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Anomalous MLE of Constrained Covariance Matrix of Multivariate Normal

Distributions. Preliminary report.

The problem of estimating the mean and covariance matrix has been studied extensively. However, estimation of the mean vector and the covariance matrix in $N_p(\mu, \Sigma)$ under the joint constraints $\Sigma \mu = \mu$, $|\Sigma| = 1$ does not appear to have been studied before. We show that the maximum likelihood estimator (MLE) of Σ is anomalous (infeasible) in the sense that it satisfies $\widehat{\Sigma}^{-1} = 0$. We rectify the situation by modifying the unconstrained MLE and Bayesian estimator. In fact, starting with any initial mean and covariance matrix estimator, a method is proposed to modify the estimators to satisfy both constraints. The key step is to re-align the mean vector and the eigenvectors of the covariance matrix using the idea of regression or the Gram-Schmidt orthogonalization procedure. A simulation study shows that the modified estimator performs better than the initial estimator. (Received August 20, 2019)