are devoted to problems in noncommutative harmonic analysis; the second to topics in commutative harmonic analysis; while the papers in the third grouping are devoted to such applications as wavelet and frame theory, and to some real-world applications.

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