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Equivariant Homotopy and Cohomology Theory

Dedicated to the Memory of Robert J. Piacenza

J. P. May

with contributions by
M. Cole, G. Comezaña, S. Costenoble,
A. D. Elmendorf, J. P. C. Greenlees,
L. G. Lewis, Jr., R. J. Piacenza,
G. Triantafillou, and S. Waner

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Abstract

This volume introduces equivariant homotopy, homology, and cohomology theory, along with various related topics in modern algebraic topology. It begins (Chapters I–VIII) with a development of the equivariant algebraic topology of spaces, culminating in a discussion of the Sullivan conjecture that emphasizes its relationship with classical Smith theory. It next (Chapters IX–XV) introduces equivariant stable homotopy theory, the equivariant stable category, and the most important examples of equivariant cohomology theories. The basic machinery that is needed to make serious use of equivariant stable homotopy theory is then presented (Chapters XVI–XXII), along with discussions of the Segal conjecture and generalized Tate cohomology. Finally (Chapters XXIII–XXVIII), the book gives an introduction to "brave new algebra", which is the study of point-set level algebraic structures on spectra, and its equivariant applications. Emphasis is placed on equivariant complex cobordism, and some related material on that topic is presented in detail.

I	Equivariant cellular and homology theory
II	Postnikov systems, localization, and completion
III	Equivariant rational homotopy theory
IV	Smith theory
V	Categorical constructions; equivariant applications
VI	The homotopy theory of diagrams
VII	Equivariant bundle theory and classifying spaces
VIII	The Sullivan conjecture
IX	An introduction to equivariant stable homotopy
X	G-$CW(V)$ complexes and $RO(G)$-graded cohomology
XI	The equivariant Hurewicz and suspension theorems
XII	The equivarinat stable homotopy category
XIII	$RO(G)$-graded homology and cohomology theories
XIV	An introduction to equivariant K-theory
XV	An introduction to equivariant cobordism
XVI	Spectra and G-spectra; change of groups; duality
XVII	The Burnside ring
XVIII	Transfer maps in equivariant bundle theory
XIX	Stable homotopy and Mackey functors
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complement $W - V$, the inclusion induces a natural transformation σ^{W-V} : $\Sigma^V \longrightarrow \Sigma^W$.

2. Stably almost complex structures and bordism

When G is the trivial group, a *stably almost complex structure* on a compact smooth manifold M is an element $[\xi] \in \tilde{K}(M)$, which goes to the class $[\nu M]$ of the stable normal bundle under the map

$$\tilde{K}(M) \longrightarrow \widetilde{KO}(M).$$

It is, of course, essentially equivalent to define this with $[\tau M]$ replacing $[\nu M]$, since these classes are additive inverses in $\widetilde{KO}(M)$.

The following definition gives the obvious equivariant generalization of this.

DEFINITION 2.1. If $[\xi] \in \tilde{K}_G(M)$ is a lift of $[\nu M] \in \widetilde{KO}_G(M)$ under the natural map, we call the pair $(M, [\xi])$ a normally almost complex G-manifold.

We will use the notation $M_{[\xi]}$ when necessary, but we will drop $[\xi]$ whenever there is no risk of confusion.

The bordism theory of these objects, denoted mu_*^G, is the "complex analog" of the unoriented theory mo_*^G discussed in Chapter XV. If V is a *complex* G-module and $(M, \partial M)_{[\xi]}$ is a G-manifold with a stably almost complex structure, then its V-suspension becomes a G-manifold after "straightening the angles", and $[\xi] - [\varepsilon_V]$ is a complex structure on $\Sigma^V(M, \partial M)$. This gives rise to a *suspension homomorphism*

$$\sigma^V : mu_*^G(X, A) \longrightarrow mu_{*+|V|}^G(\Sigma^V(X, A)),$$

which sends the class of a map $(M, \partial M) \longrightarrow (X, A)$ to the class of its suspension. Due to the failure of G-transversality, both the suspension homomorphisms and the Pontrjagin-Thom map are generally not bijective.

We construct a stabilized version of this theory as follows. Let $\mathscr{U}$ be an infinite-dimensional complex G-module equipped with a hermitian inner product whose underlying $\mathbb{R}$-linear structure is that of a complete G-universe. Define

$$MU_*^G(X, A) = \operatorname{colim}_V mu_*^G(\Sigma^V(X, A)),$$

where V ranges over all finite-dimensional complex G-subspaces of $\mathscr{U}$ and the colimit is taken over all suspension maps induced by inclusions. We should perhaps point out that MU_*^G is not a connective theory unless G is trivial. The advantage of this new theory over mu_*^G is that the bad behavior of the Pontrjagin-Thom map is corrected, and the maps induced by suspension by complex G-modules are isomorphisms by construction. This should be interpreted as a form of periodicity. Homology or cohomology theories with this property are often referred to in the literature as *complex-stable*. Other examples of such theories include equivariant complex K-theory, its associated Borel construction, etc. Complex-stability isomorphisms should not be confused with suspension isomorphisms of the form

$$\Sigma^V : h_*^G(X, A) \longrightarrow h_{*+[V]}^G(\Sigma^V(X, A)),$$

which are part of the structure of all $RO(G)$-graded homology theories.

MU_*^G or, more precisely, its dual cohomology theory was first constructed by tom Dieck in terms of a G-prespectrum TU_G, bearing the same relationship to complex Grassmanians as the G-prespectrum TO_G discussed in XV§2, does to real ones. An argument of Bröcker and Hook for unoriented bordism readily adapts to the complex case to show the equivalence of the two approaches. In what follows, we shall focus on the spectrification MU_G of TU_G. As with any representable equivariant homology theory, MU_*^G can be extended to an $RO(G)$-graded homology theory, but we shall concern ourselves only with integer gradings. We point out, however, that complex-stable theories are always $RO(G)$-gradable.

A key feature of MU_G, proven in XXVI§7, is the fact that it is a split G-spectrum; this may be seen geometrically as a consequence of the fact that the augmentation map $MU_*^G \longrightarrow MU_*$, given on representatives by neglect of structure, can be split by regarding non-equivariant stably almost complex manifolds as G-manifolds with trivial action. The splitting makes $MU_*^G = MU_*^G(S^0)$ a module over the ring MU_*.

The multiplicative structure of the ring G-spectrum MU_G can be interpreted geometrically as coming from the fact that the class of normally stably almost complex manifolds is closed under finite products. The complex-stability isomorphisms are well-behaved with respect to the multiplicative structure: in cohomology, we have a commutative diagram

$$\begin{array}{ccc} MU_G^*(X) \otimes MU_G^*(Y) & \longrightarrow & MU_G^*(X \wedge Y) \\ \downarrow{\scriptstyle \sigma^V \otimes \sigma^W} & & \downarrow{\scriptstyle \sigma^{V \oplus W}} \\ MU_G^{*+|V|}(\Sigma^V X) \otimes MU_G^{*+|W|}(\Sigma^W Y) & \longrightarrow & MU_G^{*+|V|+|W|}(\Sigma^{V \oplus W} X \wedge Y) \end{array}$$

for all based G-spaces X and Y and complex G-modules V and W. In general, for a multiplicative cohomology theory, commutativity of a diagram of the form above is assumed as part of the definition of complex-stability. K_G^* is another example of a multiplicative complex-stable cohomology theory, as is the Borel construction on any such theory.

The role of MU_G in the equivariant world is analogous to that of MU in classical homotopy theory, for its associated cohomology theory has a privileged position among those which are multiplicative, complex-stable, and have natural Thom classes (for complex G-vector bundles). We record the axiomatic definition of such theories.

DEFINITION 2.2. A G-equivariant multiplicative cohomology theory h_G^* is said to have natural Thom classes for complex G-vector bundles if for every such bundle ξ of complex dimension n over a pointed G-space X there exists a class $\tau_\xi \in h_G^{2n}(T(\xi))$, with the following three properties:

(1) *Naturality:* If $f : Y \longrightarrow X$ is a pointed G-map, then $\tau_{f^*\xi} = f^*(\tau_\xi)$.

(2) *Multiplicativity:* If ξ and η are complex G-vector bundles over X, then

$$\tau_{\xi \oplus \eta} = \tau_\xi \times \tau_\eta \in h_G^{|\xi|+|\eta|}(T(\xi \oplus \eta)).$$

(3) *Normalization:* If V is a complex G-module, then $\tau_V = \sigma^V(1)$.

The following result, which admits a quite formal proof (given for example by Okonek) explains the universal role played by MU_G.

PROPOSITION 2.3. *If h^*_G is a multiplicative, complex-stable, cohomology theory with natural Thom classes for complex G-bundles, then there is a unique natural transformation $MU^*_G(-) \longrightarrow h^*_G(-)$ of multiplicative cohomology theories that takes Thom classes to Thom classes.*

Returning to homology, for a complex G-bundle ξ of complex dimension k, the Thom class of ξ gives rise to a Thom isomorphism

$$\tau : MU^G_*(T(\xi)) \longrightarrow MU^G_{*-2k}(B(\xi)_+),$$

and similarly in cohomology. This isomorphism is constructed in the same way as in the nonequivariant case (see e.g. [LMS]), without using any feature of MU^G_* other than the existence and formal properties of Thom classes. However, in this special case, its *inverse* has a rather pleasant geometric interpretation: if $f : M \longrightarrow B(\xi)$ represents an element in $mu^G_n(B(\xi)_+)$, the map $\overline{f}$ in the pullback diagram

$$\begin{array}{ccc} E(f^*\xi) & \xrightarrow{\overline{f}} & E(\xi) \\ {\scriptstyle f^*\xi}\downarrow & & \downarrow{\scriptstyle \xi} \\ M & \xrightarrow[f]{} & B(\xi) \end{array}$$

represents an element in $mu^G_{n+2k}(T(\xi))$. This procedure allows the construction of a homomorphism which stabilizes to the inverse of the Thom isomorphism. See Bröcker and Hook for the details of a treatment of the Thom isomorphism (in the unoriented case) that uses this interpretation.

T. Bröcker and E.C. Hook. Stable equivariant bordism. Mathematische Zeitschrift 129(1972), pp. 269–277.

T. tom Dieck. Bordism of G-manifolds and integrality theorems. Topology 9(1970), pp. 345–358.

C. Okonek. Der Conner-Floyd-Isomorphismus für Abelsche Gruppen. Mathematische Zeitschrift 179(1982), pp. 201–212.

3. Tangential structures

Unfortunately, both mu^G_* and MU^G_* are rather intractable from the computational point of view. In order to address this difficulty, we shall introduce a new bordism theory, much more amenable to calculation, whose stabilization is also MU^*_G.

Consider the following variant of reduced K-theory: if X is a G-space, instead of taking the quotient by the subgroup generated by all trivial complex G-bundles, take the quotient by the subgroup generated by those trivial bundles of the form $\mathbb{C}^n \times X$, where G acts trivially on $\mathbb{C}^n$. We denote the group so obtained as $\check{K}_G$; there is an analogous construction in the real case, which we denote $\check{K}O_G$.

DEFINITION 3.1. A tangentially stably almost complex manifold is a smooth manifold equipped with a lift of the class $[\tau M] \in \check{KO}_G(M)$ to $\check{K}_G(M)$.

We shall refer to the bordism theory of these manifolds as *tangential complex bordism*, denoted $\Omega_*^{U,G}$.

We warn the reader that nowhere in the literature is the distinction between the complex bordism theories $\Omega_*^{U,G}$ and mu_*^G made clear. This is not mere pedantry on our part, as our next result will show. It was pointed out to the author by Costenoble that this result does not hold for normally stably almost complex G-manifolds.

PROPOSITION 3.2. *If M is a tangentially stably almost complex G-manifold and $H \subseteq G$ is a closed normal subgroup, then the G-tubular neighborhood around M^H has a complex structure.*

We stress the fact that no stabilization is necessary to get a complex structure on the tubular neighborhood; this lies at the heart of the calculations we shall carry out later in the chapter.

PROOF. The first thing to observe is that $\tau(M^H) = (\tau M|_{M^H})^H$ as real vector bundles. If ξ is the restriction to M^H of a complex G-vector bundle over M that represents its tangential stably almost complex structure, and the underlying real G-vector bundle of ξ is $\tau M|_{M^H} \oplus \varepsilon_{\mathbb{R}^n}$, then $(\xi^H)^\perp$ is a complex G-vector bundle. We have

$$\xi = \xi^H \oplus (\xi^H)^\perp = (\tau M|_{M^H})^H \oplus \varepsilon_{\mathbb{R}^n} \oplus \nu(M^H, M).$$

This gives the desired structure. □

We next explore the relation between mu_*^G and $\Omega_*^{U,G}$. There is a commutative square

$$\begin{array}{ccc} \check{K}_G(X) & \longrightarrow & \check{KO}_G(X) \\ \downarrow & & \downarrow \\ \widetilde{K}_G(X) & \longrightarrow & \widetilde{KO}_G(X) \end{array}$$

that yields a natural transformation of homology theories $\phi : mu_*^G \longrightarrow \Omega_*^{U,G}$. Just as we did with mu_*^G, we may stabilize $\Omega_*^{U,G}$ with respect to suspensions by finite-dimensional complex subrepresentations of a complete G-universe, obtaining a new complex-stable homology theory which we shall provisionally denote $\check{MU}_*^G$. The map ϕ stabilizes to a natural transformation $\Phi : \check{MU}_*^G \longrightarrow MU_*^G$. The following result was first proved by the author and Costenoble by a different argument and is central to the results of this chapter.

THEOREM 3.3. *Φ is an isomorphism of homology theories.*

We shall need the following standard result.

LEMMA 3.4. (Change of groups isomorphism) *If $H \subseteq G$ is a closed subgroup of codimension j, then for all H-spaces X there is an isomorphism*

$$mu_*^H(X_+) \xrightarrow{\cong} mu_{*+j}^G((G \times_H X)_+)$$

induced by application of the functor $G \times_H (-)$ to representatives of bordism classes of maps, and similarly for pairs. The analogous result holds for $\Omega_*^{U,G}$ and MU_*^G.

SKETCH PROOF. If we apply the functor $G \times_H (-)$ to a map $f : M \longrightarrow X$ that represents an element of $mu_n^H(X_+)$, we obtain an element of $mu_{n+j}^G((G \times_H X)_+)$. Conversely, if $g : N \longrightarrow G \times_H X$ represents an element of $mu_{n+j}^G((G \times_H X)_+)$ and if $\pi : G \times_H X \longrightarrow X$ is the evident H-map, we set $M = (\pi g)^{-1}(eH)$ and see that M is an H-manifold such that $N = G \times_H M$ and the restriction of g to M represents an element of $mu_n^H(X_+)$. □

PROOF OF THEOREM 3.3. We show first that the theorem is true for $G = SU(2k+1)$ and then extend the result to the general case by a change of groups argument.

We recall a few standard facts about representations of special unitary groups (e.g., from Bröcker and tom Dieck). Let M be the complex $SU(2k+1)$-module such that $M = \mathbb{C}^{2k+1}$ with the action of $SU(2k+1)$ given by matrix multiplication and let $\Lambda^i = \Lambda^i M$. Then $R(SU(2k+1))$ is the polynomial algebra over $\mathbb{Z}$ on the representations Λ^i, $1 \leq i \leq 2k$, all of which are irreducible and of complex type. Furthermore, $\Lambda^{2k-i+1} = \overline{\Lambda^i}$. This implies that any irreducible real representation of $SU(2k+1)$ is either trivial or admits a complex structure. To see this, let W be a non-trivial irreducible real $SU(2k+1)$-module. Suppose first that $W \otimes_{\mathbb{R}} \mathbb{C}$ is irreducible. Since the restriction to $\mathbb{R}$ of an irreducible complex representation of quaternionic type is irreducible, our assumptions imply that $W \otimes_{\mathbb{R}} \mathbb{C}$ is of real type and of the form $V \otimes_{\mathbb{C}} \overline{V}$, where V is a monomial in the Λ^i, $1 \leq i \leq k$. We have

$$(\overline{V} \otimes_{\mathbb{C}} V) \otimes_{\mathbb{R}} \mathbb{C} \cong (2W) \otimes_{\mathbb{R}} \mathbb{C} \cong 2(V \otimes_{\mathbb{C}} \overline{V})$$

as complex representations. On the other hand, since $2W \cong V \otimes_{\mathbb{C}} \overline{V}$, we have isomorphisms of complex $SU(2k+1)$-modules

$$(2W) \otimes_{\mathbb{R}} \mathbb{C} \cong (V \otimes_{\mathbb{C}} \overline{V}) \otimes_{\mathbb{R}} \mathbb{C} \cong V \otimes_{\mathbb{C}} (\overline{V} \otimes_{\mathbb{R}} \mathbb{C})$$

and

$$V \otimes_{\mathbb{C}} (\overline{V} \otimes_{\mathbb{R}} \mathbb{C}) \cong (V \otimes_{\mathbb{C}} V) \oplus (V \otimes_{\mathbb{C}} \overline{V})$$

(because $\overline{V} \otimes_{\mathbb{R}} \mathbb{C} \cong V \oplus \overline{V}$). So it follows that

$$2(V \otimes_{\mathbb{C}} \overline{V}) \cong (V \otimes_{\mathbb{C}} V) \oplus (V \otimes_{\mathbb{C}} \overline{V}),$$

which is absurd in view of the structure of $RSU(2k+1)$. Thus W must be reducible and so it is either of the form $V_1 \oplus V_1$, for an irreducible complex V_1 of quaternionic type, or $V_1 \oplus \overline{V_1}$, for an irreducible complex V_1 of complex type.

The first possibility is ruled out by the fact that all self-conjugate irreducible complex representations of $SU(2k+1)$ are of real type. So we must have

$$2W \cong V_1 \oplus \overline{V}_1 \cong 2V$$

as real representations, and therefore, using the uniqueness of isotypical decompositions, we may conclude that $W \cong V$ as real representations.

Now let X be a $SU(2k+1)$-space and consider a map representing an element in $MU_*^G(X)$. By complex-stability, there is no loss of generality in assuming that our map is of the form $f : M \longrightarrow X$, where $\tau M \oplus \epsilon_V \cong \xi$, V is a real representation, and ξ is a complex $SU(2k+1)$-vector bundle. By the remark above, $V = W \oplus \mathbb{R}^k$ for a complex representation W. Then $\Sigma^W(M, \partial M)$ is a tangentially stably almost complex manifold and the class of $\Sigma^W f$ is in the image of ϕ. It follows that Φ is surjective. A similar argument applied to bordisms shows that Φ is injective.

To obtain the general case, observe that any compact Lie group embeds in $U(2k)$, and $U(2k)$ embeds in $SU(2k+1)$ (via the map that sends $A \in U(2k)$ to $(det\ A)^{-1} \cdot 1_{\mathbb{R}} \oplus A$), and apply Lemma 3.4. □

T. Bröcker and T. tom Dieck. representations of compact Lie groups. Springer. 1985.
C. Okonek. Der Conner-Floyd-Isomorphismus für Abelsche Gruppen. Mathematische Zeitschrift 179(1982), pp. 201–212.

4. Calculational tools

For the remainder of the chapter, all Lie groups we consider will be Abelian.

There is a long list of names associated to the calculation of $\Omega_*^{U,G}(S^0)$ for different classes of compact Lie groups: Landweber (cyclic groups), Stong (Abelian p-groups), Ossa (finite Abelian groups), Löffler (Abelian groups), Lazarov (groups of order pq for distinct primes p and q), and Rowlett (extensions of a cyclic group by a cyclic group of relatively prime order). All of these authors rely on the study of fixed point sets by various subgroups, together with their normal bundles, through the use of bordism theories with suitable restrictions on isotropy subgroups.

The main calculational tool is the use of families of subgroups, which works in exactly the same fashion as was discussed in the real case in XV§3. Recall that, for a family $\mathscr{F}$, an $\mathscr{F}$-space is a G-space all of whose isotropy subgroups are in $\mathscr{F}$ and that we write $E\mathscr{F}$ for the universal $\mathscr{F}$-space. Recall too that, for a G-homology theory h_*^G and a pair of families $(\mathscr{F}, \mathscr{F}')$, $\mathscr{F}' \subseteq \mathscr{F}$, there is an associated homology theory $h_*^G[\mathscr{F}, \mathscr{F}']$, defined on pairs of G-spaces as

$$h_*^G[\mathscr{F}, \mathscr{F}'](X, A) = h_*^G(X \times E\mathscr{F}, (X \times \mathscr{F}') \cup (A \times E\mathscr{F})).$$

When $\mathscr{F}' = \emptyset$, we use the notation $h_*^G[\mathscr{F}]$. The theories $h_*^G[\mathscr{F}]$, $h_*^G[\mathscr{F}']$, and $h_*^G[\mathscr{F}, \mathscr{F}']$ fit into a long exact sequence. Of course, there is an analogous construction in cohomology.

In the special case of $\Omega_*^{U,G}$ (and similarly for other bordism theories), it is easy to see that $\Omega_*^{U,G}[\mathscr{F}, \mathscr{F}']$ has an alternative interpretation: it is the bordism theory of $(\mathscr{F}, \mathscr{F}')$- tangentially almost-complex manifolds, that is, com-

pact, tangentially almost complex $\mathscr{F}$-manifolds with boundary, whose boundary is an $\mathscr{F}'$-manifold.

DEFINITION 4.1. A pair of families $(\mathscr{F}, \mathscr{F}')$ of subgroups of G is called a neighboring pair differing by H if there is a subgroup H such that if $K \in \mathscr{F} - \mathscr{F}'$, then H is a subconjugate of K.

This notion was first used by Löffler, but the terminology is not standard. A special case is the more usual notion of an *adjacent* pair of families pair differing by H, which is a neighboring pair $(\mathscr{F}, \mathscr{F}')$ such that $\mathscr{F} - \mathscr{F}'$ consists of those subgroups conjugate to H.

The next proposition explains the importance of neighboring families. We introduce some terminology and notation to facilitate its discussion.

Given a subgroup H of an Abelian Lie group G, we choose a set $\mathscr{C}_{G,H}$ of finite dimensional complex G-modules whose restrictions to H form a non-redundant, complete set of irreducible, nontrivial complex H-modules. If $\mathbb{C}$ denotes the trivial irreducible representation, we let $\mathscr{C}^+_{G,H} = \mathscr{C}_{G,H} \cup \{\mathbb{C}\}$. For a nonnegative even integer k, we shall call an array of nonnegative integers $P = (p_V)_{V \in \mathscr{C}_{G,H}}$ a (G,H)-*partition* of k if

$$k = \sum_{V \in \mathscr{C}_{G,H}} 2p_V.$$

For such a partition P, we let

$$BU(P,G) = \prod_{V \in \mathscr{C}_{G,H}} BU(p_V, G).$$

We let $\mathscr{P}(k,G,H)$ denote the set of all (G,H)-partitions of k.

PROPOSITION 4.2. *If $(\mathscr{F}, \mathscr{F}')$ is a neighboring pair of families of subgroups of a compact Abelian Lie group G differing by a subgroup H, then*

$$\Omega_n^{U,G}[\mathscr{F}, \mathscr{F}'](X,A) \cong \bigoplus_{\substack{0 \le 2k \le n \\ P \in \mathscr{P}(2k,G,H)}} \Omega_{n-2k}^{U,G/H}[\mathscr{F}/H]((X^H, A^H) \times BU(P, G/H)),$$

where $\mathscr{F}/H$ denotes the family of subgroups of G/H that is obtained by taking the quotient of each element of $\mathscr{F} - \mathscr{F}'$ by H.

SKETCH OF PROOF. For simplicity, we concentrate on the absolute case. Let $f : M \longrightarrow X$ represent an element in $\Omega_n^{U,G}[\mathscr{F}, \mathscr{F}'](X_+)$ and let T be a (closed) G-tubular neighborhood of M^H. We may view T as the total space of the unit disc bundle of the normal bundle to M^H. We may also view T as an n-dimensional $\mathscr{F}$-manifold whose boundary is an $\mathscr{F}'$-manifold. Thus T represents an element of $\Omega_n^{U,G}[\mathscr{F}, \mathscr{F}'](S^0)$, and we see that $[f] = [f|_T]$ in $\Omega_n^{U,G}[\mathscr{F}, \mathscr{F}'](X_+)$. Furthermore, $[f] = 0$ if and only if there is an H-trivial G-nullbordism of $f|_T$, equipped with a complex G-vector bundle whose unit disc bundle restricts to T on M^H. Observe that M^H breaks up into various components of constant even codimension. In other words, $\Omega_n^{U,G}[\mathscr{F}, \mathscr{F}'](X_+)$ can be identified with the direct sum, with $2k$ ranging between 0 and n, of bordism of H-trivial $\mathscr{F}$-manifolds of dimension $n - 2k$ equipped with a complex G-vector bundle of dimension k,

containing no H-trivial summands. Note the twofold importance of Proposition 3.2: not only are we using that M^H is tangentially almost complex, but also that its tubular neighborhood carries a complex structure.

Consider the bundle-theoretic analog of the isotypical decomposition of a linear representation. For complex G-vector bundles E and F over a space X we may construct the vector bundle $\mathrm{Hom}_{\mathbb{C}}(E,F)$ whose fiber over $x \in X$ is $\mathrm{Hom}_{\mathbb{C}}(E_x, F_x)$; G acts on $\mathrm{Hom}_{\mathbb{C}}(E,F)$ by conjugation. If X is H-trivial, then $\mathrm{Hom}_H(E,F) = (\mathrm{Hom}_{\mathbb{C}}(E,F))^H$ is an H-trivial G-subbundle; if one regards X as a (G/H)-space, then $\mathrm{Hom}_H(E,F)$ becomes a (G/H)-vector bundle over X.

We apply this to $F = T$ and $E = \varepsilon_V$, where V is a complex G-module whose restriction to H is irreducible, thus obtaining a (G/H)-vector bundle which we call the *V-multiplicity* of E. The evaluation map

$$\bigoplus_{V \in \mathscr{C}^+_{G,H}} \mathrm{Hom}_H(\varepsilon_V, T) \otimes_{\mathbb{C}} \varepsilon_V \longrightarrow T$$

is a G-vector bundle isomorphism, and this decomposition into isotypical summands is unique. Note that in the special case we are considering, the multiplicity associated to the trivial representation is 0, so the sum really does run over $\mathscr{C}_{G,H}$.

Therefore T can be identified with a direct sum of (G/H)-vector bundles over M^H, each corresponding to an irreducible complex representation of H, and M^H breaks into a disjoint union of components on which the dimension of each multiplicity remains constant; each of these components therefore has an associated (G,H)-partition, accounting for the summation over $\mathscr{P}(2k,G,H)$ in our formula. Clearly the bundle on the component associated to a (G,H)-partition P is classified by $BU(P, G/H)$. □

Similar methods allow us to prove the following standard result.

PROPOSITION 4.3. *With the notation above, if H is a subgroup of an Abelian Lie group G, then*

$$BU(n,G)^H \cong \coprod_{P \in \mathscr{P}(n,G,H)} \prod_{V \in \mathscr{C}^+_{G,H}} BU(p_V, G/H)$$

as H-trivial G-spaces.

PROOF. It suffices to observe that the right hand side classifies n-dimensional complex G-vector bundles over H-trivial G-spaces. □

P. S. Landweber. Unitary bordism of cyclic group actions. Proceedings of the Amer. Math. Soc. 31(1972), pp. 564–570.

C. Lazarov. Actions of groups of order pq. Transactions of the Amer. Math. Soc. 173(1972), pp. 215–230.

P. Löffler. Bordismengruppen unitärer Torusmannigfaltigkeiten. Manuscripta Mathematica 12(1974), 307–327.

E. Ossa, Unitary bordism of Abelian groups. Proceedings of the American Mathematical Society 33(1972), pp. 568–571.

R.J. Rowlett. Bordism of metacyclic group actions. Michigan Mathematical Journal 27(1980), pp. 223–233.

R. Stong. Complex and oriented equivariant bordism. in Topology of Manifolds (J.C. Cantrell and C.H. Edwards, editors). Markham, Chicago 1970.

5. Statements of the main results

We come now to a series of theorems, some old, some new, that are consequences of the previous results. In all of them, we consider a given compact Abelian Lie group G.

THEOREM 5.1 (LÖFFLER). *If V is a complex G-module, and X is a disjoint union of pairs of G-spaces of the form*

$$(DV, SV) \times \prod_{i=1}^{k} BU(n_i, G),$$

then $\Omega_*^{U,G}(X)$ is a free MU_*-module concentrated in even degrees.

THEOREM 5.2. *With the same hypotheses on X, the map*

$$\Omega_*^{U,G}(BU(n,G) \times X) \longrightarrow \Omega_*^{U,G}(BU(n+1,G) \times X)$$

induced by Whitney sum with the trivial bundle $\varepsilon_{\mathbb{C}}$ is a split monomorphism of MU_*-modules.

THEOREM 5.3. *MU_*^G is a free MU_*-module concentrated in even degrees.*

THEOREM 5.4. *The stabilization map $\Omega_*^{U,G} \longrightarrow MU_*^G$ is a split monomorphism of MU_*-modules.*

Theorem 5.3 is stated in the second paper of Löffler cited below, but there seems to be no proof in the literature. Ours is a refinement of the ideas in the proof of Theorem 5.1, which yields Theorem 5.4 as a by-product, and is entirely self-contained (that is, it does not depend on results on finite Abelian groups). Tom Dieck has used a completely different method to prove a weaker version of Theorem 5.4, for G cyclic of prime order, but to the best of our knowledge nothing of the sort has previously been claimed or proved at our level of generality. Theorem 5.2, which also seems to be new, is required in the course of the proof of Theorem 5.3 and is of independent interest.

In the light of these results, it is natural to conjecture, probably overoptimistically, that MU_*^G is free over MU_* and concentrated in even degrees for any compact Lie group G. We have succeeded in verifying this for a class of non-Abelian groups that includes $O(2)$ and the dihedral groups. The statement about the injectivity of the stabilization map also holds for these groups. We hope to extend these results to other classes of non-Abelian groups; details will appear elsewhere.

The results above should be proven in the given order, but, since the proofs have a large overlap, we shall deal with all of them simultaneously.

We shall proceed by induction on the number of "cyclic factors" of the group, where, for the purposes of this discussion, S^1 counts as a cyclic group. The argument in each case is as follows: the result is either trivial or well-known for

the trivial group. Then, one shows that if the result is true for a compact Lie group G, it also holds for $G \times S^1$, and this in turn implies the same for $G \times \mathbb{Z}_n$.

T. tom Dieck. Bordism of G-manifolds and integrality theorems. Topology 9(1970), pp. 345–358.

P. Löffler. Bordismengruppen unitärer Torusmannigfaltigkeiten. Manuscripta Mathematica 12(1974), 307–327.

P. Löffler. Equivariant unitary bordism and classifying spaces. Proceedings of the International Symposium on Topology and its Applications, Budva, Yugoslavia 1973, pp. 158–160.

6. Preliminary lemmas and families in $G \times S^1$

For brevity, the subgroups $\{1\} \times S^1 \subseteq G \times S^1$ and $\{1\} \times \mathbb{Z}_n \subseteq G \times \mathbb{Z}_n$ will be denoted S^1 and $\mathbb{Z}_n$, respectively.

We shall need to consider the following families of subgroups of $G \times S^1$:

$$\begin{aligned} \mathscr{F}_i &= \{H \subseteq G \times S^1 \mid |H \cap S^1| \leq i\} \\ \mathscr{F}_\infty &= \{H \subseteq G \times S^1 \mid H \cap S^1 \neq S^1\} \\ \mathscr{A} &= \{\text{all closed subgroups of } G \times S^1\} \end{aligned}$$

These give rise to the neighboring pairs $(\mathscr{F}_{i+1}, \mathscr{F}_i)$ (differing by $\mathbb{Z}_{i+1}$) and $(\mathscr{A}, \mathscr{F}_\infty)$ (differing by S^1). Observe that $\mathscr{F}_\infty$ is the union of its subfamilies $\mathscr{F}_i$.

LEMMA 6.1. *Let G be a compact Lie group and X be a pair of $(G \times S^1)$-spaces. Then*

$$\Omega_*^{U,G\times S^1}(X \times S^1) \cong \Omega_{*-1}^{U,G}(X)$$

and

$$\Omega_*^{U,G\times S^1}((X \times S^1)/\mathbb{Z}_n) \cong \Omega_{*-1}^{U,G\times \mathbb{Z}_n}(X),$$

where $G \times S^1$ acts on S^1 and $S^1/\mathbb{Z}_n$ through the projection $G \times S^1 \longrightarrow S^1$; the same statement holds for the theories $mu_*^{G\times S^1}$ and $MU_*^{G\times S^1}$

The proofs of these isomorphisms are easy verifications and will be omitted; see Löffler. We shall also need the following result of Conner and Smith.

LEMMA 6.2. *A graded, projective, bounded below MU_*-module is free.*

LEMMA 6.3. *Consider a diagram of projective modules with exact rows*

$$\begin{array}{ccccccccc} 0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle f_1} & & \downarrow{\scriptstyle f_2} & & \downarrow{\scriptstyle f_3} & & \\ 0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & 0. \end{array}$$

If f_1 and f_3 (resp. f_2 and f_3) are split monomorphisms, so is f_2 (resp. f_1).

PROOF. Add a third row consisting of the cokernels of the f_i, which will be exact by the Snake Lemma. An easy diagram chase shows that the modules in the new row are projective, and therefore the conclusion follows. □

Note that we make no assumptions about compatibility of the splittings.

REMARK 6.4. If X is a pair of G-spaces of the kind appearing in the statement of Theorem 5.1 and H is a subgroup of G, then restricting the action to H yields an H-pair of the same kind. Moreover, by Proposition 4.3, X^H is a (G/H)-pair of the same type. This class of pairs of spaces is also closed under cartesian product with $BU(n,G)$ and with pairs of the form (DW, SW) for a complex G-module W.

P. E. Conner, L. Smith, *On the complex bordism of finite complexes*, Publications Mathématiques de l'IHES, no. 37 (1969), pp. 417–521.
P. Löffler. Bordismengruppen unitärer Torusmannigfaltigkeiten. Manuscripta Mathematica 12(1974), 307–327.

7. On the families $\mathscr{F}_i$ in $G \times S^1$

In what follows, for a G-pair X and a homology theory h_*, ψ will designate a map of the form

$$\psi : h_*(BU(n,G) \times X) \longrightarrow h_*(BU(n+1,G) \times X)$$

that is induced by taking the Whitney sum of the universal complex G-bundle over $BU(n,G)$ and the trivial G-bundle $\varepsilon_{\mathbb{C}}$.

Suppose that Theorems 5.1 – 5.4 have been proved for G. We shall deduce the following result in the case $G \times S^1$.

THEOREM 7.1. *The following statements hold for each $i \geq 1$ and for $i = \infty$.*

(1) $\Omega_*^{U,G\times S^1}[\mathscr{F}_i](X)$ *is a free MU_*-module concentrated in* odd *degrees.*

(2) *The map*

$$\psi : \Omega_*^{U,G\times S^1}[\mathscr{F}_i](BU(n, G\times S^1) \times X) \to \Omega_*^{U,G\times S^1}[\mathscr{F}_i](BU(n+1, G\times S^1) \times X)$$

is a split monomorphism of MU_*-modules.

(3) *If W is an irreducible complex $(G \times S^1)$-module, then the map*

$$\sigma^W : \Omega_*^{U,G\times S^1}[\mathscr{F}_i](X) \longrightarrow \Omega_{*+2}^{U,G\times S^1}[\mathscr{F}_i]((DW, SW) \times X)$$

is a split monomorphism of MU_*-modules.

(4) *The map $\Omega_*^{U,G\times S^1}[\mathscr{F}_i](X) \longrightarrow \Omega_*^{U,G\times S^1}(X)$ is zero.*

PROOF. We first prove this for $i = 1$, making use of a suitable model for the space $E\mathscr{F}_1$. Let $(W_i)_{i\geq 1}$ be a sequence of irreducible complex $(G \times S^1)$-modules such that S^1 acts freely on their unit circles, and every isomorphism class of such $(G \times S^1)$-modules appears infinitely many times. Let $V_k = \bigoplus_{i=1}^k W_i$ and

$$SV_\infty = \operatorname{colim}_k SV_k;$$

SV_∞ is the required space. Note also that this space embeds into the equivariantly contractible space

$$DV_\infty = \operatorname{colim}_k DV_k.$$

Using Lemma 6.1 and our assumptions about G, we see that $\Omega_*^{U,G\times S^1}(SV_1 \times X)$ is a free MU_*-module concentrated in odd degrees, and that

$$\sigma^W : \Omega_*^{U,G\times S^1}(SV_1 \times X) \longrightarrow \Omega_*^{U,G\times S^1}((DW, SW) \times SV_1 \times X)$$

and

$$\Omega_*^{U,G\times S^1}(SV_1 \times BU(n, G\times S^1)\times X) \longrightarrow \Omega_*^{U,G\times S^1}(SV_1 \times BU(n+1, G\times S^1)\times X)$$

are split monomorphisms of MU_*-modules.

We calculate $\Omega_*^{U,G\times S^1}((SV_{k+1}, SV_k)\times X)$ using the homotopy equivalence

$$(SV_{k+1}, SV_k) \simeq (SW_{k+1} * SV_k, DW_{k+1}\times SV_k),$$

and the excisive inclusion

$$SW_{k+1}\times (DV_k, SV_k) \longrightarrow (SW_{k+1} * SV_k, DW_{k+1}\times SV_k).$$

The action of $G\times S^1$ on SW_{k+1} determines and is determined by a split group epimorphism $G\times S^1 \longrightarrow S^1$ with kernel $H \subseteq G\times S^1$, $H \cong G$. This implies that SW_{k+1} is $(G\times S^1)$-homeomorphic to $(G\times S^1)/H$. By a change of groups argument and the inductive hypothesis, we see that $\Omega_*^{G,U}((SV_{k+1}, SV_k)\times X)$ is free and concentrated in odd degrees and that the maps induced respectively by suspension by an irreducible complex G-module and by addition of the bundle $\varepsilon_{\mathbb{C}}$ are split monomorphisms of MU_*-modules.

The diagram with exact columns (in which j is odd)

$$\begin{array}{ccc}
0 & & 0 \\
\downarrow & & \downarrow \\
\Omega_j^{U,G\times S^1}(SV_k\times X) & \xrightarrow{\sigma^W} & \Omega_{j+2}^{U,G\times S^1}((DW,SW)\times SV_k\times X) \\
\downarrow & & \downarrow \\
\Omega_j^{U,G\times S^1}(SV_{k+1}\times X) & \xrightarrow{\sigma^W} & \Omega_{j+2}^{U,G\times S^1}((DW,SW)\times SV_{k+1}\times X) \\
\downarrow & & \downarrow \\
\Omega_j^{U,G\times S^1}((SV_{k+1},SV_k)\times X) & \xrightarrow{\sigma^W} & \Omega_{j+2}^{U,G\times S^1}((DW,SW)\times (SV_{k+1},SV_k)\times X) \\
\downarrow & & \downarrow \\
0 & & 0
\end{array}$$

and the results above show by induction that, for all $k \geq 1$, $\Omega_*^{U,G\times S^1}(SV_k\times X)$ is free and concentrated in odd degrees and that σ^W is a split monomorphism. An analogous diagram shows the same is true for the map ψ induced by adding $\varepsilon_{\mathbb{C}}$.

To complete the proofs of (1) – (3) when $i = 1$, it suffices to observe that each step in the colimit contributes a direct summand to SV_∞. To prove (4), let $f : M \longrightarrow X\times SV_\infty$ represent an element of $\Omega_*^{U,G\times S^1}[\mathscr{F}_1](X)$. Since S^1 acts freely on M and all actions on a circle are linear, $p : M \longrightarrow M/S^1$ is the unit circle bundle of a 1-dimensional complex G-bundle E (the complex structure is given by multiplication by $i \in S^1$). Obviously, the circle bundle bounds a disc bundle, whose total space is a complex $(G\times S^1)$-manifold W. Any point $x \in W$ can be written as ty, where $t \in [0,1]$ and $y \in M$, so f extends to an equivariant

map $F : W \longrightarrow X \times DV_\infty$ defined as $F(ty) = tf(y)$, where the multiplication on the right hand side is given by the linear structure of DV_∞.

We prove the case $i \geq 1$ of Theorem 7.1 by induction on i. Observe first that the case $i = \infty$ will follow directly from the case of finite i since

$$E\mathscr{F}_\infty = \operatorname{colim}_i E\mathscr{F}_i.$$

Indeed, we shall see that each stage in the construction of $E\mathscr{F}_\infty$ as a colimit contributes a free direct summand to $\Omega_*^{U,G\times S^1}[\mathscr{F}_\infty](X)$ on which σ^W and ψ are split monomorphisms of MU_*-modules and the map to $\Omega_*^{U,G\times S^1}(X)$ is zero.

Applying Proposition 4.2 with (G, H) replaced by $(G \times S^1, \mathbb{Z}_{i+1})$ and noting that $(G \times S^1)/\mathbb{Z}_{i+1} \cong G \times S^1$ and that, under this isomorphism, the family $\mathscr{F}_{i+1}/\mathbb{Z}_{i+1}$ corresponds to the family $\mathscr{F}_1$, we find that

$$\Omega_n^{U,G\times S^1}[\mathscr{F}_{i+1}, \mathscr{F}_i](X) \cong \bigoplus_{\substack{0\leq 2k\leq n \\ P\in\mathscr{P}(2k, G\times S^1, \mathbb{Z}_{n+1})}} \Omega_{n-2k}^{U,G\times S^1}[\mathscr{F}_1](X^{\mathbb{Z}_{n+1}} \times BU(P, G\times S^1)).$$

Thus the case $i = 1$, combined with Remark 6.4, shows that the left-hand side is free and concentrated in *odd* degrees.

One then concludes, by using the long exact sequences of the pairs $[\mathscr{F}_{i+1}, \mathscr{F}_i]$, that for all i, $\Omega_*^{U,G\times S^1}[\mathscr{F}_i](X)$ is concentrated in *odd* degrees.

The diagrams with exact columns (in which j is odd)

$$\begin{array}{ccc}
0 & & 0 \\
\downarrow & & \downarrow \\
\Omega_j^{U,G}[\mathscr{F}_i](BU(n, G\times S^1)\times X) & \longrightarrow & \Omega_j^{U,G}[\mathscr{F}_i](BU(n+1, G\times S^1)\times X) \\
\downarrow & & \downarrow \\
\Omega_j^{U,G}[\mathscr{F}_{i+1}](BU(n, G\times S^1)\times X) & \longrightarrow & \Omega_j^{U,G}[\mathscr{F}_{i+1}](BU(n+1, G\times S^1)\times X) \\
\downarrow & & \downarrow \\
\Omega_j^{U,G}[\mathscr{F}_{i+1}, \mathscr{F}_i](BU(n, G\times S^1)\times X) & \rightarrowtail & \Omega_j^{U,G}[\mathscr{F}_{i+1}, \mathscr{F}_i](BU(n+1, G\times S^1)\times X) \\
\downarrow & & \downarrow \\
0 & & 0
\end{array}$$

show that, for all i, $\Omega_*^{U,G}[\mathscr{F}_i](X)$ is a free MU_*-module and the map induced by addition of $\varepsilon_{\mathbb{C}}$ is a split monomorphism of MU_*-modules.

The study of the suspension map σ^W must be broken into two cases. Since W is an irreducible representation of $G \times S^1$, its fixed point space W^{S^1} is either W or $\{0\}$ and therefore either

(1) $W^{\mathbb{Z}_{i+1}} = W$ or
(2) $W^{\mathbb{Z}_{i+1}} = \{0\}$.

In the first case, the map

$$(7.2)\sigma^W : \Omega^{U,G\times S^1}_{2j+1}[\mathscr{F}_{i+1},\mathscr{F}_i](X) \longrightarrow \Omega^{U,G\times S^1}_{2j+3}[\mathscr{F}_{i+1},\mathscr{F}_i]((DW,SW)\times X),$$

can be regarded via Proposition 4.2 as a direct sum of suspension maps

$$\Omega^{U,G\times S^1}_{2l+1}[\mathscr{F}_1](Y) \longrightarrow \Omega^{U,G\times S^1}_{2l+3}[\mathscr{F}_1]((DW,SW)\times Y),$$

where $Y = X^{\mathbb{Z}_{i+1}} \times BU(P, G\times S^1)$ for some partition P of $2(j-l)$ and we think of W as a representation of $G \times (S^1/\mathbb{Z}_{i+1}) \cong G \times S^1$. Thus it follows from the case $i = 1$ that (7.2) is a split monomorphism of MU_*-modules in this case.

For the second case consider a $(G\times S^1, \mathbb{Z}_{i+1})$-partition $P = (p_V)_{V\in\mathscr{C}_{G\times S^1,\mathbb{Z}_{i+1}}}$ of an even integer k. Let $P' = (p'_V)_{V\in\mathscr{C}_{G\times S^1}}$ denote the $(G\times S^1,\mathbb{Z}_{i+1})$-partition of $k+2$ defined by

$$p'_V = \begin{cases} p_V + 1 & \text{if } V = W \\ p_V & \text{otherwise.} \end{cases}$$

Since $W^{\mathbb{Z}_{i+1}} = \{0\}$, Proposition 4.2 implies that the map (7.2) can be interpreted as a direct sum of maps of the form

$$\psi : \Omega^{U,G\times S^1}_{2l+1}[\mathscr{F}_1](X^{\mathbb{Z}_{i+1}} \times BU(P,G)) \longrightarrow \Omega^{U,G\times S^1}_{2l+3}[\mathscr{F}_1](X^{\mathbb{Z}_{i+1}} \times BU(P',G))$$

induced by addition of $\varepsilon_{\mathbb{C}}$ to the multiplicity bundle corresponding to the V in the decomposition. We know already that maps of this kind are split monomorphisms of MU_*-modules, and we conclude that (7.2) is always a split monomorphism of MU_*-modules.

Now the following diagram with exact columns implies inductively that, for all i, σ^W is a split monomorphism of MU_*-modules on $\Omega^{U,G\times S^1}_*[\mathscr{F}_i](X)$.

$$\begin{array}{ccc}
0 & & 0 \\
\downarrow & & \downarrow \\
\Omega^{U,G\times S^1}_{2j+1}[\mathscr{F}_i](X) & \xrightarrow{\sigma^W} & \Omega^{U,G\times S^1}_{2j+3}[\mathscr{F}_i]((DW,SW)\times X) \\
\downarrow & & \downarrow \\
\Omega^{U,G\times S^1}_{2j+1}[\mathscr{F}_{i+1}](X) & \xrightarrow{\sigma^W} & \Omega^{U,G\times S^1}_{2j+3}[\mathscr{F}_{i+1}]((DW,SW)\times X) \\
\downarrow & & \downarrow \\
\Omega^{U,G\times S^1}_{2j+1}[\mathscr{F}_{i+1},\mathscr{F}_i](X) & \xrightarrow{\sigma^W} & \Omega^{U,G\times S^1}_{2j+3}[\mathscr{F}_{i+1},\mathscr{F}_i]((DW,SW)\times X) \\
\downarrow & & \downarrow \\
0 & & 0
\end{array}$$

Finally, to prove (4) of Theorem 7.1, let $f : M \longrightarrow X$ represent an element of $\Omega^{U,G\times S^1}_*[\mathscr{F}_i](X)$, $i > 1$, and suppose that we have already proved that

$$\Omega^{U,G\times S^1}_*[\mathscr{F}_j](X) \longrightarrow \Omega^{U,G\times S^1}_*(X)$$

is zero for all $j < i$. We shall construct a bordism with no isotropy restrictions from f to a map $f' : M' \longrightarrow X$ where M' is an $\mathscr{F}_{i-1}$-manifold. By the induction hypothesis, this will complete the proof.

Let us pause for a moment to explain informally how the bordism will be constructed. The idea is based on a standard technique in geometric topology known as "attaching handles". Any sphere S^k is the boundary of a disc D^{k+1}; if $S^k \subset N^n$ is embedded with trivial normal bundle in a manifold N and has a tubular neighborhood T, we can obtain a bordism of N to a new manifold by crossing N with the unit interval and pasting $D^{k+1} \times D^{n-k-1}$ (a *handle* with *core* D^k) to $N \times I$ by identifying $T \times \{1\}$ with $S^k \times D^{n-k-1}$. Our construction will be basically "attaching a generalized handle" to our manifold M. Instead of an embedded sphere, we shall use $M^{\mathbb{Z}_i}$, which bounds a manifold W; this will be the "core" of our "handle". The "handle" itself will be the total space of a disc bundle over W. The total space of its restriction to $M^{\mathbb{Z}_i}$ will be equivariantly diffeomorphic to a tubular neighborhood of $M^{\mathbb{Z}_i}$ in M, so we may take $M \times I$ and glue the "handle" in the obvious way, thus obtaining the desired bordism. Of course, all the required properties of the bordism have to be checked, and an extension of f to the bordism has to be constructed. We give the details next.

Consider a tubular neighborhood T of $M^{\mathbb{Z}_i}$, regarded as the total space of a disc bundle over $M^{\mathbb{Z}_i}$. We shall use the notation ST for the corresponding unit circle bundle, and T° for $T - ST$. We remark that $M - T^\circ$ and ST are $\mathscr{F}_{i-1}$-manifolds. When there is no danger of confusion, we shall make no notational distinction between a bundle and its total space.

Let λ denote a generator of $\mathbb{Z}_i \subset S^1 \subset \mathbb{C}$, and let V_k, $0 < k < i$, be 1-dimensional representations of $\mathbb{Z}_i$ such that λ acts by multiplication by λ^k. These form a complete, non-redundant set of nontrivial irreducible representations, and each of the V_k's obviously extends to $G \times S^1$ (an element $(g, s) \in G \times S^1$ acts by multiplication by s^k). We use these to obtain an isotypical decomposition of T. Let T_k denote the bundle $\mathrm{Hom}_{\mathbb{Z}_i}(\varepsilon_{V_k}, T)$.

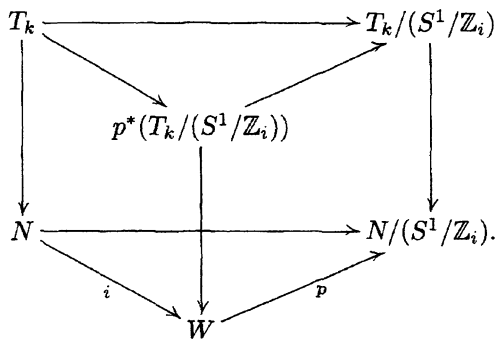
Since $M^{\mathbb{Z}_i}$ is $(S^1/\mathbb{Z}_i)$-free, our proof in the case $i = 1$ shows that $f|_{M^{\mathbb{Z}_i}}$ bounds a map $\tilde{f} : W \longrightarrow X$, where W is the total space of a $\mathbb{Z}_i$-trivial 1-dimensional $(G \times S^1)$-disc bundle over $Z = M^{\mathbb{Z}_i}/(S^1/\mathbb{Z}_i)$ whose unit circle bundle is $M^{\mathbb{Z}_i}$.

Passage to orbits gives a pull-back diagram

$$\begin{array}{ccc} T_k & \longrightarrow & T_k/(S^1/\mathbb{Z}_i) \\ \downarrow & & \downarrow \\ N & \longrightarrow & N/(S^1/\mathbb{Z}_i), \end{array}$$

for each k, where the right vertical arrow is a G-disc bundle, which may also be thought of as a $(G \times (S^1/\mathbb{Z}_i))$-bundle with trivial $(S^1/\mathbb{Z}_i)$-action. This makes the diagram above a pull-back of $(G \times (S^1/\mathbb{Z}_i))$-vector bundles. Since the zero-section of this bundle can be identified with $Z = SW/(S^1/\mathbb{Z}_i)$, we have a diagram

of $(G \times (S^1/\mathbb{Z}_i))$-bundles



Clearly the bundle $\hat{T} = \bigoplus_k p^*(T_k/(S^1/\mathbb{Z}_i)) \otimes \varepsilon_{V_k}$ extends T to W; we claim that its unit sphere bundle is an $\mathscr{F}_{i-1}$-manifold. To prove this, observe that

$$W - Z \cong M^{\mathbb{Z}_i} \times [0,1),$$

where $[0,1)$ has trivial action, and so $S\hat{T}|_{W-Z}$ is equivariantly homeomorphic to $S\hat{T}|_{W-Z} \times [0,1)$. Therefore, S^1-stabilizers of points in $S\hat{T} - ST$ not already present in ST can only appear in $S\hat{T}|_Z$, but since there is no component associated to the trivial representation (recall our remark in the course of the proof of Proposition 4.2) all these are *proper* subgroups of $\mathbb{Z}_i$, so the claim follows.

Let

$$M' \cong (M - T^\circ) \cup_{ST} S\hat{T};$$

by construction, this is an $\mathscr{F}_{i-1}$-manifold. Since $T \cup W$ is a $(G \times S^1)$-deformation retract of $\hat{T}$, there is a map $\hat{f} : W \longrightarrow X$ with $\hat{f}|_T = f|_T$ and $\hat{f}|_W = \tilde{f}$. We obtain a bordism by crossing M with the closed unit interval, pasting $\hat{T}$ to $M \times \{1\}$ along $T \times \{1\}$, and extending f in the obvious way to a map F from the bordism into X. The maps $f' = F|_{M'}$ and f represent the same element in the bordism of X with no isotropy restrictions, as required. □

8. Passing from G to $G \times S^1$ and $G \times \mathbb{Z}_k$

To complete the proofs of our theorems, it suffices to prove the following result, in which we again assume that we have proven Theorems 5.1 – 5.4 for G.

THEOREM 8.1. *Let $C = S^1$ or $C = \mathbb{Z}_k$. The following statements hold.*

(1) $\Omega_*^{U,G\times C}(X)$ *is a free MU_*-module concentrated in* even *degrees.*

(2) *The map*

$$\psi : \Omega_*^{U,G\times C}(BU(n, G \times S^1) \times X) \longrightarrow \Omega_*^{U,G\times C}(BU(n+1, G \times S^1) \times X)$$

is a split monomorphism of MU_*-modules.

(3) *If W is an irreducible complex $(G \times C)$-module, then*

$$\sigma^W : \Omega_*^{U,G\times C}(X) \longrightarrow \Omega_{*+2}^{U,G\times C}((DW, SW) \times X)$$

is a split monomorphism of MU_*-modules.

We first show that $\Omega_*^{U,G\times S^1}[\mathscr{A},\mathscr{F}_\infty](X)$ is a free MU_*-module concentrated in even degrees and that σ^W and ψ here are split monomorphisms of MU_*-modules. By Proposition 4.2, we have

$$\Omega_n^{U,G\times S^1}[\mathscr{A},\mathscr{F}_\infty](X) \cong \bigoplus_{\substack{0\le 2k\le n \\ P\in\mathscr{P}(2k,G\times S^1,S^1)}} \Omega_{n-2k}^{U,G}(X^{S^1}\times BU(P,G)).$$

Thus, by the induction hypothesis, $\Omega_n^{U,G\times S^1}[\mathscr{A},\mathscr{F}_\infty](X)$ is free over MU_* and concentrated in even degrees, and the maps ψ induced by addition of $\varepsilon_{\mathbb{C}}$ are split monomorphisms of MU_*-modules.

Theorem 7.1(4) implies that the long exact sequence of the pair $(\mathscr{A},\mathscr{F}_\infty)$ breaks into short exact sequences. In particular, the map

$$\Omega_*^{U,G\times S^1}(X) \longrightarrow \Omega_*^{U,G\times S^1}[\mathscr{A},\mathscr{F}_\infty](X)$$

is a monomorphism, hence $\Omega_*^{U,G\times S^1}(X)$ is concentrated in even degrees.

In order to study the effect of σ^W on $\Omega_n^{U,G\times S^1}[\mathscr{A},\mathscr{F}_\infty](X)$, it is necessary to distinguish two cases:

(1) $W^{S^1} = W$ and
(2) $W^{S^1} = \{0\}$.

The analysis is similar to the one carried out in the previous section and will be omitted; it yields the expected conclusion: σ^W is a split monomorphism of MU_*-modules on $\Omega_n^{U,G\times S^1}[\mathscr{A},\mathscr{F}_\infty](X)$.

The diagram with exact columns

$$\begin{array}{ccc}
0 & & 0 \\
\downarrow & & \downarrow \\
\Omega_{2j}^{U,G\times S^1}(X) & \xrightarrow{\sigma^W} & \Omega_{2j+2}^{U,G\times S^1}((DW,SW)\times X) \\
\downarrow & & \downarrow \\
\Omega_{2j}^{U,G\times S^1}[\mathscr{A},\mathscr{F}_\infty](X) & \xrightarrow{\sigma^W} & \Omega_{2j+2}^{U,G\times S^1}[\mathscr{A},\mathscr{F}_\infty]((DW,SW)\times X) \\
\downarrow & & \downarrow \\
\Omega_{2j-1}^{U,G\times S^1}[\mathscr{F}_\infty](X) & \xrightarrow{\sigma^W} & \Omega_{2j+1}^{U,G\times S^1}[\mathscr{F}_\infty]((DW,SW)\times X) \\
\downarrow & & \downarrow \\
0 & & 0
\end{array}$$

together with Lemmas 6.2 and 6.3 shows that $\Omega_*^{U,G\times S^1}(X)$ is projective, and therefore free, and σ^W is a split monomorphism of MU_*-modules on $\Omega_*^{U,G\times S^1}(X)$. A similar diagram gives the corresponding conclusion for ψ.

This completes the proof of Theorem 8.1 for $C = S^1$, and it remains to deal with the case $C = \mathbb{Z}_k$. Let V denote the 1-dimensional complex representation of

$G \times S^1$ on which G acts trivially and an element $e^{2\pi it} \in S^1$ acts by multiplication by $e^{2\pi itk}$. Since S^1 acts without fixed points on $SV \times X$,

$$\Omega_*^{U,G\times S^1}[\mathscr{A}, \mathscr{F}_\infty](SV \times X) = 0.$$

Therefore, by the long exact sequence of the pair (DV, SV),

$$\Omega_*^{U,G\times S^1}[\mathscr{A}, \mathscr{F}_\infty](X) \longrightarrow \Omega_*^{U,G\times S^1}[\mathscr{A}, \mathscr{F}_\infty]((DV, SV) \times X)$$

is an isomorphism, and, by the long exact sequence of the pair $(\mathscr{A}, \mathscr{F}_\infty)$,

$$\Omega_*^{U,G\times S^1}[\mathscr{F}_\infty](SV \times X) \longrightarrow \Omega_*^{U,G\times S^1}(SV \times X)$$

is an isomorphism.

By Theorem 7.1, we conclude that $\Omega_*^{U,G\times S^1}(SV \times X)$ is a free MU_*-module concentrated in odd degrees. This being so, the long exact sequence of the pair (DV, SV) breaks up into short exact sequences

$$0 \longrightarrow \Omega_{2j}^{U,G\times S^1}(X) \xrightarrow{\alpha} \Omega_{2j}^{U,G\times S^1}((DV, SV) \times X) \longrightarrow \Omega_{2j-1}^{U,G\times S^1}(SV \times X) \longrightarrow 0.$$

Since SV can be identified with $S^1/\mathbb{Z}_k$, we conclude from Lemma 3.4 that

$$\Omega_*^{U,G\times\mathbb{Z}_k}(X) \cong \operatorname{coker} \alpha.$$

Now apply the Snake Lemma to the diagram with exact columns

$$\begin{array}{ccc}
0 & & 0 \\
\downarrow & & \downarrow \\
\Omega_{2j}^{U,G\times S^1}(X) & \xrightarrow{\alpha} & \Omega_{2j}^{U,G\times S^1}((DV, SV) \times X) \\
\downarrow & & \downarrow \\
\Omega_{2j}^{U,G\times S^1}[\mathscr{A}, \mathscr{F}_\infty](X) & \xrightarrow{\cong} & \Omega_{2j}^{U,G\times S^1}[\mathscr{A}, \mathscr{F}_\infty]((DV, SV) \times X) \\
\downarrow & & \downarrow \\
\Omega_{2j-1}^{U,G\times S^1}[\mathscr{F}_\infty](X) & \xrightarrow{\beta} & \Omega_{2j-1}^{U,G\times S^1}[\mathscr{F}_\infty]((DV, SV) \times X) \\
\downarrow & & \downarrow \\
0 & & 0.
\end{array}$$

Since α is a monomorphism and β is an epimorphism, we see that $\operatorname{coker} \alpha \cong \ker \beta$. Since $\ker \beta$ is a free MU_*-module concentrated in odd degrees, $\Omega_*^{U,G\times\mathbb{Z}_k}(X)$ is free and concentrated in even degrees.

To show that σ^W is a split monomorphism, let $Y = (DW, SW) \times X$ and consider the maps

$$\alpha' : \Omega_{2j+2}^{U,G\times S^1}(Y) \longrightarrow \Omega_{2j+2}^{U,G\times S^1}((DV, SV) \times Y)$$

and

$$\beta' : \Omega_{2j+1}^{U,G\times S^1}[\mathscr{F}_\infty](Y) \longrightarrow \Omega_{2j+1}^{U,G\times S^1}[\mathscr{F}_\infty]((DV, SV) \times Y)$$

that fit into the diagram obtained from the previous one by raising all degrees by two and replacing X by Y. Then σ^W induces a map from the original diagram to the new diagram, and there results a commutative square

$$\begin{array}{ccc} \operatorname{coker} \alpha & \xrightarrow{\sigma^W} & \operatorname{coker} \alpha' \\ \cong \downarrow & & \downarrow \cong \\ \ker \beta & \xrightarrow[\sigma^W]{} & \ker \beta'. \end{array}$$

By Lemma 6.2, the bottom arrow is a split monomorphism of MU_*-modules, hence so is the top arrow. The proof that ψ is a split monomorphism is similar.

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