1171-05-243 Lei Xue\* (lxue@uw.edu). A Proof of Grünbaum's Lower Bound Conjecture for polytopes, lattices, and strongly regular normal pseudomanifolds. Preliminary report.

In 1967, Grünbaum conjectured that any d-dimensional polytope with  $d + s \leq 2d$  vertices has at least

$$\phi_k(d+s,d) = \binom{d+1}{k+1} + \binom{d}{k+1} - \binom{d+1-s}{k+1}$$

k-faces. In the talk, we will prove this conjecture and discuss equality cases. We will then extend our results to lattices with diamond property (the inequality part) and to strongly regular normal pseudomanifolds (the equality part). We will also talk about recent results on d-dimensional polytopes with 2d + 1 or 2d + 2 vertices. (Received August 17, 2021)