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New Trends in Noncommutative Algebra

A Conference in Honor of Ken Goodearl's 65th Birthday
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University of Washington, Seattle, WA

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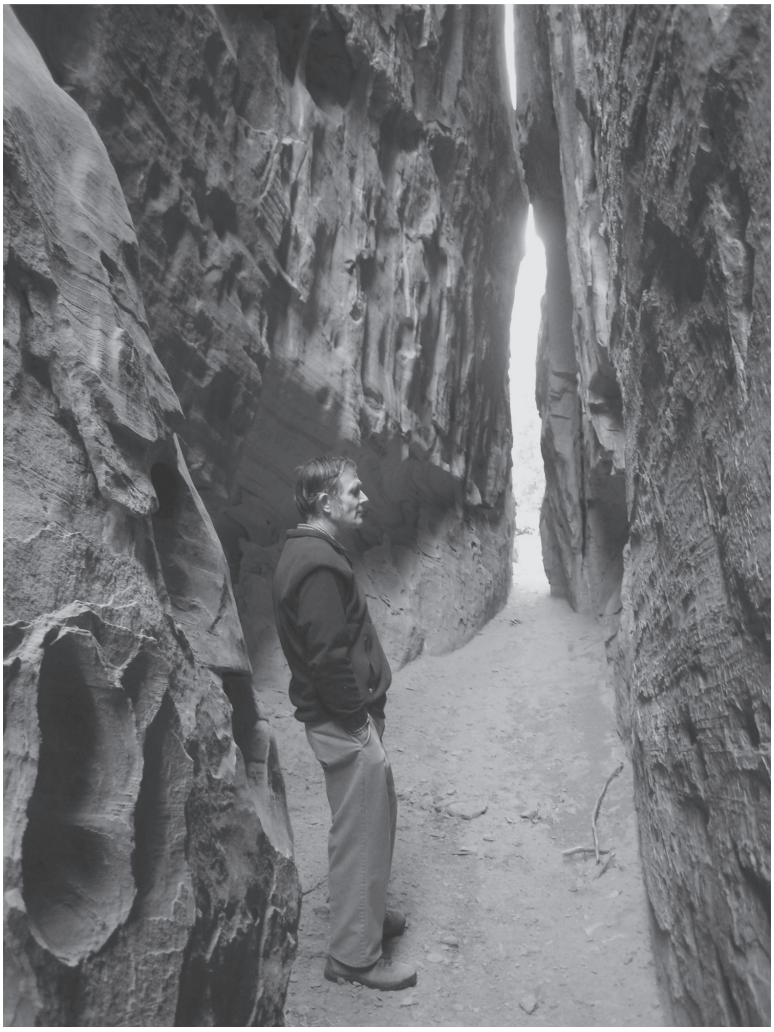

This proceedings is dedicated to Ken Goodearl
on the occasion of his 65th birthday.

Contents

Preface

This volume forms the proceedings of a conference entitled "New Trends in Noncommutative Algebra," that was held in honor of the 65th birthday of Ken Goodearl, at the University of Washington, Seattle, in August 2010.

The first article outlines some of Ken's contributions to noncommutative ring theory. The subsequent articles in the volume, by some of the leading workers in these areas, reflect the wide interests of Ken. Specific topics include: noncommutative algebraic geometry, representation theory, Calabi-Yau algebras, quantum algebras and deformation quantization, Poisson algebras, growth of algebras, group algebras, and noncommutative Iwasawa algebras.

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Ken Goodearl's contributions to ring theory

It is hard to overstate Ken Goodearl's role in the development of noncommutative ring theory, beginning in the early 1970s and continuing as strongly as ever today. His impact has been threefold: First, through his many collaborations. Second, through a vast body of research results. Third, through highly influential textbooks, lectures, and expository articles.

In this essay we'll attempt to provide an overview of Ken's specific contributions to C*-algebras, partially ordered structures, von Neumann regular rings and noetherian ring theory.

First, however, we, the authors of this essay, thank Ken for his many years as mentor, inspiration, generous colleague, and friend.

1. C*-algebras and von Neumann regular rings

From its very beginning, Ken Goodearl's work on C*-algebras and (von Neumann) regular rings has influenced (and has been influenced by) the most active lines of research on these topics. Trained as a ring theorist, though with a solid background in functional analysis, Ken has always been attracted by the relationship among both subjects. Regular rings were invented by John von Neumann [**102**] in his studies on continuous geometries and operator algebras. They are defined by a very simple axiom: for any element x of the ring R, there is y in R such that $x = xyx$. Regular rings have many idempotents, since for $x \in R$ as before, xy and yx are idempotent elements of R with $xR = (xy)R$ and $Rx = R(yx)$. If $\mathcal{N}$ is a finite von Neumann algebra of operators on a complex Hilbert space $\mathcal{H}$, then the set $\mathcal{U}$ of all unbounded closed operators affiliated to $\mathcal{N}$ is a regular ring, containing the same projections–self-adjoint idempotents–as $\mathcal{N}$. An important example of this situation is provided by the von Neumann algebra $\mathcal{N}\Gamma$ of a discrete group Γ, which can be defined as the set of all operators T in $B(l^2(\Gamma))$ which satisfy that $T(\xi g) = T(\xi)g$ for all $\xi \in l^2(\Gamma)$ and all $g \in \Gamma$. The corresponding regular ring $\mathcal{U}\Gamma$ is an interesting algebraic object, as for instance it plays a crucial role in the resolution of Atiyah's Conjecture for several classes of groups, see [**99**, Chapter 10].

The theory of von Neumann algebras was algebraized by Kaplansky, who introduced AW*-algebras and Baer *-rings [**93**]. In close relationship with them we find the self-injective von Neumann regular rings, the continuous or just $\aleph_0$-continuous regular rings, and the rank-complete regular rings. Ken's initial contributions dealt with these classes of regular rings, determining various aspects of their algebraic structure. Many of these advances were included in his authoritative book [**35**], which became the standard reference for the theory of regular rings.

Elliott's work [**29**] on the classification of approximately finite dimensional (AF) algebras through the structure of their ordered K_0 represented an important highlight in the area, being the start of a successful program aimed at the classification of separable nuclear C*-algebras by using K-theoretic invariants. Elliott's original approach was not in terms of K-theory, but it was soon realized that K-theory provides the best setting for his result. Although the main application that Elliott had in mind was to C*-algebras, his result includes a purely algebraic classification theorem. Namely, every direct limit of a sequence of finite-dimensional semisimple algebras over an algebraically closed field is completely classified by its ordered K_0 group, together with the position of the unit in this group. An AF-algebra is just a C*-algebra obtained by completing the above direct limit (taking $\mathbb{C}$ as the coefficient field) under a suitable norm. Elliott's paper initiated a very fruitful research line in C*-algebras, and several other classification results having Elliott's original paper as a model appeared. Ken Goodearl has largely contributed to this program, through various important papers and also through his monograph [**39**], which covers all the basic material related to the structure of partially ordered abelian groups entering into classification theory of C*-algebras. In 1987, Ken and David Handelman gave in [**56**] a classification theorem for ring and C^*-algebra sequential inductive limits of finite-dimensional semisimple real algebras in a similar spirit to Elliott's work. In another impressive paper [**55**], the same authors studied the classification problem for *extensions* of AF-algebras. This work was extended and completed by Ken in [**42**], where various important results on existence of extensions were proven. For instance, it is shown that, given two AF-algebras A and C, such that C is unital and A has no nonzero unital homomorphic images, there is a unital essential extension of A by C ([**42**, Corollary 8.4]). The methods developed in [**42**] were successfully applied by Ken in the paper [**48**] to obtain a description of the Grothendieck groups of the multiplier algebras of certain C*-algebras.

Ken's contributions to classification include [**46**], in which he proves the Riesz decomposition property and the cancellation property for projections in matrices over inductive limits of finite products of matrix algebras over continuous functions on compact Hausdorff spaces with slow dimension growth. These are important facts, frequently used by the researchers working in classification. He also constructed an interesting class of simple C*-algebras in [**45**], some of them having real rank zero and the others having real rank one, this being distinguished by a numerical invariant attached to the construction. These algebras are nowadays known as *Goodearl algebras*, cf. [**104**, Section 8].

A particular aspect of AW*-algebras and AF-algebras is that they are filled with lots of projections, and that, in different ways, they are very close to regular rings. In 1957, Berberian [**14**] showed that any finite AW*-algebra admits a regular overring R, to which the involution can be extended, which contains the same projections as A. It was later shown that R is the classical as well as the maximal ring of quotients of A; see for example [**15**]. In the case of AF-algebras, the regular ring appears as a subalgebra of A, namely A is the *norm completion* of the algebraic inductive limit R of a countable direct limit of finite-dimensional semisimple complex algebras, which is clearly a regular ring. The conceptual completion of both classes of C*-algebras came from the work of Brown and Pedersen [**22**], who introduced the concept of *real rank* for C*-algebras. The case where the C*-algebra

has real rank zero is characterized by the presence of many projections, and includes all AW*-algebras and all AF-algebras. There is not any obvious regular ring associated to a C*-algebra of real rank zero. However, the connection with a purely algebraic concept appeared in a natural way thanks to a joint work of Ken with Pere Ara, Kevin O'Meara and Enrique Pardo [**5**], through a notion invented by Crawley and Jónsson [**26**] in order to study refinements in general algebraic structures. This concept was adapted to the world of rings and modules by Warfield [**111**]. Ken [**78**, p. 167] and Nicholson [**103**, Theorem 2.1] obtained independently an elementwise characterization of this class of rings. Namely, a unital ring R is an *exchange ring* provided that for every x in R there exists an idempotent e such that $e \in xR$ and $1 - e \in (1 - x)R$. It was proven in [**5**] that the unital C*-algebras of real rank zero are precisely the ones which are exchange rings. In the same paper, one of the most influential papers in the area, the concept of separative cancellation was introduced, and the fundamental question of whether all exchange rings are separative is formulated. Recall that a ring R is said to be *separative*, or to satisfy *separative cancellation*, in case for every finitely generated projective right R-modules A, B, if $A \oplus A \cong A \oplus B \cong B \oplus B$ then $A \cong B$. Various open questions on the structure of exchange rings have a positive answer in the presence of separativity. For instance, if R is a separative exchange ring then the Bass stable rank of R can only take the values $1, 2$ or ∞; moreover, the dichotomy principle holds for simple exchange rings: any simple separative exchange ring is either purely infinite or has stable rank one. One of the novelties introduced in [**5**] was the systematic use of the monoid $V(R)$ of isomorphism classes of finitely generated projective modules as a key ingredient in the study of large classes of rings and algebras, prominently exchange rings and regular rings. The Grothendieck group $K_0(R)$ is obtained as the enveloping group of $V(R)$, and its order structure is precisely determined by the image of the natural map $V(R) \to K_0(R)$. However it turns out that the monoid $V(R)$ contains in general much more information than the group $K_0(R)$ does, and its essential properties are many times masked by the passage to the enveloping group. The key question was soon realized by Ken, and was formulated in his survey paper [**47**], as a fundamental open problem: Which monoids arise as $V(R)$'s for a von Neumann regular ring R?

The same question can be formulated for exchange rings. There are a few obvious conditions that an abelian monoid must satisfy in order to be the V-monoid of an exchange ring, including the so-called *Riesz refinement property*, and the natural question is whether any monoid satisfying these necessary conditions can be realized by a regular ring (or an exchange ring or a C*-algebra of real rank zero). Fred Wehrung [**112**] answered the question in the negative for abelian monoids of size $\aleph_2$ or bigger, but the question remains widely open even for countable monoids. Realization results have been obtained recently through the consideration of some algebras (called Leavitt path algebras) associated to directed graphs and certain overrings of them [**3**], as well as certain algebras associated to finite posets [**4**]. The recent work by Ara and Goodearl in [**1**] and [**2**] produces a new class of algebras, the *algebras of separated graphs*, having quite general V-monoids, with the aim of developing from them much larger classes of examples of regular rings and C*-algebras of real rank zero, which would ideally solve the realization problem in the countable case, or would at least produce exotic examples of exchange rings.

In the second part of this essay, we'll focus primarily on a narrative that begins in Ken's study of differential operator rings, progresses through his work on iterated skew polynomial rings, and ultimately leads to his current research on homological and Poisson-geometric aspects of quantum groups and related algebras.

2. Differential operator rings.

This chapter spans the 1970s and 1980s, starting with Ken's studies of global and Krull dimension of differential operator rings $T = R[\theta, \delta]$, for associative rings R and derivations δ. Ken's first paper in this program [**32**] established that the right global dimension of R is no greater than that of T. Ken's subsequent work on global dimension of differential operator rings included [**33, 36, 40**], and [**70**] (with Tom Lenagan and Paul Roberts). And Ken's work on Krull dimension of differential operator rings and related algebras included collaborations with (his thesis advisor) the late Bob Warfield [**79**], with Tom Lenagan [**60, 61, 62**], and with Tim Hodges and Tom Lenagan [**57**]. To give one significant application of the above studies: If T is a ring of several commuting differential operators over a commutative noetherian coefficient ring of finite Krull dimension, and if T has finite global dimension, then the Krull and global dimensions of T coincide [**40**].

Ken's other work on differential operator rings included [**6**] (with Allen Bell), [**19**] (with Ken Brown and Tom Lenagan), [**38, 41, 43**], and [**80**] (with Bob Warfield). These studies of prime ideals, along with their localization-theoretic and representation theoretic link structure (in the sense of Jategaonkar [**89**]), helped prepare the way for analogous later work on quantum algebras.

3. Skew and q-skew polynomial rings. Quantum coordinate rings.

Ken's contribution to the theory of algebraic quantum groups began in the early 1990s in [**44**], with his introduction of *q-skew derivations* and *q-skew polynomial rings*, generalizing behavior found in differential operator rings and quantized Weyl algebras (see, e.g.,[**100**]). Specifically, a skew polynomial ring $R[y; \tau, \delta]$ (in the formal variable y, defined over a coefficient ring R via an automorphism τ and τ-derivation δ), is q-skew if the operator equation $\delta\tau = q\tau\delta$ holds for some central scalar q. It turned out that iterated q-skew polynomial constructions could also be used to describe the quantum coordinate rings of $n \times n$ matrices $\mathcal{O}_q(M_n)$, as well as the quantum groups $\mathcal{O}_q(SL_n)$ and $\mathcal{O}_q(GL_n)$. These and related algebras had been described by Reshetikhin, Takhtadzhyan, and Faddeev in [**106**], and popularized for ring theorists by Paul Smith [**107**] (also cf., e.g.,[**101**]).

Ken's study of q-skew polynomial rings continued (with Ed Letzter) in [**71, 72**]; the latter paper providing a proof that the prime factors of $\mathcal{O}_q(M_n)$, $\mathcal{O}_q(GL_n)$, and $\mathcal{O}_q(SL_n)$ were all integral domains, assuming q is not a root of unity. (Note: We'll assume below, unless otherwise stated, that the quantizing parameters mentioned are sufficiently generic and in particular not roots of unity.)

Ken also studied quantum coordinate rings from a Hopf theoretic perspective, establishing with Lenagan and Rigal [**69**] a quantum analogue of the classical First Fundamental Theorem of Invariant Theory for $\mathcal{O}_q(GL_n)$ and with Lenagan [**64, 65**] an analogue of the Second Fundamental Theorem. Goodearl and Lenagan continued their joint research in this direction with notable results on quantum minors and

winding-automorphism-invariant prime ideals; see [**66, 67**]. Work on quantum minors by Lenagan and co-authors Domokos and Rigal should also be included here [**28, 98**], and a later approach by Goodearl and Lenagan appeared in [**68**].

This Hopf theoretic line of study also began, and helped prepare the way for subsequent approaches to, the counting of orbits of primitive ideals; see §5.

4. Stratification.

In the early-to-mid 1990s, a description of prime and primitive ideals in the quantum coordinate rings $\mathcal{O}_q(G)$ of semisimple algebraic groups G emerged in the work of Tim Hodges and Thierry Levasseur [**86, 87, 88**] and Tony Joseph [**90, 91, 92**]. (Earlier work of Yan Soibleman should also be noted here; see [**108, 109**].) Key to the theory are natural actions on $\mathcal{O}_q(G)$ and its spectra by a maximal torus H of G. In [**16**], Ken Brown and Ken Goodearl presented an axiomatic approach, and they further described the link structure (as in [**89**]) of the prime spectrum of $\mathcal{O}_q(G)$.

Some key features of the above theory: (1) There are only finitely many H-prime (i.e., H-invariant prime) ideals, indexed by the double Weyl group $W \times W$ of G. These H-prime ideals stratify (and so partition) the prime and primitive spectra. (2) Each stratum of primitive ideals consists of a single H-orbit, and the primitive ideals are exactly those prime ideals maximal within their strata. (3) The prime spectrum is "normally separated" – that is, for each pair of primes $P \subsetneq P'$ there exists an element in P' that modulo P is normal and nonzero. (4) There is a natural H-equivariant bijection between the primitive ideals of $\mathcal{O}_q(G)$ and the symplectic leaves in G (under the associated Poisson structure presented in [**108, 109**]). (5) Each prime ideal is completely prime (i.e., each prime factor is an integral domain).

In [**73**], Goodearl and Letzter established analogues of much of the preceding stratification theory for suitably conditioned iterated q-skew polynomial rings equipped with a compatible action by a torus H. (A more general version [**18**] was later obtained using Goodearl's and Stafford's "Graded Goldie Theorem" [**76**].) In particular, these iterated q-skew polynomial rings had only finitely many H-prime ideals, providing a (finite) stratification of the prime and primitive spectra, each stratum of primitive ideals consisted of a single H-orbit, and a prime ideal was primitive exactly when it was maximal within its stratum. (Complete primality already followed from [**72**].) Examples fitting the necessary hypotheses (but not covered in the preceding two paragraphs) included $\mathcal{O}_q(M_n)$ and other quantum coordinate rings. Still open, however, were exact counts of H-prime ideals, normal separation, and a suitably "geometric" description of primitive spectra. In lectures and survey articles, Ken conjectured positive solutions to all three of these problems; see, e.g. [**49, 50, 51**]. (Ken had already made partial progress on the second and third of these conjectures in, e.g., [**49**] and with Letzter in [**74**].)

Two papers of G. Cauchon [**24, 25**] then pushed the program forward (and settled one of Ken's conjectures). Cauchon added the hypothesis of nilpotency of skew derivations to the iterated q-skew model, allowing for his method of "deleting derivations" [**24**]. This additional hypothesis refined the model but retained its applicability to quantum matrices and related algebras. Second, in [**25**] he proved normal separation for $\mathcal{O}_q(M_n)$. Third, in [**25**] he initiated a combinatorial approach to counting H-prime ideals via what are now known as Cauchon Diagrams.

5. H-prime ideals, totally nonnegative grassmanians, and symplectic leaves in matrix Poisson varieties.

Building on the preceding work of Cauchon, and on the stratification theory of Goodearl and Letzter, there now followed a concentration of effort in the study of H-prime spectra of quantum coordinate rings (particularly in the case of generic quantizing parameters). Goodearl and Lenagan [**66, 67**] identified explicitly the 230 H-prime ideals in $\mathcal{O}_q(M_3)$. Launois and Lenagan [**95**] identified the condition for rectangular quantum matrices to be primitive, and, in subsequent work with Bell and other authors, Launois [**7, 8, 9, 10, 11, 12**] obtained much more information about the dimension of the strata and the primitive spectra of quantum matrices and other quantum coordinate rings.

A study of the ring theoretic properties of the quantum grassmannian was initiated by Kelly, Lenagan, and Rigal [**94**]. A suprising connection between H-prime ideals in the quantum grassmannian and the cell decomposition of the totally nonnegative grassmannian was noticed by comparing diagrams of the H-prime spectrum of $\mathcal{O}_q(G(2,4))$ [**97**] and the diagram of the poset of nonempty cells in the 2×4 totally nonnegative grassmannian [**113**]. This led to further work by Goodearl, Launois and Lenagan [**58, 59, 96**] which has made precise the connection between the H-prime spectrum of quantum matrices, the H-orbits of symplectic leaves in matrix Poisson varieties and the cell decomposition of totally nonnegative matrices. In particular, the quantum minors belonging to an H-prime in quantum matrices can be described. Casteels [**23**] also solves this problem. Recent work by Yakimov [**115, 116, 117, 118**] has extended much of this work to quantum coordinate rings of simple algebraic groups, quantum partial flag varieties, and the algebras $U_q^+(w)$.

We conclude this section with a fundamental problem underlying much of the present and preceding discussion, originally formulated by Hodges and Levasseur in [**86**], and popularized by Ken over many years (c.f. [**50, 54**]): To what extent does the Kostant-Kirillov-Souriau orbit method work for $\mathcal{O}_q(G)$ and related algebras? Put slightly more precisely, can the natural bijection described in §4, between primitive ideals of $\mathcal{O}_q(G)$ and symplectic leaves in G, be suitably adjusted to obtain a homeomorphism between these two spaces? Believing that a clearer picture of the leaves was required in order to solve this problem, Brown, Goodearl, and Yakimov [**20**] developed a detailed description of the torus orbits of leaves in $n\times n$ matrices; this project was then continued by Goodearl and Yakimov for flag varieties in [**82**]. Recently, Yakimov has produced an explicit candidate for the conjectured homeomorphism in the case of $\mathcal{O}_q(G)$; see [**118**].

6. Homological aspects of quantum groups and noetherian Hopf algebras.

Gabber famously proved that enveloping algebras of finite dimensional complex solvable Lie algebras are catenary [**30**]; that is, the saturated chains of primes connecting two fixed prime ideals must all have the same length. Gabber's approach combined homological properties, GK-dimension (Gel'fand-Kirillov dimension), and normal separation. In [**63**], Ken Goodearl and Tom Lenagan abstracted this approach to obtain the following powerful formulation: If R is an affine, Noetherian, Auslander-Gorenstein, Cohen-Macaulay algebra with finite GK-dimension, and if the prime spectrum of R is normally separated, then R is catenary. This

theorem made catenarity accessible to a wide range of algebras, including quantum matrices (via Cauchon's proof of normal separation [**25**]) and, for example, the rings with Auslander dualizing complexes studied by Amnon Yekutieli and James Zhang in [**119**]. Furthermore, in [**83**], Goodearl and Zhang combined the approaches of [**63**] and [**119**] to study the homological properties of $\mathcal{O}_q(G)$; they established catenarity in this case as well.

On a separate track, Ken Brown and Ken Goodearl in [**17**] initiated a study of homological properties of noetherian PI Hopf algebras, with results applicable to quantum groups at roots of unity and restricted enveloping algebras. They proved that an affine noetherian Hopf algebra, finite over its center, with finite global dimension, is Auslander-regular and Macaulay. They also showed that for certain prime noetherian algebras Λ, finite over their centers, and with the above homological properties, the simple modules of maximal dimension are in bijective correspondence with the smooth points of the center $Z(\Lambda)$. Using this bijection, Brown and Goodearl were able to answer questions of C. De Concini, V. Kac [**27**] and A. Premet [**105**]. Moreover, a question first raised by Brown and Goodearl in [**17**], on the extent to which the aforementioned homological properties hold for general noetherian Hopf algebras, has stimulated considerable subsequent research; see for example [**114**].

Most recently, in [**84**], Ken Goodearl and James Zhang classified noetherian Hopf algebra domains H of GK-dimension 2 (over algebraically closed fields k of characteristic zero) satisfying the condition that $\mathrm{Ext}^1_H(k,k) \neq 0$. (Recent work of Wang, Zhang, and Zhuang [**110**] shows that the last condition is not redundant.) This research continues a fascinating program on the classification of noetherian Hopf algebras of low GK-dimension satifying homological regularity conditions, a program started by Brown and Zhang [**21**] in the case of GK-dimension 1.

7. Gems off the path.

Of course there are numerous wonderful results by Ken not in the above story. For instance, there is his theorem asserting that every ring between a hereditary noetherian prime ring and its quotient ring is a (torsion theoretic) localization of the ring. And there is Ken's example with Aidan Schofield [**75**] showing that a finitely generated essential extension of a simple module over a noetherian ring of Krull dimension 1 need not be artinian. Or Ken's and Birge Zimmerman-Huisgen's study of torsion direct products [**85**]. There is also Ken's proof with Dave Benson, for group rings RG over finite groups G, that if M is a flat RG-module projective as an R-module, then M is projective as an RG-module. Lastly we'll note Ken's remarkable theorem with Toby Stafford [**77**] that a finitely generated affine Dedekind domain over an uncountable field k is either simple or commutative.

8. Expository works.

A discussion of Ken's contributions to ring theory cannot be complete without mentioning some of his expository works. An early, influential book is *Ring theory. Nonsingular rings and modules* [**34**]. This was followed by *Von Neumann Regular Rings* [**35**], which, as commented earlier, became the standard reference for the theory of regular rings. Ken's book *An introduction to noncommutative Noetherian rings* (co-authored with Bob Warfield) [**81**], now in its second edition, remains after more than 20 years both a classic text and a standard reference. More

recently, *Lectures on algebraic quantum groups* (co-authored with Ken Brown) [**18**], has become a highly cited source on ring theoretic aspects of algebraic quantum groups and related algebras.

In addition, Ken's interest in the related areas of C*-algebras and partially ordered structures led to two books *Notes on real and complex C*-algebras* [**37**] and *Partially ordered abelian groups with interpolation* [**39**] which, while not strictly in ring theory, have lots of connections with ring theory.

Lastly, Ken's (almost) innumerably many lectures, accompanied by his expository and programmatic survey articles (some already noted above), have also played a large role over the years. These papers have been typically filled with conjectures, detailed examples, and reworkings of his research papers; recent examples include [**47, 49, 50, 51, 53, 54**].

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